INDUSTRIAL ARITHMETIC

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1 Arithmetic - Text-books, 1916.

2. Mathematics (Engineering).

S.D.
T.C.

Industrial Arithmetic

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WEBB PUBLISHING CO.
ST. PAUL, MINN.
1916

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PREFACE

This book is complete in itself for the upper grades and first year of high school. It gives a review of notation and numeration, common and decimal fractions, percentage, interest, denominate numbers, square and cube root, ratio and proportion and other subjects usually taught in the grades. It gives the fundamental principles of algebra, the use of equations, ratio, proportion and percentage as stepping stones to the future school work in algebra, physics, etc., and as working tools for manual and vocational training.

Evidence is abundant that there is a place in the school curriculum for an arithmetic leading up to the mathematics of manual training and the sciences and to the problems in vocational fields.

The pupil from the school of to-day finds himself sadly deficient in ability to apply his arithmetical knowledge to the mechanical operations in this vocational age. If he enters any of the trades or industries of to-day, he finds the same inability to apply or make practical use of his grade arithmetic.

This is a very opportune time for an arithmetic such as this book purports to be. We are all giving more attention than ever before to the mechanical application of the power of air, water, heat, light, and electricity.

The informational side of this book in these branches is extremely interesting and valuable. The mathematical side is certainly as useful and instructive. The table of contents shows the subjects treated.

THE AUTHORS.

July, 1916.

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INDUSTRIAL ARITHMETIC

DEFINITIONS AND NUMBERING

1. **A unit** is one thing; as, one man, one pupil, etc.

2. **A number** is a unit or a collection of units; as, 1, 5, 276, etc.

3. **Notation** is the art of writing numbers by figures or letters.

4. **Numeration** is the art of reading numbers expressed by figures or letters.

5. In the Arabic or decimal system of notation ten figures are used in writing numbers. These are 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

6. **Write in figures:**

1. Five hundred twenty thousand eighty-five.

2. Two hundred seventy-five thousand one hundred twenty-five.

3. Nine hundred one thousand seven hundred two.

4. Six hundred twenty-eight and twenty-five hundredths.

5. Seventy-four hundred and twenty-eight hundredths.

6. Two hundred twenty million seventy thousand, four hundred six.

7. Nine hundred eight million sixteen thousand four hundred seven.

8. Four hundred ninety-nine million seven hundred eighty-seven.

9. Seven hundred eighty million, four hundred seventy-three thousand.

10. Thirteen million four hundred fifty thousand, five hundred nine.

7. Write the following in words:

1. 8,111,100. *2.* 5,070,606. *3.* 466,000,405.
4. 7,890,124,002. *5.* 8,074,016,050. *6.* 902,705,612.
7. 6,010,100,700. *8.* 8,503,004,234.

8. In the Roman system of notation seven letters are used in writing numbers. These are: I, V, X, L, C, D, M. Their values are: 1, 5, 10, 50, 100, 500, 1,000.

The numbers from one to twenty are the following:

1. I	2. II	3. III	4. IV	5. V
6. VI	7. VII	8. VIII	9. IX	10. X
11. XI	12. XII	13. XIII	14. XIV	15. XV
16. XVI	17. XVII	18. XVIII	19. XIX	20. XX

The tens and hundreds are the following:

30. XXX	40. XL	50. L	60. LX
70. LXX	80. LXXX	90. XC	100. C
200. CC	300. CCC	400. CD	500. D
600. DC	700. DCC	800. DCCC	900. CM
1,000. M	2,000. MM	3,000. MMM	4,000. M$\overline{\text{V}}$

When one letter is followed by another of the same value, or by a letter of less value, their values are added. As, XX = 20; XV = 15.

When one letter is followed by another of greater value, its value is subtracted from the value of the greater. As, IX = 9; XL = 40.

When a letter is between two letters each of greater value, its value is subtracted from their sum. As, XIV = 14; XIX = 19; CVL = 145.

A dash over a letter multiplies its value by 1,000, $\overline{\text{V}}$ = 5,000; $\overline{\text{C}}$ = 100,000; $\overline{\text{M}}$ = 1,000,000; $\overline{\text{L}}$ = 50,000.

9. Write in Roman notation:

1.	87	11.	1269	21.	1916	31.	4326
2.	500	12.	2146	22.	1875	32.	3427
3.	177	13.	1870	23.	2472	33.	1089
4.	76	14.	2768	24.	3472	34.	1189
5.	1907	15.	1254	25.	2349	35.	1287
6.	276	16.	2456	26.	1210	36.	1376
7.	4567	17.	8532	27.	1311	37.	1456
8.	1917	18.	3459	28.	1412	38.	1525
9.	5673	19.	2378	29.	1513	39.	1600
10.	8902	20.	2436	30.	1618	40.	1700

ADDITION

10. Addition is the process of uniting two or more numbers into one number. The resulting number in addition is called the *sum*. *Like* numbers are numbers that express the same kind of units, as 4 miles and 7 miles, or 17 bushels and 9 bushels. *Unlike* numbers are numbers that express different kinds of units, as 7 pounds and 5 years, or 6 men and $14.

Only like numbers can be added.

Facility in addition depends upon a thorough mastery of the forty-five combinations given below. These should become so familiar that the sight of any combination should bring the sum to mind instantly.

11. Memorize so that you can give results as fast as you can talk:

1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	4	4	4	4	4	4
1	2	3	4	5	6	7	8	9	2	3	4	5	6	7	8	9	3	4	5	6	7	8	9	4	5	6	7	8	9

5	5	5	5	5	6	6	6	6	7	7	7	8	8	9
5	6	7	8	9	6	7	8	9	7	8	9	8	9	9

(The teacher should make "flash cards" with these combinations to develop speed on the part of pupils in giving results. See Figure 1.)

12. Add:

36	84	29	56	37	69	25	89
7	8	3	4	14	18	16	16

13. When we have a column of numbers to add, the quickest way is to look for combinations which will make 10 and so count up by 10's. By practice you will be able to make combinations of tens by taking numbers out of their regular order, always being careful not to omit any number entirely. The following column will serve to illustrate:

5612	65
6434 4372	60
3429 1638	50
4365 8291 2584	40
1266	30
9377 4297	20
8651 2263 5046	10

$$\begin{array}{r} 9 \\ 2 \\ 4 \\ 3 \\ 5 \\ 6 \\ \hline \end{array}$$

14. Practice until you can add the following in one minute or less:

(1)	(2)	(3)	(4)	(5)	(6)
4926	3429	5897	4116	9824	1106
3478	1406	3462	8429	3224	1359
5220	5320	9876	3414	946	8429
9651	9427	294	1029	8651	7295
4827	3800	4420	3042	397	8276
5916	8086	8046	9664	4829	5842
1324	5231	9274	537	1463	9463
5265	4097	3812	5863	1892	7006
4931	6403	5621	2974	1892	5731
2114	1265	9046	5114	297	9426
4632	4324	3092	9870	1069	4863
5976	9896	5563	2942	437	9204
1072	4142	2497	3792	8499	3046
5008	1658	124	4863	5455	1994
[illegible]	[illegible]	70929	61644	48515	83040

Check your work by adding each column in the opposite direction after you have completed the example.

Make up other exercises in addition and time yourself to see how quickly you can add them. Exchange your paper with a classmate and see whether he can add them more quickly than you could.

Addition is indicated briefly as follows: 4863 + 9264.

SUBTRACTION

15. Subtraction is the process of finding how much greater one number is than another number. The resulting number is called the *difference*, or *remainder*. Only like numbers can be subtracted. Facility in subtraction depends upon ability to recognize quickly what digit added to another digit will produce a given number; as, 3 and what are 5? 9 and what are 14?

Find the difference between 2,452 and 9,873.

9873	Solution:	2 and **1** are 3; 5 and **2** are 7;
2452		4 and **4** are 8; 2 and **7** are 9.
7421		

16. Subtract: 318 from 482.

482	Solution:	8 and **4** are 12 (one ten from 8 tens)
318		1 and **6** are 7 (one less)
164		3 and **1** are 4

17. Subtract:

(1)	(2)	(3)	(4)
4896427	5864924	94632706	1632748
2435214	3421718	52411508	541529

(5)	(6)	(7)	(8)
800437	7596320	40001	50000
610328	4384608	38004	4329

Test your results by adding the remainder to the smaller number given to see if the sum equals the larger number given.

Subtraction is indicated briefly by 984 − 367.

Make up other exercises and see how long it takes you to solve and test ten of them. Can you do them more quickly than any one else in your class?

MULTIPLICATION

18. Multiplication is a short process of adding a certain number of groups of equal numbers.

Thus: By addition—

146 bu.
146 bu.
146 bu.
———
438 bu.

By multiplication—

3 groups of 146 bu. = 438 bu.

The three is called the *multiplier* and tells how many groups there are. The 146 bu. is called the *multiplicand* and tells what is in each group. The 438 bu. is called the *product* and tells how much is in all the groups added together. The multiplicand and product are always *like* numbers.

19. Facility in multiplication depends upon a quick application of the multiplication tables. These tables should be thoroughly memorized.

2 × 1 = 2	3 × 1 = 3	4 × 1 = 4
2 × 2 = 4	3 × 2 = 6	4 × 2 = 8
2 × 3 = 6	3 × 3 = 9	4 × 3 = 12
2 × 4 = 8	3 × 4 = 12	4 × 4 = 16
2 × 5 = 10	3 × 5 = 15	4 × 5 = 20
2 × 6 = 12	3 × 6 = 18	4 × 6 = 24
2 × 7 = 14	3 × 7 = 21	4 × 7 = 28
2 × 8 = 16	3 × 8 = 24	4 × 8 = 32
2 × 9 = 18	3 × 9 = 27	4 × 9 = 36
2 × 10 = 20	3 × 10 = 30	4 × 10 = 40
2 × 11 = 22	3 × 11 = 33	4 × 11 = 44
2 × 12 = 24	3 × 12 = 36	4 × 12 = 48

5 × 1 = 5	6 × 1 = 6	7 × 1 = 7
5 × 2 = 10	6 × 2 = 12	7 × 2 = 14
5 × 3 = 15	6 × 3 = 18	7 × 3 = 21
5 × 4 = 20	6 × 4 = 24	7 × 4 = 28
5 × 5 = 25	6 × 5 = 30	7 × 5 = 35
5 × 6 = 30	6 × 6 = 36	7 × 6 = 42
5 × 7 = 35	6 × 7 = 42	7 × 7 = 49
5 × 8 = 40	6 × 8 = 48	7 × 8 = 56
5 × 9 = 45	6 × 9 = 54	7 × 9 = 63
5 × 10 = 50	6 × 10 = 60	7 × 10 = 70
5 × 11 = 55	6 × 11 = 66	7 × 11 = 77
5 × 12 = 60	6 × 12 = 72	7 × 12 = 84

8 × 1 = 8	9 × 1 = 9	10 × 1 = 10
8 × 2 = 16	9 × 2 = 18	10 × 2 = 20
8 × 3 = 24	9 × 3 = 27	10 × 3 = 30
8 × 4 = 32	9 × 4 = 36	10 × 4 = 40
8 × 5 = 40	9 × 5 = 45	10 × 5 = 50
8 × 6 = 48	9 × 6 = 54	10 × 6 = 60
8 × 7 = 56	9 × 7 = 63	10 × 7 = 70
8 × 8 = 64	9 × 8 = 72	10 × 8 = 80
8 × 9 = 72	9 × 9 = 81	10 × 9 = 90
8 × 10 = 80	9 × 10 = 90	10 × 10 = 100
8 × 11 = 88	9 × 11 = 99	10 × 11 = 110
8 × 12 = 96	9 × 12 = 108	10 × 12 = 120

11 × 1 = 11	12 × 1 = 12
11 × 2 = 22	12 × 2 = 24
11 × 3 = 33	12 × 3 = 36
11 × 4 = 44	12 × 4 = 48
11 × 5 = 55	12 × 5 = 60
11 × 6 = 66	12 × 6 = 72
11 × 7 = 77	12 × 7 = 84
11 × 8 = 88	12 × 8 = 96
11 × 9 = 99	12 × 9 = 108
11 × 10 = 110	12 × 10 = 120
11 × 11 = 121	12 × 11 = 132
11 × 12 = 132	12 × 12 = 144

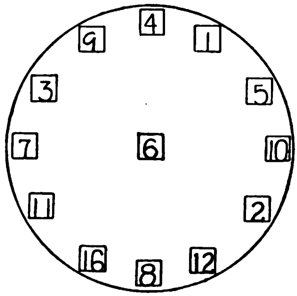

FIGURE 1. Clock face with interchangeable figures for practice.

Flash cards, clock face with number in the center, and other common devices should be used for drill until perfect accuracy and desired speed are obtained.

When these multiplication tables are applied to problems, care should be taken to name the multiplicand and product every time. To do this will insure a better understanding of the problem and make division easier to comprehend.

The sign for multiplication (×) may be read *times* or *of*. It is customary to say *times* when the multiplier is a whole number and *of* when the multiplier is a fraction. In every case the multiplier is a group or a part of a group, thus:

3 × 18 cattle means 3 groups with 18 cattle in each group. $\frac{2}{3}$ × 18 cattle means $\frac{2}{3}$ of a group of 18 cattle.

PROBLEMS

20. Work the following problems, writing out the equation for each, being careful to name both multiplicand and product in each one.

1. How much will 6 lbs. of butter cost at 35c. per pound? (It will require 6 × 35c. to pay for this amount of butter; hence—6 × 35c. = ______c.).

2. What is the cost of 640 acres of land at $50 per acre?

3. Find the value of 2,500 bales of cotton, each weighing 480 lbs. at 20c. per lb.

4. A man bought 68 head of cattle at $50 per head. Find the total cost.

5. If a man walks 4 miles per hour, how far can he go in 18 days, if he travels 8 hours each day?

6. 30 dozen eggs at 32c. per dozen. Find the cost.

7. How many peaches in 24 crates, if there are 84 in each crate?

8. There are 60 minutes in an hour, 24 hours in a day, 7 days in a week and 52 weeks in a year. How many minutes in a year?

9. Use the answer to example 8 and find how many minutes since you were born.

10. If you could save 25 cents each day, how many cents would you have at the end of a year of 365 days?

11. If a boy works two hours each school day for 4 years of 10 months each, how many hours would he work altogether?

12. Sound travels about 1,120 ft. per second. How far will it travel in 15 seconds?

13. There are 60 seconds in a minute and it takes light about 8 minutes to travel from the sun to the earth. If light travels 186,000 miles per second, how far is the sun from the earth? Then how far is it from the earth to the sun?

14. If it takes 160 railroad rails to a mile, how many rails are needed to make the track from your town to Chicago?

15. A pin-making machine makes 160 pins per minute. How many pins can a factory turn out in one day of 8 hours, if the factory has 10 such machines running?

16. A barrel of flour contains 196 lbs. of flour. What is the weight of flour produced per day by a mill that turns out 1,200 bbls.?

Make up other problems after ascertaining some facts from the stores or factories in your town.

21. Notice that 35 × 842 equals 842 × 35. Therefore, for convenience the smaller number is used as the multiplier in doing the mechanical work; but in writing the equation care should be used to keep the proper one for the multiplicand. Thus, in the problem to find the cost of 842 acres of land at $35 per acre, we have:

Mechanical work—

```
  842
   35
 ----
 4210
2526
-----
29470
```

Equation—

842 × $35 = $29,470.

22. The best method of checking multiplication is by "casting out nines." Drop from the sum a nine whenever a nine is reached in adding separately the digits of the multiplicand and multiplier. Cast out the nines from the product of these remainders. Note whether the result thus obtained agrees with the remainder after casting the nines out of the sum of the digits of the product of the example. If so, the work is correct.

```
   5644
    276
 ------
  33864
 39508
11288
-------
1557744
```

Adding the digits in the multiplicand and casting out the nines we have—

$5 + 6 + 4 + 4 = 19$—2 nines $= 1$

Doing the same in the multiplier we have—

$2 + 7 + 6 = 15$—1 nine $= 6$

$6 \times 1 = 6$

Adding the digits in the product and casting out the nines we have—

$1 + 5 + 5 + 7 + 7 + 4 + 4 = 33$—3 nines $= 6$.

$6 = 6$.

23. Muitiply and check:

(1). 94,364 × 563
(2). 5,821 × 321
(3). 8,483 × 1,042
(4). 1,017 × 1,032
(5). 1,009 × 9,428
(6). 8,932 × 274
(7). 86,405 × 105
(8). 320,070 × 68
(9). 648 × 546

DIVISION

24. Division may be one of two processes. In one case, *division is the process of finding how many groups of a certain number of units are found in another number of like units,* as, How many groups of 8 ft. are in 56 ft.? Or, *division is the process of finding how many units are in each group, if a number of units are placed in a given number of equal groups,* as, How many peaches will each boy have if 56 peaches are divided equally among 7 boys? In both kinds of division the *mechanical work* is the same, so that, from a purely mathematical standpoint, *division is the process of finding how many times one number is contained in another.*

The number to be divided is called the *dividend.*

The number to divide by is called the *divisor.*

The result of the division is called the *quotient.*

25. In division we have one of the two factors of a number given to find the other factor.

2 × *what number* = 12	*what number* × 7 = 28
4 × *what number* = 20	*what number* × 5 = 55
9 × *what number* = 63	*what number* × 6 = 48
2 × ___ = 2	___ × 1 = 2
2 × ___ = 4	___ × 2 = 4
3 × ___ = 6	___ × 3 = 6
2 × ___ = 8	___ × 4 = 8
2 × ___ = 10	___ × 5 = 10
2 × ___ = 12	___ × 6 = 12
2 × ___ = 14	___ × 7 = 14
2 × ___ = 16	___ × 8 = 16
2 × ___ = 18	___ × 9 = 18
2 × ___ = 20	___ × 10 = 20
2 × ___ = 22	___ × 11 = 22
2 × ___ = 24	___ × 12 = 24

Arrange the entire multiplication table in this manner leaving out one of the factors. Practice until the missing factor can be given instantly. Flash cards should be made for this drill also.

26. In dividing we commence at the left of the dividend so that any remainder may be reduced to units of the next smaller denomination.

Divide 684 by 2—

$$\text{Short division form:}\quad 2\overline{)684}^{\;342} \qquad 9\overline{)4564}^{\;507\text{———1 Rem.}}$$

27. Work the following exercises and test in each case by multiplying the result obtained by the divisor and adding in the remainder, if any. This will equal the dividend, if correct.

$8\overline{)462514}$ $6\overline{)1040653}$ $4\overline{)900072}$ $9\overline{)1134658}$ $12\overline{)50436}$

$5\overline{)10150060}$ $4\overline{)10006}$ $7\overline{)409007}$ $11\overline{)1001032}$

LONG DIVISION

```
    635
73)46355
   438
   ---
    255
    219
    ---
     365
     365
     ---
```

28. Use the same form as in short division, being careful to place the figure first in the quotient directly over the last figure of the first partial dividend. Place each succeeding figure in the quotient directly over the next figure in the dividend. Neatness and accuracy in the form of work will be a great help in all long division work.

29. The shortest method of checking division is to cast out nines from the quotient and divisor. Find the product of the remainders, add the division remainder and compare this result with the remainder after casting out the nines from the dividend.

```
     254
17)4329
    34
    —
    92
    85
    —
     79
     68
     —
     11
```

Adding the digits in the quotient and casting out the nines we have: $2 + 5 + 4 = 11 -$ 1 nine $= 2$. Adding the digits in the divisor and casting out the nines we have: $1 + 7 = 8 - 0$ nine $= 8$. $8 \times 2 = 16 + 11 = 27$.

Casting out 3 nines $= 0$.

Adding the digits in the dividend and casting out the nines we have: $4 + 3 + 2 + 9 = 18$.

Casting out 2 nines $= 0$

$0 = 0$.

30. Divide and test by casting out nines:

(1) 6,429,874 by 362	(6) 1,437,580 by 306
(2) 5,980,043 by 946	(7) 4,434,288 by 637
(3) 8,632,057 by 429	(8) 4,556,632 by 580
(4) 1,404,267 by 234	(9) 7,000,000 by 398
(5) 4,000,062 by 392	(10) 1,000,406 by 700

FACTORS

31. The *factors* of a number are the integers whose product is the number.

32. An *integer* is a whole number.

33. A factor of a number is an *exact divisor* of the number.

34. Give the exact divisors of 18, 32, 14, 45, 60, 48, 66, 84, 63, 99.

35. Many numbers have the same factor repeated as in the following:

$$2 \times 2 = 4$$
$$3 \times 3 = 9$$
$$5 \times 5 = 25$$
$$7 \times 7 = 49$$
$$2 \times 2 \times 2 = 8$$
$$3 \times 3 \times 3 = 27$$
$$5 \times 5 \times 5 = 125$$

36. Instead of writing the same factor over and over, we indicate the number of times it is used by a small figure called an *exponent* written to the right and a little above the number, thus—

2^2 means 2×2.

3^4 means $3 \times 3 \times 3 \times 3$.

37. The same factor taken twice is called the *square* of the number.

38. The same factor taken three times is called the *cube* of the number.

39. Memorize the squares of all numbers from 1 to 20.

40. Memorize the cubes of all numbers from 1 to 10.

41. A *prime number* is one that has no exact divisor except itself and one.

42. A *prime factor* is a prime number used as a factor.

43. A *composite number* is one that has other exact divisors than itself and one.

44. (1) Give all the composite numbers to 60.

(2) Give the prime factors of all numbers to 20.

45. Test the truth of the following statements:

(a) A number is evenly divisible by 2, if the one's figure is 2, 4, 6 8, or 0.

(b) A number is evenly divisible by 3, if the sum of the digits is evenly divisible by 3.

(c) A number is evenly divisible by 5, if its one's figure is 5 or 0.

(d) A number is evenly divisible by 9, if the sum of its digits is evenly divisible by 9.

46. Select from the following all the numbers evenly divisible by 2; by 3; by 5; by 9.

648	345	414	48,762
76	123	642	59,007
580	197	334	86,420
792	556	175	55,477

47. Factoring is the process of separating a number into its factors. We usually separate a number for convenience into its prime factors.

$$\begin{array}{r} 3\underline{)315} \\ 3\underline{)105} \\ 5\underline{)\ 35} \\ 7 \end{array}$$

Form of written work:

Test: $3 \times 3 \times 5 \times 7 = 315$

This may be written $3^2 \times 5 \times 7 = 315$.

48. Resolve the following into prime factors and test:

(1) 184	(5) 175	(9) 864
(2) 225	(6) 324	(10) 5,600
(3) 108	(7) 800	(11) 180
(4) 96	(8) 144	(12) 1,000

GREATEST COMMON DIVISOR

49. A *common divisor* of two or more numbers is a number that evenly divides each of them.

Name a common divisor of 16 and 24; of 15 and 40; of 27, 36 and 60.

50. The *greatest common divisor* of two or more numbers is the greatest number that will evenly divide each of them. It is the product of all their common prime factors.

51. Find the greatest common divisor of 36, 63 and 54.

$$\begin{array}{r|rrr} 3) & 36 & 63 & 54 \\ \hline 3) & 12 & 21 & 18 \\ \hline & 4 & 7 & 6 \end{array}$$

G. C. D. = 3×3 or 9.

52. Find the G. C. D. of:

(1) 14, 49	(8) 8, 12, 42	(15) 64, 128, 256
(2) 17, 51	(9) 9, 33, 48	(16) 63, 126, 999
(3) 20, 185	(10) 14, 21, 63	(17) 150, 125, 375
(4) 44, 121	(11) 36, 54, 90	(18) 81, 729, 2187
(5) 24, 246	(12) 26, 65, 91	(19) 42, 48, 36, 84
(6) 19, 133	(13) 63, 84, 126	(20) 64, 160, 640, 320
(7) 16, 72	(14) 32, 48, 128	(21) 32, 90, 360, 160

LEAST COMMON MULTIPLE

53. A *common multiple* of two or more numbers is a number that is evenly divisible by each of them. Thus 36 is a common multiple of 4 and 6.

54. The *least common multiple* of two or more numbers is the least number that is evenly divisible by each of them. Thus 12 is the l. c. m. of 4 and 6.

55. Rule: *To find the l. c. m. of two or more numbers, resolve each of the numbers into its prime factors. Take all the factors of the largest number and such factors of the other numbers as are not found in the largest number. The product of these factors is the l. c. m.*

56. 1. Find the l. c. m. of 8. 28, 36 and 52.

$$\begin{array}{rl} 8 &= 2 \times 2 \times 2 \\ 28 &= 2 \times 2 \times 7 \\ 36 &= 2 \times 2 \times 3 \times 3 \\ 52 &= 2 \times 2 \times 13 \\ \hline \end{array}$$
$$2 \times 2 \times 13 \times 3 \times 3 \times 7 \times 2 = 6{,}552 \text{ l. c. m.}$$

Note: A number is a multiple of another number, if its prime factors include all the prime factors of the other number.

How many numbers can you find of which 6,552 is a multiple?

2. Find the l. c. m. of:

(1)	45, 63, 72, 99	(6)	20, 28, 56, 80
(2)	6, 18, 30, 42	(7)	14, 35, 42, 28
(3)	16, 24, 64, 96	(8)	120, 225, 270
(4)	98, 42, 126	(9)	39, 65, 195
(5)	180, 432, 160	(10)	38, 95, 100

FRACTIONS

57. A *unit* is a single quantity by which another quantity of the same kind is measured, as 1 inch is the unit of 7 inches; 1 barrel is the unit of 18 barrels; 1 acre is the unit of 40 acres, etc.

58. These integral units are often divided into equal parts known as *fractional units*, as, $\frac{1}{2}$ ft.; $\frac{1}{3}$ inch; $\frac{1}{8}$ bbl., etc.

59. A *fraction* is one or more fractional units; as, $\frac{1}{7}$ yd.; $\frac{3}{4}$ mile; \$$\frac{5}{6}$; $\frac{7}{8}$ sq. ft., etc.

The number above the line in the expression of a fraction is called the numerator; the number below the line is called the denominator.

60. The *denominator* indicates the number of parts into which the integral unit is divided.

61. The *numerator* indicates the number of these parts taken.

62. Arrange the following fractional units in the order of their size, placing the largest first:

$$\frac{1}{16},\ \frac{1}{8},\ \frac{1}{24},\ \frac{1}{2},\ \frac{1}{5},\ \frac{1}{10},\ \frac{1}{20},\ \frac{1}{60},\ \frac{1}{4}.$$

63. A *proper fraction* is a fraction whose value is less than 1; as $\frac{1}{5}$, $\frac{7}{8}$, $\frac{9}{10}$, $\frac{1}{4}$.

64. An *improper fraction* is a fraction equal to or greater than 1; as $\frac{4}{4}$, $\frac{9}{8}$, $\frac{5}{5}$, $\frac{7}{3}$.

65. A *mixed number* is a number expressed by a whole number and a fraction; as, $3\frac{1}{2}$, $16\frac{3}{8}$, $4\frac{5}{9}$.

REDUCTION OF FRACTIONS

66. Reduction is the process of changing the numerator and denominator of a fraction without changing the value of the fraction.

67. Principle: Multiplying or dividing both numerator and denominator of a fraction by the same number does not change its value.

68. Change:—

1.	$\frac{1}{2}$ to 8ths	5.	$\frac{2}{3}$ to 6ths	9.	$\frac{4}{5}$ to 20ths
2.	$\frac{1}{5}$ to 15ths	6.	$\frac{3}{8}$ to 24ths	10.	$\frac{5}{16}$ to 48ths
3.	$\frac{1}{6}$ to 24ths	7.	$\frac{5}{9}$ to 36ths	11.	$\frac{3}{8}$ to 32nds
4.	$\frac{1}{4}$ to 8ths	8	$\frac{11}{12}$ to 36ths	12.	$\frac{3}{2}$ to 12ths

69. Reduce:—

1. $\frac{9}{12}$ to 4ths	6. $\frac{60}{80}$ to 20ths
2. $\frac{30}{36}$ to 6ths	7. $\frac{18}{24}$ to 8ths
3. $\frac{24}{60}$ to 10ths	8. $\frac{96}{144}$ to 12ths
4. $\frac{4}{28}$ to 7ths	9. $\frac{50}{100}$ to 10ths
5. $\frac{15}{45}$ to 3rds	10. $\frac{30}{110}$ to 11ths

70. Lowest Terms. A fraction is reduced to its lowest terms when there is no common divisor of its numerator and denominator.

71. Rule: *To reduce a fraction to its lowest terms, divide both numerator and denominator by their greatest common divisor.*

72. Reduce to lowest terms:

1. $\frac{8}{16}$	9. $\frac{125}{75}$	17. $\frac{25}{800}$	25. $\frac{724}{836}$
2. $\frac{25}{30}$	10. $\frac{39}{65}$	18. $\frac{756}{924}$	26. $\frac{64}{1728}$
3. $\frac{16}{24}$	11. $\frac{144}{288}$	19. $\frac{312}{440}$	27. $\frac{150}{750}$
4. $\frac{40}{48}$	12. $\frac{150}{475}$	20. $\frac{729}{846}$	28. $\frac{360}{540}$
5. $\frac{36}{60}$	13. $\frac{64}{256}$	21. $\frac{342}{819}$	29. $\frac{365}{576}$
6. $\frac{42}{49}$	14. $\frac{130}{390}$	22. $\frac{837}{945}$	30. $\frac{2754}{3483}$
7. $\frac{72}{96}$	15. $\frac{400}{1000}$	23. $\frac{420}{600}$	31. $\frac{480}{640}$
8. $\frac{45}{80}$	16. $\frac{664}{812}$	24. $\frac{525}{2000}$	32. $\frac{1000}{2500}$

73. Equivalents. It is sometime desirable to change mixed numbers to improper fractions, or, conversely, to change improper fractions to whole or mixed numbers.

74. Rule: *To change a mixed number to an improper fraction, multiply the whole number by the denominator of the fraction, add the numerator and place the sum over the denominator.*

75. Change to improper fractions mentally:

1. $1\frac{1}{2}$	5. $4\frac{1}{8}$	9. $2\frac{3}{16}$	13. $5\frac{5}{9}$
2. $3\frac{2}{3}$	6. $6\frac{1}{4}$	10. $4\frac{5}{8}$	14. $7\frac{3}{10}$
3. $5\frac{3}{4}$	7. $8\frac{3}{5}$	11. $10\frac{1}{6}$	15. $11\frac{1}{3}$
4. $1\frac{7}{8}$	8. $9\frac{3}{4}$	12. $2\frac{5}{7}$	16. $20\frac{4}{5}$

76. Rule: *To change an improper fraction to a whole or mixed number, divide the numerator by the denominator.*

77. Change to whole or mixed numbers mentally:

1.	$\frac{5}{2}$	9.	$\frac{24}{7}$	17.	$\frac{22}{10}$	25.	$\frac{14}{6}$
2.	$\frac{8}{5}$	10.	$\frac{40}{6}$	18.	$\frac{35}{5}$	26.	$\frac{76}{8}$
3.	$\frac{12}{7}$	11.	$\frac{74}{12}$	19.	$\frac{160}{20}$	27.	$\frac{52}{5}$
4.	$\frac{16}{3}$	12.	$\frac{50}{20}$	20.	$\frac{49}{20}$	28.	$\frac{96}{11}$
5.	$\frac{20}{8}$	13.	$\frac{20}{16}$	21.	$\frac{120}{80}$	29.	$\frac{64}{7}$
6.	$\frac{32}{6}$	14.	$\frac{42}{30}$	22.	$\frac{144}{70}$	30.	$\frac{18}{5}$
7.	$\frac{48}{8}$	15.	$\frac{70}{30}$	23.	$\frac{360}{80}$	31.	$\frac{420}{14}$
8.	$\frac{12}{5}$	16.	$\frac{100}{80}$	24.	$\frac{45}{11}$	32.	$\frac{17}{3}$

ADDITION AND SUBTRACTION OF FRACTIONS

78. *Similar fractions* are fractions that have a common denominator. Only similar fractions can be added.

79. *A common denominator* of two or more fractions is a number that contains all the denominators of the fractions a whole number of times. The least common denominator is the least common multiple of the denominators. Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ to similar fractions having the least common denominator: The least common multiple of the denominators 2, 3, and 4 is 12. Reduce each fraction to 12ths.

$$\frac{1 \times 6}{2 \times 6} = \frac{6}{12}$$

$$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

$$\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

80. Reduce the following to fractions having the least common denominator, factoring the denominators as in finding the least common multiple whenever you cannot get the least common denominator by inspection:

1.	$\frac{3}{8}, \frac{7}{10}$	7.	$\frac{9}{25}, \frac{7}{10}, \frac{4}{15}$
2.	$\frac{5}{6}, \frac{4}{9}$	8.	$\frac{7}{16}, \frac{5}{12}, \frac{5}{8}, \frac{3}{4}$
3.	$\frac{3}{4}, \frac{2}{3}, \frac{5}{8}$	9.	$\frac{7}{18}, \frac{8}{9}, \frac{3}{14}, \frac{5}{7}$
4.	$\frac{4}{5}, \frac{1}{2}, \frac{7}{10}$	10.	$\frac{19}{24}, \frac{7}{8}, \frac{8}{15}$
5.	$\frac{8}{9}, \frac{2}{3}, \frac{5}{6}$	11.	$\frac{24}{25}, \frac{11}{15}, \frac{9}{10}, \frac{5}{6}$
6.	$\frac{7}{8}, \frac{3}{4}, \frac{5}{12}$	12.	$\frac{11}{27}, \frac{31}{54}, \frac{7}{18}$

13. $\frac{9}{10}, \frac{3}{5}, \frac{2}{3}$
14. $\frac{5}{7}, \frac{3}{14}, \frac{11}{28}$
15. $\frac{2}{3}, \frac{4}{5}, \frac{7}{15}$
16. $\frac{3}{5}, \frac{1}{3}, \frac{7}{30}$
17. $\frac{5}{16}, \frac{7}{8}, \frac{3}{4}, \frac{1}{2}$
18. $\frac{3}{7}, \frac{4}{9}$
19. $\frac{4}{33}, \frac{5}{18}, \frac{6}{11}$
20. $\frac{3}{14}, \frac{9}{16}, \frac{7}{8}, \frac{3}{7}$
21. $\frac{2}{9}, \frac{5}{21}, \frac{11}{14}$
22. $\frac{14}{15}, \frac{9}{25}, \frac{17}{30}$
23. $\frac{7}{84}, \frac{5}{36}, \frac{27}{64}$
24. $\frac{47}{100}, \frac{83}{90}, \frac{41}{45}$

81. *To add fractions* reduce the fractions to similar fractions having a common denominator and add the numerators.

82. *To subtract fractions* reduce the fractions to similar fractions having a common denominator and subtract the numerators.

83. Add:

1. $\frac{1}{2}, \frac{3}{4}$
2. $\frac{1}{2}, \frac{1}{8}$
3. $\frac{1}{2}, \frac{3}{8}$
4. $\frac{1}{4}, \frac{7}{8}$
5. $\frac{3}{4}, \frac{5}{8}$
6. $\frac{1}{4}, \frac{1}{16}$
7. $\frac{3}{4}, \frac{5}{16}$
8. $\frac{3}{4}, \frac{11}{32}$
9. $\frac{5}{6}, \frac{4}{9}$
10. $\frac{8}{9}, \frac{5}{12}$
11. $\frac{7}{9}, \frac{5}{6}$
12. $\frac{1}{2}, \frac{2}{3}, \frac{1}{4}$
13. $\frac{1}{6}, \frac{1}{8}, \frac{3}{4}$
14. $\frac{1}{2}, \frac{3}{4}, \frac{3}{8}$
15. $\frac{9}{10}, \frac{4}{15}, \frac{3}{5}$
16. $\frac{7}{8}, \frac{3}{4}, \frac{7}{12}$
17. $\frac{8}{9}, \frac{5}{18}, \frac{2}{3}$
18. $\frac{3}{4}, \frac{11}{36}$
19. $\frac{5}{18}, \frac{3}{14}, \frac{4}{7}$
20. $\frac{6}{11}, \frac{5}{22}, \frac{1}{2}$
21. $\frac{5}{27}, \frac{8}{9}, \frac{17}{54}$
22. $\frac{5}{64}, \frac{7}{16}, \frac{3}{8}, \frac{3}{4}$
23. $\frac{7}{8}, \frac{13}{24}, \frac{11}{16}, \frac{5}{32}$
24. $\frac{23}{42}, \frac{19}{56}, \frac{4}{35}$
25. $\frac{15}{16}, \frac{17}{24}, \frac{11}{14}$
26. $\frac{41}{48}, \frac{35}{36}, \frac{11}{12}$
27. $\frac{17}{18}, \frac{9}{16}, \frac{5}{27}$
28. $\frac{1}{36}, \frac{5}{12}, \frac{7}{18}, \frac{4}{15}$
29. $\frac{5}{9}, \frac{2}{11}, \frac{15}{33}, \frac{7}{66}$
30. $\frac{1}{144}, \frac{5}{16}, \frac{7}{12}, \frac{17}{18}$
31. $2\frac{7}{8}, 5\frac{1}{2}$
32. $17\frac{3}{4}, 9\frac{2}{3}$
33. $16\frac{5}{8}, 8\frac{3}{4}$
34. $10\frac{2}{3}, 14\frac{3}{16}$
35. $21\frac{4}{5}, 18\frac{9}{10}$
36. $42\frac{5}{8}, 3\frac{11}{16}$
37. $19\frac{1}{2}, 7\frac{3}{14}$
38. $48\frac{1}{16}, 9\frac{7}{12}$
39. $7\frac{5}{11}, 18\frac{19}{22}$
40. $4\frac{15}{16}, 11\frac{8}{9}$

84. Subtract as indicated:

1.	$\frac{3}{4} - \frac{5}{16}$	9.	$\frac{5}{7} - \frac{3}{5}$	17.	$8\frac{1}{4} - \frac{5}{8}$
2.	$\frac{7}{8} - \frac{1}{2}$	10.	$\frac{9}{10} - \frac{4}{15}$	18.	$9\frac{1}{2} - 3\frac{5}{6}$
3.	$\frac{1}{2} - \frac{3}{32}$	11.	$\frac{7}{9} - \frac{1}{6}$	19.	$18\frac{5}{6} - 14\frac{7}{8}$
4.	$\frac{1}{4} - \frac{5}{32}$	12.	$\frac{3}{4} - \frac{5}{18}$	20.	$12\frac{5}{32} - 7\frac{3}{4}$
5.	$\frac{7}{8} - \frac{35}{64}$	13.	$3\frac{1}{2} - 1\frac{1}{4}$	21.	$19\frac{4}{5} - 6\frac{7}{8}$
6.	$\frac{9}{16} - \frac{3}{14}$	14.	$5\frac{7}{8} - 2\frac{1}{2}$	22.	$27\frac{5}{32} - 6\frac{1}{2}$
7.	$\frac{5}{8} - \frac{7}{72}$	15.	$6\frac{5}{6} - 4\frac{2}{3}$	23.	$19\frac{7}{8} - 4\frac{15}{16}$
8.	$\frac{11}{12} - \frac{5}{16}$	16.	$7\frac{5}{8} - \frac{5}{16}$	24.	$17\frac{7}{24} - 1\frac{9}{16}$

MULTIPLICATION OF FRACTIONS

85. *To multiply a fraction by a whole number,* multiply the numerator or divide the denominator of the fraction by the whole number.

86. Give results at sight:

1.	$6 \times \frac{5}{12}$	4.	$9 \times \frac{5}{18}$	7.	$8 \times \frac{5}{24}$	10.	$11 \times \frac{5}{33}$
2.	$4 \times \frac{3}{8}$	5.	$12 \times \frac{13}{48}$	8.	$7 \times \frac{15}{28}$	11.	$16 \times \frac{5}{32}$
3.	$5 \times \frac{2}{7}$	6.	$15 \times \frac{7}{30}$	9.	$30 \times \frac{18}{60}$	12.	$14 \times \frac{2}{11}$

In more difficult problems cancellation is often used, thus:

$$\overset{4}{\cancel{12}} \times \frac{4}{\underset{5}{\cancel{15}}} = \frac{16}{5} \text{ or } 3\tfrac{1}{5}$$

We divide by some common factor *before* multiplying, as this shortens the work.

87. Solve, using cancellation whenever possible:

1.	$24 \times \frac{3}{4}$	7.	$48 \times \frac{5}{16}$	13.	$94 \times \frac{32}{47}$
2.	$68 \times \frac{5}{8}$	8.	$32 \times \frac{7}{8}$	14.	$256 \times \frac{3}{64}$
3.	$34 \times \frac{9}{17}$	9.	$8 \times \frac{5}{8}$	15.	$800 \times \frac{9}{40}$
4.	$16 \times \frac{3}{5}$	10.	$21 \times \frac{10}{21}$	16.	$729 \times \frac{5}{81}$
5.	$150 \times \frac{4}{25}$	11.	$9 \times \frac{5}{180}$	17.	$42 \times \frac{84}{63}$
6.	$360 \times \frac{20}{90}$	12.	$144 \times \frac{7}{96}$	18.	$320 \times \frac{5}{16}$

88. *To multiply a whole number by a fraction,* multiply the whole number by the numerator of the fraction and write the product over the denominator. Cancel when possible.

89. Multiply:

1.	$\frac{2}{5}$ of 35	9.	$\frac{3}{7}$ of 21	17.	$\frac{5}{11}$ of 88
2.	$\frac{3}{8}$ of 64	10.	$\frac{5}{11}$ of 33	18.	$\frac{7}{9}$ of 63
3.	$\frac{5}{8}$ of 68	11.	$\frac{3}{16}$ of 48	19.	$\frac{3}{16}$ of 14
4.	$\frac{5}{6}$ of 24	12.	$\frac{7}{64}$ of 128	20.	$\frac{4}{9}$ of 24
5.	$\frac{3}{4}$ of 15	13.	$\frac{4}{9}$ of 36	21.	$\frac{7}{8}$ of 12
6.	$\frac{2}{7}$ of 14	14.	$\frac{5}{12}$ of 16	22.	$\frac{5}{21}$ of 14
7.	$\frac{5}{8}$ of 20	15.	$\frac{3}{5}$ of 75	23.	$\frac{3}{5}$ of 25
8.	$\frac{9}{10}$ of 18	16.	$\frac{7}{16}$ of 24	24.	$\frac{5}{64}$ of 32

90. *To multiply a fraction by a fraction,* multiply the numerators together and the denominators together. Cancel when possible.

91. Find the product of:

1.	$\frac{2}{3}$ of $\frac{15}{16}$	5.	$\frac{3}{5}$ of $\frac{5}{16}$	9.	$\frac{9}{10}$ of $\frac{16}{25}$
2.	$\frac{7}{8}$ of $\frac{32}{35}$	6.	$\frac{11}{12}$ of $\frac{2}{33}$	10.	$\frac{8}{9}$ of $\frac{45}{64}$
3.	$\frac{4}{5}$ of $\frac{25}{28}$	7.	$\frac{14}{23}$ of $\frac{46}{7}$	11.	$\frac{4}{5}$ of $\frac{9}{16}$
4.	$\frac{6}{7}$ of $\frac{42}{56}$	8.	$\frac{7}{16}$ of $\frac{5}{7}$	12.	$\frac{3}{8}$ of $\frac{14}{27}$

13. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{7}{8}$ ft. = ________ft.

14. $\frac{5}{16}$ of $\frac{9}{10}$ of $\frac{8}{9}$ yd. = ________yd.

15. 3 of $\frac{3}{8}$ of $\frac{5}{6}$ acre = ________acre.

16. $\frac{2}{3}$ of 5 of $\frac{3}{8}$ mile = ________mile.

17. $\frac{5}{6}$ of $\frac{3}{4}$ of $\frac{6}{7}$ bu. = ________bu.

18. 4 of $\frac{3}{8}$ of 7 lbs. = ________lbs.

19. $\frac{3}{5}$ of $\frac{5}{9}$ of 10 gal. = ________gal.

20. 10 of $\frac{1}{8}$ of 16 sq. in. = ________sq. in.

21. If you take $\frac{3}{4}$ of $\frac{5}{8}$ of a foot, what part of a foot have you?

22. If a man earns $\$1\frac{3}{4}$ per day, how much will he earn in 12 days? 21

23. What will $\frac{3}{8}$ of a yard of lace cost at $\$\frac{5}{8}$ per yard?

24. Find the cost of $18\frac{3}{4}$ bu. potatoes at $\$\frac{3}{4}$ per bu.

25. I wish to draw a line $\frac{2}{3}$ as long as a line $\frac{15}{16}$ of an inch long. How long shall I draw it?

92. *To multiply a mixed number by a mixed number,* it is generally more convenient to reduce each to an improper fraction and multiply, cancelling when possible.

93. Multiply:

1.	$3\frac{1}{2} \times 4\frac{1}{8}$	5.	$8\frac{3}{8} \times 4\frac{5}{6}$	9.	$7\frac{3}{4} \times 8\frac{4}{7}$
2.	$4\frac{1}{3} \times 9\frac{1}{5}$	6.	$7\frac{2}{3} \times 9\frac{3}{4}$	10.	$3\frac{5}{16} \times 2\frac{1}{2}$
3.	$6\frac{7}{8} \times 5\frac{1}{4}$	7.	$15\frac{3}{4} \times 10\frac{2}{3}$	11.	$8\frac{4}{5} \times 2\frac{1}{2}$
4.	$24\frac{1}{2} \times 32\frac{2}{3}$	8.	$6\frac{2}{5} \times 5\frac{1}{8}$	12.	$7\frac{4}{5} \times 8\frac{4}{5}$

DIVISION OF FRACTIONS

94. *To divide a fraction by a whole number,* divide the numerator or multiply the denominator by the whole number.

95. Divide:

1.	$\frac{25}{28}$ by 5	5.	$\frac{12}{19}$ by 5	9.	$\frac{18}{35}$ by 3
2.	$\frac{32}{63}$ by 16	6.	$\frac{11}{16}$ by 2	10.	$\frac{17}{33}$ by 11
3.	$\frac{4}{5}$ by 3	7.	$\frac{36}{43}$ by 9	11.	$\frac{14}{23}$ by 7
4.	$\frac{9}{10}$ by 3	8.	$\frac{17}{20}$ by 10	12.	$\frac{111}{288}$ by 11

96. *To divide any quantity by a fraction,* invert the fraction and multiply.

97. Divide:

1.	8 by $\frac{2}{3}$	5.	$16\frac{3}{4}$ by $\frac{4}{5}$	9.	$7\frac{1}{2}$ by $\frac{5}{16}$
2.	$6\frac{1}{2}$ by $\frac{3}{4}$	6.	$18\frac{4}{5}$ by $\frac{9}{5}$	10.	$11\frac{2}{5}$ by $\frac{3}{5}$
3.	$5\frac{2}{3}$ by $\frac{7}{8}$	7.	$22\frac{2}{3}$ by $\frac{2}{3}$	11.	$4\frac{1}{8}$ by $\frac{3}{16}$
4.	$\frac{5}{8}$ by $\frac{3}{8}$	8.	$12\frac{5}{9}$ by $\frac{1}{6}$	12.	$2\frac{1}{2}$ by $\frac{25}{4}$

98. In the solution of problems, a clear, concise form of analysis is often a great help to clearness in thinking. This is particularly true in the two kinds of division of fractions. The following forms are suggested for use in connection with problems only.

1. At $\$\frac{4}{5}$ per pound, how many pounds of tea can be bought for $5?

Notice that the two numbers given have the *same unit value.*

Analysis:—

How many of $\$\frac{4}{5} = \$5$?

$\frac{5}{4}$ of $\$\frac{4}{5} = \$1$

$\therefore$ $\frac{5}{4}$ lbs. can be bought for $1?

$5 \times \frac{5}{4}$ lb. $= \frac{25}{4}$ lb. or $6\frac{1}{4}$ lbs.

$\therefore$ $6\frac{1}{4}$ lbs. can be bought for $5?

2. James has $28. This is $\frac{4}{7}$ as much money as Henry has. How much money has Henry?

Notice that the two numbers given have a *different unit value.*

Analysis:

$\frac{4}{7}$ of *Henry's money* = $28

$\frac{1}{7}$ of *Henry's money* = $\frac{1}{4}$ *of* $28 or $7

$\frac{7}{7}$ of *Henry's money* = 7 × $7 or $49.

$\therefore$ Henry has $49.

99. Write out the analysis for the following:

1. How many pieces of board each $\frac{5}{8}$ ft. long can be cut from a board 16 ft. long?

2. An oak table 4 ft. long is $\frac{2}{3}$ as long as a pine table we wish to make. How long is the table we wish to make?

3. How many weights each weighing $2\frac{1}{2}$ lbs. can be cast from 120 lbs. of pig iron, allowing $\frac{1}{8}$ of the mass for waste? (Reduce the $2\frac{1}{2}$ to an improper fraction.)

4. Wishing pieces of wire each $8\frac{3}{4}$ in. long, how many pieces can I cut from 40 ft. of wire?

5. If $\frac{2}{3}$ of a ton of coal costs \$4, what is the cost of a ton?

6. I have a drawing in which 1 in. equals 12 ft. This scale is $\frac{2}{3}$ of the size I wish to use. In the scale I wish to use, 1 in. will equal how many feet?

7. A circle 18 in. in diameter is $\frac{3}{8}$ of the diameter of a certain buggy wheel. What is the diameter of the wheel?

8. A room is 14 ft. wide. This is $\frac{2}{3}$ of its length. How long is the room?

9. A room is $20\frac{5}{8}$ ft. long and $16\frac{1}{2}$ ft. wide. The width of the room is what part of the length of the room?

100. There are how many fourths of a foot in 1 foot? Then $\frac{1}{4}$ *more* than a foot equals how many fourths of a foot? $\frac{1}{4}$ *less* than a foot equals how many fourths of a foot?

1. A carpenter has a board which is $\frac{1}{4}$ longer than he needs. If the board he has is 20 ft. long, how long is the board he needs?

2. A boy earns $\frac{2}{5}$ less money in a day than his father. If the boy earns 90c per day, how much does his father earn in a day?

3. After working a month, a boy found that he could make $\frac{2}{3}$ more boxes in a day than he made the first day. If he now makes 15 boxes a day, how many boxes did he made the first day?

4. What is the entire value of a man's potato crop if he sold $\frac{3}{8}$ of it for \$720?

5. Make up similar problems from some work you are doing in school and see whether your classmates can solve them.

MISCELLANEOUS PROBLEMS

1. At \$$2\frac{1}{4}$ per volume, what will a set of 18 law books cost?

2. Find the cost of $27\frac{1}{2}$ cords of wood at \$$5\frac{3}{4}$ per cord.

3. A boy gets \$$\frac{1}{5}$ per hour. How much can he earn in $18\frac{1}{2}$ hrs.?

4. A train runs 75 miles in $2\frac{1}{3}$ hours. What is the average rate per hour?

5. A desk cost \$$37\frac{1}{2}$. This was $\frac{3}{4}$ of the cost of a table. Find the cost of the table.

6. A farmer sold $\frac{3}{5}$ of his potatoes to one man and $\frac{2}{5}$ of them to another. He then had 36 bu. left. How many bushels had he in all?

7. What is the cost of $7\frac{1}{2}$ tons of coal at \$$8\frac{1}{4}$ per ton?

8. A house and lot are worth \$3,740. If the lot is worth $\frac{3}{8}$ as much as the house, what is the value of each?

9. If a man can saw a cord of wood in $\frac{3}{8}$ of a day, how many cords can he saw in $5\frac{1}{2}$ days?

10. A and B work together making hand sleds. If A makes $\frac{5}{6}$ as many sleds as B and they receive \$132 for all the sleds, how much should each receive?

11. A kept account of his working time for one week, which was as follows: Monday, $7\frac{1}{2}$ hrs.; Tuesday, $8\frac{3}{4}$ hrs.; Wednesday, $5\frac{1}{4}$ hrs.; Thursday, $7\frac{5}{6}$ hrs.; Friday, 8 hrs.; Saturday, $6\frac{2}{3}$ hrs. How many hours did he work during the week? How much did he earn at \$$\frac{3}{8}$ per hour?

12. A man bought $8\frac{3}{4}$ lbs. of glue and used $3\frac{1}{3}$ lbs. How many pounds had he left?

13. A draftsman drew a line $18\frac{3}{4}$ in. long. He then wanted a second line $\frac{2}{3}$ as long. How long should he make the second line?

14. If $1\frac{2}{3}$ ft. are allowed to each person, how many persons can be seated on two sides of a table 20 ft. long?

15. A boy spent $\frac{4}{15}$ of his study time getting his arithmetic. What is his entire study time if he spends $1\frac{1}{3}$ hrs. on arithmetic?

16. A newsboy's earnings each day for ten days were as follows: $\$\frac{3}{5}$, $\$\frac{3}{4}$, $\$\frac{16}{25}$, $\$\frac{41}{50}$, $\$\frac{9}{25}$, $\$\frac{1}{2}$, $\$\frac{9}{10}$, $\$1\frac{1}{4}$, $\$\frac{18}{25}$ and $\$\frac{7}{10}$. How much did he earn in all during the ten days?

17. After piling up $\frac{2}{3}$ of $2\frac{3}{4}$ thousand bricks, there are how many bricks left to pile?

18. $\frac{2}{3}$ of the lumber needed to make a bookcase cost $\$7\frac{1}{2}$. Find the cost of all the lumber needed.

19. A man earns $\$2\frac{3}{4}$ per day. How much can he earn in 40 days?

20. If $\frac{3}{4}$ of a barrel of pork sold for \$12, what is the price per barrel?

21. A room is 24 ft. long. It is $\frac{1}{3}$ longer than it is wide. How wide is it?

22. Draw a line $\frac{1}{5}$ longer than another line.

23. Draw a line $\frac{1}{5}$ shorter than another line.

24. Draw a line $\frac{1}{5}$ as long as another line.

25. Draw a line $\frac{4}{5}$ as long as another line.

26. Draw a line $\frac{6}{5}$ as long as another line.

Which ones of the last five exercises are alike?

27. From a bolt of 50 yds. of cloth a merchant sold $7\frac{1}{2}$ yds. to one customer and $12\frac{3}{4}$ to another customer. How many yds. are there left?

28. If a man earns $\$2\frac{3}{4}$ per day, how long will it take him to earn $\$46\frac{3}{4}$?

29. I sold a horse for \$250, thereby gaining $\frac{1}{9}$ of the cost. How many dollars did I gain?

30. If a train runs $\frac{7}{8}$ mile in $1\frac{3}{8}$ min., how long will it take to run 20 miles?

31. In a certain stairway each step rises $6\frac{3}{4}$ in. If there are 16 steps, how high is the second floor above the first?

32. A owns $79\frac{11}{16}$ acres of land. If he sells a $\frac{1}{3}$ interest in this land to B, how many acres will each own?

33. If you walk $3\frac{3}{8}$ miles per hour, how long will it take you to walk 20 miles?

34. If a man spends $\frac{9}{32}$ of his income for board and room, $\frac{1}{6}$ of it for clothes and $\frac{5}{12}$ for other things, what part of his income will he have left?

35. Paid $\$13\frac{1}{2}$ for $3\frac{3}{5}$ yds. of cloth. At this rate how much will $12\frac{2}{3}$ yds. cost?

36. If a man can mow $\frac{5}{8}$ of a field of $3\frac{1}{2}$ acres in $\frac{3}{5}$ of a day, how long will it take him to mow the entire field?

37. I own $\frac{3}{16}$ of a tract of land. If I sell $\frac{4}{5}$ of my share, what is the value of what I then own, the tract being worth $3,200?

38. A has $640 which is $2\frac{2}{3}$ times as much as B has. How much money has B?

39. If a student can make a taboret in $\frac{2}{7}$ of a week, how many can he make in a month?

40. $\frac{3}{5}$ of a certain ore is iron. How much of this ore does it take to make 10 tons of iron?

41. What is the difference in inches between $\frac{2}{3}$ of $\frac{1}{2}$ of a foot and $\frac{5}{6}$ of a foot?

42. $\frac{3}{4}$ of my money is invested in fruit land, $\frac{1}{3}$ of the remainder is in bank. The balance I have just used to buy a house and lot for $4,200. How much money have I in the bank?

43. A tank was $\frac{3}{5}$ full when $\frac{3}{4}$ of its contents spilled out. It will require how many times the present contents to fill the tank full?

44. Henry paid $\$9\frac{1}{4}$ for his sweater and John paid $\$5\frac{1}{2}$ for his sweater. Henry's sweater cost what part more than John's?

45. A board was divided into two equal parts. Each of these parts was then divided into 6 parts. If one of the small pieces is $1\frac{5}{8}$ ft. long, how long was the original board?

46. A wholesale firm shipped 320 packages, each weighing $28\frac{3}{4}$ lbs. What was the entire weight?

47. In a case of 30 dozen eggs, three out of every eight were found to be bad. How many bad eggs were in the case?

48. A wheel makes $116\frac{2}{3}$ revolutions in a minute. How many revolutions will it make in $10\frac{2}{5}$ minutes?

49. Bought a carload containing 1,200 crates of peaches at $\$\frac{3}{5}$ per crate. Sold $\frac{1}{4}$ of them for $\$\frac{3}{4}$ per crate; $\frac{5}{8}$ of them for $\$\frac{4}{5}$ per crate and the remainder for $\$\frac{1}{2}$ per crate. How much was the profit?

50. A cubic foot of water weighs $62\frac{1}{2}$ lbs. Approximately $7\frac{1}{2}$ gallons = 1 cu. ft. Estimate the weight of water a 10-gallon keg will contain.

DECIMAL FRACTIONS

101. A *decimal fraction* is a fraction whose denominator is some power of ten. The denominator of a decimal fraction is not written, but is indicated by a period known as a *decimal point* placed just to the right of the unit's place.

NOTATION AND NUMERATION

102. Our entire system of notation is based upon the decimal system. It requires ten units to make one ten, ten tens to make one hundred, ten hundreds to make one thousand, etc. Or, going the other way, one hundred is one tenth of a thousand, one ten is one tenth of a hundred, one

unit is one tenth of a ten. Now, taking one tenth of a unit, we have one tenth, one tenth of a tenth and we have one hundreth, one tenth of a hundredth and we have one thousandth, etc. The following table will serve to show the names of places in our system. These should be thoroughly memorized with reference to one another and to the decimal point.

Billions	Hundred millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Ones	Decimal point	Tenths	Hundredths	Thousandths	Ten-thousandths / Tenths of thousandths	Hundred-thousandths / Hundredths of thousandths	Millionths	Ten-millionths / Tenths of millionths	Hundred-millionths / Hundredths of millionths	Billionths
0	0	0	0	0	0	0	0	0	0	.	0	0	0	0	0	0	0	0	0

103. We read a decimal precisely as though it were a whole number and then give it the name of the decimal place in which the right hand figure stands. Read the number to the left of the decimal point first. Say "and" when you come to the decimal point, thus— 126.045 is read, one hundred twenty-six *and* forty-five thousandths.

104. Read the following:

1.	.027	6.	480.0064	11.	7428.000463
2.	.49	7.	96.8	12.	906.104
3.	3.0043	8.	400.04	13.	80.00203
4.	92.04	9.	7.0096	14.	.000264
5.	.008	10.	84.397	15.	1001.001

105. In the decimal system of notation any unit represents what part of a unit in the place next to the left? Two places to the left? Three places to the left?

One hundredth is what part of one tenth? Then in .41 we have 4 tenths and 1 tenth of a tenth. In .46 we have 4 tenths and 6 tenths of a tenth.

One thousandth is one hundredth of a tenth. Therefore, in .401 we have 4 tenths and 1 hundredth of a tenth. In .329 we have 3 tenths and 29 hundredths of a tenth.

We may read decimal numbers in a similar manner referring to hundredths, thousandths or any other unit.

Again any unit represents 10 units of the place next to the right, 100 units of the second place to the right, 1000 units of the third place to the right, etc.

2.63 = 26 tenths and 3 tenths of a tenth.
2.63 = 263 hundredths.
.428 = 4 tenths and 28 hundredths of a tenth.
.428 = 42 hundredths and 8 tenths of a hundredth.
.428 = 428 thousandths.
56.32 = 5632 hundredths.

106. Read the following as tenths; as hundredths; as thousandths; (always locate the place of the unit to which you refer before reading).

1.	.61	11.	98.2	21.	864.3
2.	.601	12.	.04	22.	3.
3.	1.61	13.	.6052	23.	8.246
4.	.62	14.	384.2	24.	.0703
5.	1.67	15.	75.43	25.	543.
6.	1.601	16.	70.03	26.	10.05
7.	.621	17.	.007	27.	6.25
8.	.625	18.	7.3	28.	.7525
9.	3.625	19.	6.065	29.	3.752
10.	3.605	20.	.7924	30.	123.5462

A good exercise is to take a number such as the last one given and see in how many different ways you can read it. It may be read correctly as units of any denomination.

107. Write in figures:

1. One hundred twenty-five ten-thousandths.
2. Six hundred and four hundredths.
3. Five thousand four and four tenths.
4. Eighty-five hundred-thousandths.
5. Nine thousand eight hundred sixty-four and seventeen thousandths.
6. Eight hundred-thousandths.
7. Eight hundred thousandths.
8. Sixty-four thousand and ninety-six millionths.
9. One hundred twenty-five ten-thousandths.
10. Sixteen and four tenths.
11. Eight hundred six and eight hundred six thousandths.
12. One hundred-thousandth.
13. One thousand and two thousandths.
14. One thousand two thousandths.
15. Sixteen hundreds.
16. Sixteen hundredths.
17. Ninety-eight hundred-thousandths.
18. One million and one thousandth.
19. Four tenths of thousandths.
20. Four ten-thousandths.

108. *To reduce a decimal to a common fraction*, write the denominator of the decimal, omit the decimal point, and reduce to lowest terms.

109. Reduce to common fractions in lowest terms or to mixed numbers:

1.	.2	9.	.025	17.	5.5
2.	.75	10.	.08	18.	.375
3.	.45	11.	.500	19.	6.25
4.	.25	12.	.0050	20.	1.05
5.	.8	13.	.0006	21.	7.75
6.	.60	14.	.64	22.	2.5
7.	.12	15.	.0025	23.	4.125
8.	.07	16.	.625	24.	1.005

ADDITION AND SUBTRACTION OF DECIMALS

110. *To add or subtract decimals* write units of the same size under each other and proceed as in whole numbers.

111. Add:

1. 6.3, 4.72, 98.46, 54.3, 78.25
2. 8.462, 94.03, 1.0047, 51.06
3. 118.26, 49.7423, 9427.8436
4. 100.42, 94.63, .007, .0045, 9.416
5. 10.486, .04297, 6.035, .00049
6. 98.02, 49.732, .0078, .05632

112. Subtract:

1.	8.64 − 1.28	6.	82. − .92
2.	50.06 − 7.08	7.	47.02 − 18.9
3.	6.004 − 2.38	8.	3.904 − 2.8
4.	.9 − .0482	9.	166.5 − 48.05
5.	.043 − .0096	10.	90. − .097

113. Exercise:

Place a decimal point so as to make the number 12 that number of tenths; hundredths; thousandths; ten-thousandths. Do the same with the following: 163, 405, 6824, 3006, 10429, 12953.

MULTIPLICATION OF DECIMALS

114.

1. Multiply 16 horses by 3.
2. Multiply 16 tenths by 3. Point off result.
3. Multiply 1.6 by 3. Point off result.
4. Multiply 16 by $\frac{3}{10}$.
5. Multiply 16 by .3. Point off result.
6. Multiply $\frac{16}{10}$ by $\frac{3}{10}$.
7. Multiply 1.6 by .3.
8. Multiply .16 by .3.

9. Multiply .16 by .03. Compare the number of decimal places in the product with the sum of the decimal places in the multiplicand and multiplier.

115. *To multiply decimal fractions*, multiply as in whole numbers and point off as many decimal places in the product as there are in both factors.

116. Multiply:

1. 4.06 × .8	6. 1.863 × 90.
2. 73.42 × .09	7. 50. × .005
3. 6.073 × .7	8. 763 × 9.02
4. 90.008 × .04	9. 100.3 × .00405
5. 16.3 × 9.8	10. 82.6 × 3.008

DIVISION OF DECIMALS

117. In division two things have to be considered, viz., the *size* of the units in the dividend and divisor and the *number* of units in each. Dividing units by other units of the same size will produce a whole number in the quotient if the number of these units in the dividend equals or exceeds the number in the divisor. Compare .1 of an inch with any other .1 of an inch as to size; .1 mile and another .1 mile; .1 minute and another .1 minute. Compare .01 of a yd. and another .01 yd. as to size. Compare .001 sq. ft. and another .001 sq. ft. as to size.

118. Principle: In division we will have integral numbers in the quotient as long as the units used in the dividend equal or exceed the units in the divisor in size. When the units used in the dividend are smaller than the units in the divisor, the quotient will be a fraction. For example, 784.632 divided by 5.2. The divisor is 52 tenths. The quotient will be a whole number until in the work we have used the 6 (the figure in tenths place).

119. *To point off the result in division of decimals*:

1st, Read the divisor as units of the denomination of its right hand figure.

2nd, Locate the figure in the dividend which stands in the place of the same denomination as you have just read the divisor.

3rd, Place a small vertical line (called separatrix) just to the right of the figure just located in the dividend.

4th, Say to yourself, "The result will be whole numbers to this point" and place a decimal point just over the separatrix.

5th, Divide as in whole numbers, being careful to place the first figure in the quotient directly over the last figure used in the partial dividend.

6th, Whenever the first figure of the quotient comes to the *right* of the decimal point, fill in with ciphers between the decimal point and the first figure of the quotient.

120. Form of work:

```
                    5 2.2 0 1
6 2.7 4 ) 3 2 7 5.0 9'7 6
          3 1 3 7 0
            1 3 8 0 9
            1 2 5 4 8
              1 2 6 1 7
              1 2 5 4 8
                  6 9 6 0
                  6 2 7 4
                    6 8 6
```

The divisor is 6,274 hundredths. Therefore the result will be a whole number out to and including hundredths in the dividend. Place the separatrix between the figure 9 and the figure 7 and the decimal point directly over it. Proceed as in division of whole numbers being careful to place the first figure (5) of the quotient properly.

121. An excellent exercise in "pointing off" the result in decimal division is to locate the decimal point in the result and merely indicate the number of figures in the result by crosses or ciphers, thus:

```
            × × .                          . × × × ×
3 6.4 ) 9 8 4.6'5 7          7.8 ) .0'0 4 2 9 7 5

                  × × × .                  × × × × .
9.6 2 4 ) 2 3 4 5.8 2 9'6    .0 0 6 3) 7.4 0 0 0'
```

122. Merely indicate the "pointing off":

(1) 4.6) 732.59	(7) 1.107) 5.2
(2) 9.43) 2458.72	(8) 643.) 9.43275
(3) .076) 5670.432	(9) .00782) .06
(4) 296.5) 349810.26	(10) .05934) .08040973
(5) 32.74) 5684.329	(11) 1.007) 2.3
(6) 65.004) 3829.654302	(12) 59.1) .0002

A student should be able, with practice to indicate the result in ten problems like the above in one minute.

MISCELLANEOUS EXERCISES

1. What is the sum of the following: 1.0036; .097; 56.2; .00024; 5000; .0604; 26.428.

2. A boy bought a suit of clothes for $14.50, a hat for $1.75, a pair of suspenders for $.49 and a pair of shoes for $2.75. He gave the merchant a twenty-dollar bill. How much change should he receive?

3. What is the cost of 7.5 lbs. of steak at 28.25c per lb.?

4. A man earned $236.25 in 17.5 weeks. How much are his average weekly wages?

5. On a line 10 inches long, a draftsman made a mark 3.06 in. from one end and another mark 2.75 in. from the other end. What is the distance between these two marks?

6. A blacksmith has 42 iron bars each 2.7 ft. long. If placed end to end, how far would they all reach?

7. A tank contains 1842.5 gal. How many barrels in this? (31.5 gal. = 1 bbl.)

8. In three piles of wood there are respectively 18.6 cords, 14.75 cords and 2.29 cords. How much is this wood all worth at $7.40 per cord?

9. A man paid a coal bill amounting to $69. If the coal was $5.75 per ton, how many tons did he pay for?

10. Land is worth $112½ per acre. How much must be paid for 160 acres?

PERCENTAGE

123. Percentage. *Per cent* refers to hundredths. The sign % is used to denote the phrase *per cent.* Six per cent is six hundredths, $\frac{6}{100}$, or .06, or 6%.

124. Read the following decimals as hundredths:

1. .60	11. 2.25	21. .0075	31. 5.
2. .25	12. 2.025	22. .0475	32. .5
3. .42	13. 2.0025	23. .3775	33. .05
4. 1.	14. 1.4	24. .0001	34. .005
5. 1.25	15. 1.425	25. .0009	35. .0005
6. .001	16. .375	26. .0005	36. .505
7. .045	17. .75	27. .00025	37. .0505
8. .006	18. .5	28. .906	38. .055
9. .0625	19. 2.5	29. 1.703	39. 1.7525
10. .0025	20. .005	30. 1.42	40. 1.07375

125. Now read the above as hundredths but call each one per cent instead of hundredths.

126. Write decimally:

1. 12%	9. $\frac{7}{10}$%	17. $7\frac{1}{2}$%	25. $\frac{4}{5}$%
2. 37%	10. 6.7%	18. $2\frac{1}{4}$%	26. .8%
3. 8%	11. $\frac{1}{2}$%	19. $\frac{3}{4}$%	27. 80%
4. 100%	12. 50%	20. $12\frac{1}{2}$%	28. $1\frac{1}{2}$%
5. 125%	13. $\frac{1}{4}$%	21. $\frac{1}{8}$%	29. .03%
6. 92%	14. 25%	22. $\frac{3}{8}$%	30. 2.7%
7. 204%	15. .2%	23. $100\frac{1}{2}$%	31. 8.04%
8. 15%	16. $\frac{1}{5}$%	24. $14\frac{1}{8}$%	32. .28%

127. Reduce to per cent and memorize results:

$\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{11}$, $\frac{1}{12}$, $\frac{1}{16}$, $\frac{1}{20}$, $\frac{1}{25}$, $\frac{1}{50}$.

128. Rule: *To find any per cent of a number, multiply the number by the per cent expressed as hundredths, or as a common fraction.*

Find 40% of 200.

$$200 \times .40 = 80.00. \quad \text{Or, } 200 \times \tfrac{40}{100} = 80.$$

Find 5% of 80.

$$80 \times .05 = 4.00. \quad \text{Or, } 80 \times \tfrac{5}{100} = 80 \times \tfrac{1}{20} = 4.$$

129. In the following use the per cent either as a common fraction, or as a decimal fraction, depending upon which one is the easier.

Find:

1.	24% of \$60.	11.	$\frac{1}{2}$% of \$18.
2.	50% of 5280 ft.	12.	50% of \$18.
3.	7% of \$540.25.	13.	$\frac{1}{4}$% of 140 lbs.
4.	92% of 846 lbs.	14.	$\frac{3}{8}$% of 400 gal.
5.	10% of 180 bu.	15.	$16\frac{2}{3}$% of 636 books.
6.	20% of 45 yds.	16.	$\frac{1}{6}$% of 84 ft.
7.	120% of 15 sq. ft.	17.	$104\frac{3}{4}$% of \$1000.
8.	$16\frac{2}{3}$% of 24 mi.	18.	$14\frac{1}{5}$% of 60 in.
9.	$37\frac{1}{2}$% of 16 yrs.	19.	$\frac{7}{8}$% of \$4000.
10.	80% of 90pp.	20.	$3\frac{2}{5}$% of 180 lbs.

PROBLEMS

1. A pile containing 600 bu. contained $2\frac{2}{3}$% weed seed. How many bushels of weed seed in the pile?

2. What is the distance already traveled in a 500-mile race when $33\frac{1}{3}$% has been finished?

3. A man spent 40% of his money for a suit of clothes, 15% of it for an overcoat, $3\frac{1}{2}$% of it for a pair of shoes. How many dollars did he have left if he had \$80 at first?

4. In a school with an enrollment of 1,600 pupils, 35% were sick. How many pupils were sick?

5. If you cut off $6\frac{1}{4}$% of a board 16 ft. long, how many feet will remain?

6. A man bought 12 tons of coal and used $83\frac{1}{3}$% during the winter. How many tons did he use?

7. A boy cut off $8\frac{1}{3}$% from his 24-foot fish-pole. How long is it now?

8. A fruit dealer bought 1,500 bbls. of apples and sold 72% of them at $4.25 per bbl. How much did he get for the part sold?

9. A man agreed to haul 60% of a stack of hay containing 18 tons. How many loads of one ton each must he haul?

10. Find 18% of a line 16 in. long.

11. A man has 185 bu. of corn and wishes to buy 40% more. How many bushels will he then have?

12. When eggs are selling for 40c. per dozen, what will be the selling price after a reduction of 15%?

13. John worked 32 problems and Henry worked $87\frac{1}{2}$% of that number. How many did Henry work?

14. There are 80 volumes on one shelf and 125% of this number on the shelf below. How many volumes on both shelves?

15. A man received $12\frac{1}{2}$% for selling a diamond for $200. How much did he receive?

130. *To find what per cent one number is of another*, make that number the numerator of a fraction and the other number the denominator. Reduce this fraction to hundredths.

131. What per cent of: (Work mentally)—

1. 12 is 9	11. 56 is 7	21. 150 is 50
2. 72 is 24	12. 100 is 17	22. 50 is 100
3. 45 is 9	13. 24 is 18	23. 50 is 20
4. 80 is 30	14. 75 is 24	24. 80 is 120
5. 400 is 200	15. 40 is 2	25. 12 is 20
6. 64 is 8	16. 80 is 40	26. 180 is 120
7. 120 is 100	17. 60 is 50	27. 48 is 32
8. 90 is 10	18. 125 is 25	28. 225 is 25
9. 150 is 25	19. 48 is 6	29. 360 is 90
10. 1000 is 250	20. 70 is 10	30. 4800 is 600

132. What per cent of: (Written work)—

1. 840 is 193.2	4. 790.8 is 197.7	7. $\frac{3}{8}$ is $\frac{1}{4}$
2. 78 is 2.34	5. 3.65 is .146	8. $18\frac{1}{4}$ is $9\frac{1}{8}$
3. 660 is 82.5	6. 1642 is 207	9. 5890 is 29.45
	10. $\frac{7}{8}$ is $\frac{3}{5}$	

PROBLEMS

1. If a man's taxes in 1914 were $640 and in 1915 they were $520, what per cent were his taxes in 1915 of his taxes in 1914?

2. A man hauled 3,200 lbs. of coal the first load and 2,800 lbs. the second load. His second load was what per cent as heavy as his first load?

3. John saved $2.40 in January and Henry saved $1.80 the same month. Henry saved what per cent as much as John?

4. A pupil spelled 125 words correctly out of 140 words. What per cent did he miss?

5. A man owned 640 acres of land and sold 40 acres. What per cent did he sell?

6. A man bought 4,500 ft. of lumber and used 3,300 ft. What per cent did he have left?

7. A salesman bought a 2,000-mile ticket. He used up coupons for 840 miles. What per cent of his ticket is used?

8. A man weighing 160 lbs. was taken sick and now weighs 130 lbs. What per cent of his weight did he lose?

9. Wages were reduced from $2.75 to $2.50 per day. What per cent were they decreased?

10. One half of a field was fertilized and yielded 180 bu. of potatoes per acre. The other half yielded 120 bu. per acre. What was the per cent of increase on the fertilized half?

11. A boy increased in weight from 121 lbs. to 130 lbs. What per cent did he increase?

12. There were 2,860 pupils in school in October and 3,040 pupils in school in November. What was the per cent of increase?

13. A balloon carried 400 lbs. of ballast. When 90 lbs. had been thrown out, what per cent of the ballast remained?

14. A rectangle is 20 in. long and 14 in. wide. The length is what per cent of the width? The width is what per cent of the length?

15. A steel beam is 30 ft. long. An oak beam is 12 ft. long. The length of the oak beam is what per cent of the length of the steel beam?

133. *To find a number when a given per cent of it is known,* divide the known number by the per cent expressed as a decimal or a common fraction.

40% of what number is 26. What is the number?

Analysis:

$$\begin{aligned} 40\% \text{ of what number} &= 26 \\ 1\% \text{ of the number} &= \tfrac{1}{40} \text{ of } 26 \text{ or } .65 \\ 100\% \text{ of the number} &= 100 \times .65 \text{ or } 65. \end{aligned}$$

$\therefore$ 40% of 65 is 26.

Mechanical work:

$$\begin{array}{r} 65 \\ .40\overline{)\,26.00} \\ \underline{24\,0} \\ 2\,00 \\ \underline{2\,00} \end{array}$$

PROBLEMS

1. A man spends 5% of his salary for clothes. If he spends \$80 for clothes, what is the amount of his salary?

2. Twelve years is 60% of Mary's age. How old is Mary?

3. An architect received \$3,400 which was 5% of the cost of the building. What was the cost of the building?

4. After driving 15 miles a man said he had gone $37\frac{1}{2}\%$ of his journey. How many miles was his entire journey?

5. One player succeeded in making $16\frac{2}{3}\%$ of all the points made in a basketball game. If he made 12 points, how many points were made in all?

6. A class was so large that $12\frac{1}{2}\%$ could not be seated. If 6 were unable to find seats, how many pupils were in the class?

7. It required $\frac{1}{2}\%$ of a man's money to pay his taxes amounting to \$8. How much money had he?

8. What is the value of an entire property if 35% of it is worth \$700 ?

9. Seven days of the month have passed, or 25% of it. Which month is it?

10. Eighteen per cent of a boy's weight is 9 pounds. How much does he weigh?

134. The whole of anything is how many per cent of it?

Then 10% *less* than the whole of anything is how many per cent of it?

10% *more* than the whole of anything is how many per cent of it?

PROBLEMS

1. After increasing the length of a kite string 10% it was 220 ft. long. How long was it at first?

2. Draw a line. Draw a second line 25% shorter than the first line. Draw a third line $66\frac{2}{3}$% longer than the second line. The third line is now how many per cent of the length of the first line.

3. How long would you make a board if you wanted it $33\frac{1}{3}$% longer than a board 8 ft. long?

4. Wages this year are 25% lower than they were last year. This year they are $1.80 per day. How much were they last year?

5. A company which is paying 20c. per hour announced an increase of 8c. per hour. What was the per cent of increase?

6. Stock which was selling for $126 per share went up to $142 per share. What was the per cent of increase?

7. A new engine in a vessel increased its speed from $18\frac{1}{2}$ knots per hour to $20\frac{3}{4}$ knots per hour. What was the per cent of increase?

8. In 1910, there were 1,968 Chinese immigrants to the U. S. In 1911, there were but 1,460 Chinese immigrants to the U. S. What was the per cent of decrease?

9. In 1909, Pennsylvania produced 137,966,791 tons of bituminous coal valued at 94c per ton. In the same year, Oklahoma produced 3,119,377 tons valued at $2 per ton. The value of the coal production in Oklahoma in 1909 was how many per cent less than the value of the coal production in Pennsylvania that year?

10. "A" walked a certain distance in $7\frac{1}{2}$ hrs. "B" walked the same distance in $5\frac{3}{4}$ hrs. "A" took what per cent more time than "B" to walk the distance?

MISCELLANEOUS PROBLEMS

1. If a man saves $200 on a salary of $1,200, what per cent does he save? ($200 is what per cent of $1,200?)

2. 36 is 9% of what number?

3. The window surface in a schoolroom should be 20% of the floor surface. If a floor is 28 ft. x 40 ft., what should be the window surface?

4. A schoolroom contains 4,000 cu. ft. If 40% of the air is changed every 10 minutes, how many cubic feet of air will be changed every 30 minutes?

5. A bicycle cost $50 and was sold at a gain of 30%. What was the selling price? (30% of $50 added to $50).

6. A bicycle sold for $70 at a gain of 40%. What was the cost?

7. In an 800 acre farm 8,000 sq. rds. are pasture. What percentage of the farm is pasture? (1 acre = 160 sq. rds.)

8. The U. S. silver dollar contains $371\frac{1}{4}$ grains of pure silver; this is 90% of the weight of a silver dollar. What does a silver dollar weigh?

9. 25 is 200% of what number.

10. An automobile costs $3,500. If sold at a loss of 15%, at what price must it be sold?

11. During the month of December a merchant's sales amounted to \$36,510 which was $37\frac{1}{2}\%$ of his sales for the entire year. What was the amount of his average monthly sales throughout the year?

12. A man spent 50% of his time in Minnesota, 20% of his time in Iowa, one half the remainder in Wisconsin, and what time he had left at his home in Chicago. How many days during the year did he spend at home?

13. After a man's wages had been increased $6\frac{1}{4}\%$ he received \$850. How much was his former salary?

14. A farmer raised 1,200 bu. of wheat and 1,600 bu. of oats. He raised what per cent more oats than wheat?

15. A man had a cow that gave 650 lbs. of milk testing 3.9% butter fat during one month. How many pounds of butter fat did the cow produce that month?

16. A farmer had to throw off 20% of his load of hay. The remainder of his load weighed 2,200 lbs. What was the weight of his original load?

17. A man has 40% of his money invested in a farm, 25% of it in a house and lot in town, and the balance which is \$7000 is deposited in a bank. How much is invested in the farm?

18. A man rented a house for \$45 per month. This was $12\frac{1}{2}\%$ more than he had been paying. How much had he been paying?

19. The temperature in a room is 60 degrees. What per cent is it raised when the temperature is $68\frac{1}{2}$ degrees?

20. In a running broad jump, a boy, by practice, increased his jump from $14\frac{2}{3}$ ft. to $16\frac{1}{2}$ ft. What was the per cent of increase?

21. A man lost his pocketbook containing $\frac{1}{2}\%$ of his money. If it contained \$16, how much money had he?

22. A farmer raised 1,200 bushels of wheat. He sold an elevator company 400 bushels; $\frac{1}{4}$ of the remainder he kept for seed. What per cent of the whole crop had he left?

23. Twenty gallons of water were spilled into a tank containing 16 gallons of nitric acid. The water was what per cent of the mixture? There were how many per cent less of nitric acid than of water?

24. A man owning 20% of a factory sold 10% of his share for $1,600. At this rate what is the entire factory worth?

INTEREST

135. Interest is the compensation paid for the use of money.

1. A farmer wishes to borrow $3,000 for 2 yr. 6 mo. and 10 days from a bank, which charges 5% interest. How much interest must he pay for the use of the money?

Interest on \$3,000 for 2 yr. at $5\% = (150 \times 2) = \$300.$
Interest on \$3,000 for 6 mo. at $5\% = (150 \times \frac{1}{2}) = 75.$
Interest on \$3,000 for 10 da. at $5\% = (150 \times \frac{1}{36}) = \underline{4.16\frac{2}{3}}$
Interest on \$3,000 for 2 yr. 6 mo. 10 da. at $5\% = \$379.16\frac{2}{3}$

2. Which will be cheaper for me to borrow, $2,000 at 6% and build a house, or pay rent at $15 per month?

3. If I receive 3% interest on a deposit of $4,000, how much interest can I draw out at the end of 4 yr. 6 mo.?

4. I have $8,000 to invest. Which will be the better, to lend it at 6% or buy a property on which I must pay $160 a year insurance and $100 a year taxes and receive $70 a month rent?

5. A man borrowed $3,400 on Aug. 15, 1914, giving his note at 7%. What must he pay to take up his note May 15, 1915 ?

6. What principal will produce \$45 in 2 yr. 6 mo. at 6%?

If \$45 = the interest for $2\frac{1}{2}$ yr. \$45 ÷ $2\frac{1}{2}$ = \$18 interest for 1 yr. Since 6% of the principal = \$18, 1% of the principal = $\frac{1}{6}$ of \$18 = \$3. And 100% of the principal = 100 × \$3 = \$300.

7. What principal will produce \$304.80 in 4 yr. at 6%?

8. What principal will produce \$127.65 in 2 yr. 6 mo. at 6%?

9. In what time will \$240 produce \$36 at 6%?

6% of \$240 = \$14.40 interest for 1 yr. \$36 ÷ \$14.40 = $2\frac{1}{2}$ yr.

10. How long will it take \$360 to produce \$360 at 6%?

11. At what rate will \$300 produce \$28 in 2 yr. 4 mo.?

The interest on \$300 at 1% for $2\frac{1}{3}$ yr. is \$7, and \$28 ÷ \$7 = 4, the rate per cent.

12. At what rate will \$300 produce \$27 in 3 yr.?

INSURANCE

136. Insurance is an agreement to pay an indemnity in case of loss by death, fire or accident.

1. Find the premium on a fire insurance policy for \$2,500 for two years at \$1.50 a hundred.

\$2,500 = 25 hundreds. 25 × \$1.50 = \$37.50 the premium.

2. What premium must be paid on a policy for \$2,760 at \$1.50 per hundred?

3. My house is worth \$400 and is insured for $\frac{3}{4}$ of its value, for three years, at \$1.50 per hundred. How much is the premium?

4. A house worth \$36,000 is insured for $\frac{3}{4}$ of its value for three years at \$2.25 per hundred. How much is the premium?

5. A merchant has a stock of goods worth $36;000 insured for $\frac{3}{4}$ of its value, at $18 per $1,000. How much is his premium?

6. A barn and house worth $27,600 are insured for $\frac{2}{3}$ of their value. The premium is $414. What is the rate of insurance?

7. A house worth $3,600 is insured for $\frac{2}{3}$ its value and household goods worth $2,400 are insured for $\frac{3}{4}$ their value. What is the premium at $1\frac{1}{2}\%$?

8. A farmer takes out a 20-payment life policy for $5,000, annual premium $70 per thousand. How much will he have paid when his policy has matured?

9. Find the total cost at maturity of a 20-payment life policy for $10,000 at $32 per thousand.

10. Find the total cost at maturity of a 15-payment life policy for $5,000 at $52.50 per thousand.

11. What is the premium at 2.3% on a house valued at $3,600?

12. What is the premium at 3.5% on a house valued at $6,000 if it is insured for $\frac{2}{3}$ of its value?

13. A house worth $3,500 is insured for $\frac{3}{5}$ of its value. The premium is $220. What is the rate?

COMMERCIAL DISCOUNT

137. Commercial discount is the deduction made to dealers from a published list price. Is also called trade discount.

1. What is the net price of a bill of goods listed at $240, less a discount of 15%?

List price	=	$240
Discount (15%)	=	36
Net price	=	$204

2. What is the net price of a bill of goods listed at \$193, discount 10%?

3. What is the net price of a bill of goods if the catalogue price is \$480, rate of discount being 25%?

4. When the list price is \$400, and the net price is \$380, what is the rate of discount?

5. A bill of goods listed \$750 is offered at 20% and 10% off. What is the net cash price?

20 % of \$750 = \$150. \$750 − \$150 = \$600.

10% of \$600 = \$60. \$600 − \$60 = \$540, net cash price.

6. A merchant gives a discount of 40% and 25% from his list price. What is the net cash price of a bill of goods listed at \$960?

7. A man sells a machine for \$2,880, which is 40% and 25% below the list price. What is the list price?

8. The net cost of a bill of goods was \$1,200. What was the list price, if the cost was 25% and 20% below the list?

9. A bill of \$72.50 was discounted at 20%, 12½% and 8%. What was the net bill?

10. A suit of clothes sells for \$49 at a profit of 40%. What was the cost?

11. If I pay 24 cents a pound for meat, and 33⅓% of it is waste, what is the cost of the meat per pound?

12. What is the net cost of a piano listed at \$390, with a discount of 20%, 10% and 2½%?

13. What is the net cost of an automobile listed at \$1,450, with a discount of 40%, 25% and 10%?

14. A merchant was able to obtain 5% discount on a bill of \$720 by borrowing the money at the bank for 90 days at 6% interest. How much was he able to save?

TAXES

138. A **tax** is a sum of money levied on a person, his property, income or business by some branch of the government for public purposes.

A school district wishes to raise a tax of $10,000. The assessed value of the property in the district is $2,000,000. What is the tax rate? What is John Smith's tax whose property is valued at $5,000?

10,000 ÷ 2,000,000 = .005, the rate of taxation.

Each property owner must pay .005 or 5 mills on a dollar of his assessed valuation. John Smith's valuation is $5,000, and $5,000 × .005 = $25.

Real estate, such as lands, buildings, railroads, mines, etc., is taxed in this way.

Personal property is: cattle, horses, household goods, money, machinery, etc., and sometimes is not taxed.

The amount of money to be raised for any purpose is first determined. In the example above it was $10,000.

An assessment roll is made out, which is a list of all the tax-payers in the community with the estimated value of each person's property set opposite his name. The rate of taxation, or the amount of tax to be levied on each dollar of each man's property, is found by dividing the amount of tax to be raised by the total assessed valuation of all the property.

PROBLEMS

1. If the rate of taxation is .005 on the dollar, and a man's property valuation is $8,000, what is his tax?

2. The assessed valuation of a man's property is $10,000, and his tax is $25. What is his rate of taxation?

3. The assessed valuation in a township is $5,000,000. How much income will be received from a 5-mill tax?

4. The assessed valuation in a certain town is $2,400,000, and to build a schoolhouse will cost $9,600. What rate of taxation will meet this levy? What must Mr. Miller pay, if his property is valued at $8,000 ?

5. A man has a farm of 160 acres valued at $85 per acre, which is assessed at one fourth of its value, stock worth $1,100 assessed at 35% of their value, and $650 in the bank. The rate of taxation is one half of one per cent. What is the total amount of his tax, if he is allowed an exemption of $100 ?

6. If a street car company should pay a city 5% on its gross earnings of $2,347,684, what would the tax be?

DENOMINATE NUMBERS

139. A **denominate number** is one composed of one or more concrete units, as, 3 days, 2 pounds and 7 ounces, or 2 feet and 4 inches. Day is a simple unit and pounds and ounces represent a compound one.

WEIGHTS AND MEASURES

140. Troy Weight.

24 grains (gr.)	=	1 pennyweight (pwt.)
20 pennyweights	=	1 ounce (oz.)
12 ounces	=	1 pound (lb.)
5,760 grains	=	1 troy pound (lb.)

PROBLEMS

1. How many ounces are there in 24.45 lbs.?

2. How many grains in a 3-carat diamond? (1 carat = 3.168 gr.)

3. The weight of a large diamond is 150 carats. What is its weight in ounces?

4. A jeweler used 7 ounces of gold for rings, 10 ounces for pins and 12 ounces for earrings. How many pounds did he use in all?

5. How many grains in 20 ounces of silver?

6. Reduce 5 pounds to grains.

7. A silversmith used 4.3 ounces of silver in making a set of teaspoons, 11.4 ounces for a set of tablespoons, and 13.7 ounces for a set of forks. How many grains of silver did he use for all? What was the silver worth at 60 cents an ounce?

8. If the diamond of a ring contains 32 grains, how many carats does it weigh?

9. A diamond weighs 1 carat, an emerald 2 carats and an opal ½ carat. How many grains in all?

10. Reduce 8,462 grains to pounds.

141. Apothecaries' Weight.

20 grains (gr.)	=	1 scruple (℈)
3 scruples	=	1 dram (ʒ)
8 drams	=	1 ounce (℥)
12 ounces	=	1 pound (lb.)
5,760 grains	=	1 pound (lb.)

PROBLEMS

1. Reduce 4 pounds to scruples.

2. How many drams in two pounds of camphor? What would it cost at 10 cents an ounce?

3. What is the cost of 2,240 ounces of sulphur at 5½ cents a pound?

4. Reduce 14 drams to grains.

5. Reduce 984 drams to pounds.

142. Avoirdupois Weight.

16 ounces	=	1 pound (lb.)
100 pounds	=	1 hundredweight (cwt.)
2,000 pounds	=	1 ton (T.)
20 hundredweights	=	1 ton (T.)
2,240 pounds	=	1 long ton
7,000 grains	=	1 pound

PROBLEMS

1. What is the weight of a cake of ice 3 ft. long, 2 ft. wide and $1\frac{1}{2}$ ft. thick? (1 cu. ft. of ice = 62 lbs.)

2. What is the weight of a boat which displaces 156 cu. ft. of water?

(An object floating in water weighs the same as the weight of the volume of water displaced.)

3. How many tons in 400 bales of hay? (1 bale = about 70 lbs.)

4. If a boarding-house keeper uses, on an average, 15 lbs. of flour a day, how many barrels will he use in 18 weeks? (196 lbs. of flour = 1 bbl.)

5. How many ounces in two tons of flour?

6. How much will $18\frac{1}{2}$ lbs. of butter cost at 35 cents per pound?

7. How many ounces in a cake of ice 2 ft. long, 1 ft. wide and 4 in. thick? (1 cu. ft. weighs 62 lbs.)

8. If a family received 50 lbs. of ice each day for 27 days, how many ounces did they receive in all?

143. Dry Measure

2 pints (pt.)	=	1 quart (qt.)
8 quarts	=	1 peck (pk.)
4 pecks	=	1 bushel (bu.)

PROBLEMS

1. Express 486 pints as bushels and quarts.

2. How many quart-boxes will be required to hold $2\frac{1}{2}$ bu. of berries?

3. A wagon box $10\frac{1}{2}$ ft. long, $3\frac{1}{2}$ ft. wide and 2 ft. deep will contain how many bushels? (1 cu. ft. = $\frac{4}{5}$ of a bushel.)

4. If a grocer buys apples at $2.00 a bushel, how much will he make on 10 bu., if he sells them at 80 cents a peck?

5. A bushel of strawberries cost \$2.56. How much were they a quart?

6. A merchant bought 5 bu. of apples at \$1.50 per bushel, and sold them at 50 cents per peck. How much did he make on each quart?

7. Reduce 10 bushels to pints.

8. How many bushels in 84 quarts?

144. Liquid Measure

4 gills (gi.)	=	1 pint (pt.)
2 pints	=	1 quart (qt.)
4 quarts	=	1 gallon (gal.)

PROBLEMS

1. How many gallons in 125 cu. ft. of water?

2. A rectangular tank is 6 ft. long, 4 ft. wide and 8 ft. deep. How many gallons will it contain? (1 cu. ft. = 7.48 gal. or 7.5 gal.)

3. A milk man sold during the morning 12 gal., 3 qts., 1 pt. of milk at 5 cents per pint. How much did he receive for his milk?

4. A tank contains 600 gal. of water. How many cu. ft. of water in the tank?
(1 cu. ft. = 7.48 gal. of water.) (1 gal. = 231 cu. in.)

5. What part of a barrel is 6 gal. and 1 qt? ($31\frac{1}{2}$ gal. = 1 bbl.)

6. What will 8 gallons of milk cost at 4 cents a pint?

7. What will 4 quarts of molasses cost at 75 cents a gallon?

8. If a family uses 4 gal., 3 qts., $\frac{1}{2}$ pt. of milk a week, how many pints will they use in 5 weeks?

9. How many pint bottles will be required to hold 45 gal., 3 qts. of wine?

10. If it takes ½ lb. of sugar to can one pint of berries, how much will it take to can 10 gallons?

11. Reduce 19 gallons to gills.

145. Linear Measure

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
16.5 feet	= 1 rod (rd.)
320 rods	= 1 mile (mi.)
5,280 feet	= 1 mile

A nautical mile, used for distances at sea, equals 1.15 statute miles.

PROBLEMS

1. Express 4,345 inches as rods, feet and inches.

2. How many inches in 1 yd., 2 ft. and 7 inches?

3. A wire fence 6 wires high is 24 yds., 2 ft. 6 in. long. How many feet of wire are needed for the fence?

4. If a man travels 70 rods in 6 minutes, how long will it take him to travel 3 miles?

5. A lot is 90 yards long and 70 yards wide. What is the length in feet around the four sides?

6. How many bricks 6 inches long will be required for 4 rows, each row being 30 rds., 4 yds., 2 ft. and 9 in. long?

7. A girl bought a bolt of ribbon containing 50 yards for $2.00. How much did it cost a foot?

8. How many miles in 2,240 rods?

9. Reduce one mile to inches.

10. Reduce 1,320 rods to miles.

146. Square Measure

144 square inches	= 1 square foot
9 square feet	= 1 square yard
30¼ square rods	= 1 square rod
160 square rods	= 1 acre
640 acres	= 1 square mile

PROBLEMS

1. How many acres and square rods in 5,080 square yards?

2. How many acres in 1,440 square rods?

3. How many square yards of surface in a blackboard 12 ft. long and 4 ft. wide?

4. A man owns a ranch containing 1,600 acres. How many square miles in his land?

5. How many square rods in 640 acres?

6. Into how many building lots can 12 acres of land be divided, each lot being 5 rods front and 12 rods deep?

7. How many square inches in $\frac{4}{9}$ of a square foot?

8. Reduce 34 square rods to square feet.

9. Reduce 4 square miles to square rods.

10. How many square feet in $\frac{3}{4}$ of a square mile?

147. Cubic Measure

1,728 cubic inches (cu. in.)	=	1 cubic foot (cu. ft.)
27 cubic feet	=	1 cubic yard (cu. yd.)
128 cubic feet	=	1 cord

In measuring excavations, 1 cu. yd. = 1 load.

PROBLEMS

1. How many cubic feet in 13 cubic yards?

2. How many cubic inches in $7\frac{1}{2}$ gallons?

3. Reduce 10 cubic yards to cubic inches.

4. Reduce 8,640 cubic inches to cubic feet.

5. How many cubic inches in 50 cu. yds., 8 cu. ft. and 5 cu. inches?

6. A hall is 30 feet long, 10 feet wide, and 18 feet high. How many cubic inches does it contain?

7. A bin contains 400 cu. ft. How many bushels of wheat will it contain?

(1 bu. = about $\frac{5}{4}$ cu. ft.) (1 bu. = 2,150.42 cu. inches.)

8. Each person breathes on an average 28 cubic feet of air in an hour. How many hours will the air in a room 15 ft. long, 10 ft. wide and 8 ft. high supply two persons?

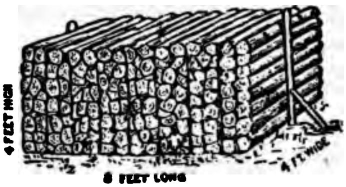

FIGURE 2.

148. In measuring wood, a pile 8 ft. long, 4 ft. wide, and 4 ft. high = 1 cord. A cord of stove wood is a pile 8 ft. long, and 4 ft. high, but 16 in. wide. (Stove length.)

PROBLEMS

1. A pile of wood is 60 ft. long, 12 ft. wide and 8 ft. high. How many cords of wood in the pile?

2. A pile of wood 20 ft. long, 8 ft. high and 4 ft. wide contains how many cords?

3. What will be the cost of a pile of wood 80 ft. long, 8 ft. high and 4 ft. thick at $7.50 per cord?

4. What is the value of a pile of cord wood, 124 ft. long, 4 ft. wide and 8 ft. high, at $8.50 per cord?

5. How long is a pile of wood which is 12 ft. wide, 8 ft. high, and contains 30 cords?

6. How many cords of 16-in. stove wood can be cut from 10 cords of 4-ft. wood?

149. Time Table.

60 seconds	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
365 or 366 days	= 1 year (yr.)
100 years	= 1 century

1. How many seconds in 4 days, 20 hours, 6 minutes and 30 seconds?

2. How many hours, minutes and seconds are there in .24 day?

3. Reduce 8 days to seconds.

4. A boy is 5 years old. How many weeks has he lived? How many days?

5. A man worked 8 hours a day for 9 months. How many days of 24 hours each did he work? How many weeks?

6. If one's pulse beats 72 times a minute, how often does it beat in a year?

7. If light travels 186,000 miles per second, how long would it be in reaching the earth (93,100,000 miles)?

8. How many hours were in the nineteenth century? How many will there be in the twentieth?

150. Table of Counting.

12 units	=	1 dozen (doz.)
12 dozen	=	1 gross (gro.)
12 gross	=	1 great gross (gt. gr.)
20 units	=	1 score
24 sheets	=	1 quire
20 quires	=	1 ream
2 reams	=	1 bundle
5 bundles	=	1 bale

PROBLEMS

1. How many reams in 10 bales?
2. How many bales in 45 bundles?
3. How many sheets in ¾ of a ream?
4. How many dozen in 14 great gross?
5. How many sheets of paper in 10 quires?
6. How many reams in 4,320 sheets of paper?
7. What will 2 quires of paper cost, if a ream costs $2.00?
8. If there are 40 children in a certain schoolroom, and each child uses 50 sheets of paper during the year, how many quires are used?

151. Weights for Bu. or Bbl.

32 pounds	=	1 bushel of oats (bu.)
48 pounds	=	1 bushel of barley
56 pounds	=	1 bushel of corn
60 pounds	=	1 bushel of wheat or potatoes (bbl.)
196 pounds	=	1 barrel of flour
200 pounds	=	1 barrel of beef or pork

LONGITUDE AND TIME

152. The earth rotates through a complete circle, 360°, or turns once on its axis in 24 hours. 360° of longitude, then, correspond to 24 hours of time.

153. Circular Measure

60 seconds (″)	=	1 minute (′)
60 minutes	=	1 degree (°)
360 degrees	=	1 circle

15° of longitude correspond to 1 hour of time. 1 hour, then = 15° of longitude.

15′ of longitude correspond to 1 minute of time. 1 minute = 15′ of longitude.

15″ of longitude correspond to 1 second of time. 1 second = 15″ of longitude.

1° of longitude corresponds to 4 minutes of time.

1′ of longitude corresponds to 4 seconds of time.

PROBLEMS

1. Reduce 1,440 seconds to degrees.

2. What part of a circle is 30 degrees?

3. How many degrees in 5½ circles? How many minutes?

4. What part of a second is 4 degrees?

5. An arc of a circle contains 45 degrees, how many seconds does it contain?

154. Standard Time. All places east of any given place have later time than at that place. All west of that place have earlier time than at that place.

Since 15° of longitude equal 1 hour of time, the earth's surface has been divided into belts 15° wide, or 1 hour wide. The sun time of the meridian extending through the middle of each belt is taken as the time for that belt.

The meridians adopted in the United States and Canada are the 75th, the 90th, the 105th and the 120th. All places at the 75th and $7\frac{1}{2}°$ east and $7\frac{1}{2}°$ west of the 75th meridian are 1 hour later than places at the 90th meridian; and 2 hours later than at the 105th; and 3 hours later than at the 120th meridian. When it is 6 a. m. at Seattle, it is 9 a. m. at Philadelphia. In traveling from New York City to Chicago, the watch must be set back 1 hour. And in traveling from Chicago to Denver, the watch must be set back again another hour.

PROBLEMS

1. What is the difference in time corresponding to a difference of 40° longitude? $1° = 4'$ and $40°$ will $= 40 \times 4'$ $= 2$ hours and 40 minutes.

2. What is the difference in longitude corresponding to a difference in time of 12 hrs. and 40 min.? 1 hr. = 15° of longitude. 1 min. = 15′ of longitude. 12 hrs. $= 12 \times 15° = 180°$. 40 min. $= 40 \times 15' = 600' = 10°$. $180° + 10° = 190°$.

3. What difference in time corresponds to a difference of 10° 40′ 20″?

4. What difference in time corresponds to a difference of 25° 17′ 19″?

5. What difference in time corresponds to a difference of 170° 0′ 40″?

6. Express the difference in longitude that corresponds to 6 hrs., 24 min., 34 sec.

7. Express the difference in longitude that corresponds to 23 hrs. 20 min.

8. When it is noon at Chicago, what is the time 30° west of Chicago?

9. If I start at Denver and travel until my time is 2 hours and 20 minutes too slow, in which direction and how far have I traveled?

10. What is the difference in time between New Orleans, 90° west and Washington $77^\circ\ 3'\ 6''$ west?

11. What is the difference in time between Paris, $2^\circ\ 20'$ east, and San Francisco, $122^\circ\ 26'\ 15''$ west?

12. What is the time in San Francisco when it is 12 noon in Paris?

13. The difference in time between Philadelphia and Cincinnati is 37 minutes, 20 seconds. What is the difference in longitude?

14. I traveled on a parallel of latitude from $80^\circ\ 24'\ 18''$ west longitude until my watch lost, 3 hrs. 25 min. What is the longitude where I stopped?

15. The longitude of New York City is $74^\circ\ 3'$ west, and Jerusalem is $35^\circ\ 13'$ east. When it is 7 hrs., 30 min. A. M. at New York, what is the time in Jerusalem?

16. The longitude of Rome is $12^\circ\ 27'$ east, and Paris is $2^\circ\ 20'$ east. When it is 5 hrs., 20 min. at Paris, what is the time at Rome?

17. London is 0° and Washington is $77^\circ\ 3'\ 6''$ west. What is the difference in time between the two places? When it is 6 A. M. at London, what is the time at Washington?

18. The longitude of Philadelphia is $75^\circ\ 10'$ west, and that of Paris is $2^\circ\ 20'$ east. When it is 6 P. M. at Paris, what is the time at Philadelphia?

LUMBER MEASURE

155. A board foot is the unit of board measure. Lumber is usually bought and sold by board measure. A board

foot is 1 ft. long, 1 ft. wide and 1 in. thick. A board 16 ft. long, 1 ft. wide, and 1 in. thick contains 16 board feet. Boards less than 1 in. thick are regarded as 1 in. thick. Over 1 in. thick, they are regarded as they are measured. $1\frac{1}{2}$ in. thick are regarded as $1\frac{1}{2}$ in. thick, etc..

156. To find the number of board feet in a board, *multiply the number of feet in length by the number of feet in width by the number of inches in thickness.*

A board 18 ft. long, 8 in. wide, and 1 in. thick contains $18 \times \frac{2}{3} \times 1 = 12$ board feet. (8 in. $= \frac{2}{3}$ of a foot.)

Or, to find the number of board feet in lumber, multiply the number of feet in length by the number of inches in width by the number of inches in thickness, and divide this product by 12.

A piece of lumber 16 ft. long, 8 in. wide, and 3 in. thick contains $\frac{16 \times 8 \times 3}{12} = 32$ board feet.

Lumber is sold by the 1,000 ft., meaning board feet. One thousand feet is designated by "M."

Twenty boards 12 ft. long, 8 in. wide, and 2 in. thick at \$60 per thousand, or per M, would cost $\frac{20 \times 12 \times 8 \times 2 \times 60}{12}$ = \$19.20. (Three decimal places for the M, the 1,000.)

PROBLEMS

1. Find the cost of 18 boards $16' \times 9''$ at \$40 per M.

2. Find the cost of 40 boards $16' \times 6''$ at \$60 per M.

3. Find the cost of 24 planks $16' \times 8'' \times 2''$ at \$40 per M.

4. At \$40 per M., what will 60 flooring boards cost, if they are 16 ft. long, 4 in. wide, and $\frac{3}{4}$ in. thick?

5. Find the cost of 81 planks $14' \times 6'' \times 2''$ at \$40 per M.

6. How many board feet in a piece of lumber $1\frac{1}{2}$ in. thick, 8 in. wide, and 16 ft. long?

$$\frac{16 \times 8 \times 1\frac{1}{2}}{12} = 16.$$

7. How many board feet in a piece of lumber 16 ft. long by 4 in. wide by $2\frac{1}{2}$ in. thick?

8. What will 20 pieces of lumber, each 12 ft. long by 6 in. wide by $3\frac{1}{2}$ in. thick, cost at $50 per M.?

9. At $30 per M., what is the value of a pile of lumber 12 ft. long by 12 ft. wide and 8 ft. high?

10. How many board feet of lumber can be put into a space 12 ft. long by 8 ft. wide by 6 ft. high, if the lumber is $\frac{3}{4}$ in. thick?

11. How many board feet in a pile of 100 boards, 12 ft. long, $8\frac{1}{4}$ in. wide, and $1\frac{3}{4}$ in. thick? (In width, $\frac{1}{2}$ in. or over is counted as 1 in., and less than $\frac{1}{2}''$ is not counted. Thus $7\frac{1}{2}$ in. wide is counted as 8 in. wide and $8\frac{1}{4}$ in. wide is counted as 8 in.)

12. How many board feet in a plank 16 ft. long, $7\frac{3}{4}$ in. wide, and 2 in. thick?

13. How many board feet in a piece of lumber 18 ft. long, $6\frac{1}{4}$ in. wide, and $\frac{3}{4}$ in. thick?

14. How many board feet in four pieces of lumber 4, 5, 6 and 8 in. wide respectively, and each piece is 12 ft. long and 3 in. thick? ($4'' + 5'' + 6'' + 8'' = 23''$ wide.)

15. Six pieces of lumber are 4, 5, 6, 7, 8 and 10 in. wide respectively. The pieces are 12 ft. long, and 2 in. thick. How many feet of lumber in the lot?

16. Find the cost at $20 per M. of the following bill of lumber:

463 posts $4'' \times 4'' \times 9'$	3,900 boards $1'' \times 3'' \times 16'$
266 stringers $2'' \times 6'' \times 16'$	450 boards $1'' \times 12'' \times 16'$

17. How many board feet will be contained in 10 wooden pillars 10 ft. high and 14 in. square?

18. What is the cost of 2,000 sq. ft. of flooring at \$38 per M., adding $\frac{1}{5}$ for waste in matching?

19. I wish to lay a plank walk 6 ft. wide in front of my city lot. The lot is 50 ft. front. I shall use plank 12 ft. long, 4 in. wide, dressed on two sides only, and before being surfaced, $1\frac{1}{4}$ in. thick. I shall use 3 sleepers, placed lengthwise of the walk. The sleepers will be 2″ × 4″ and 12 ft. long. Six 4 in. spikes, 29 to the pound, at 4 cents a pound will be used in each plank. What will be the cost of the material? (Lumber at \$40 per M.)

20. Find the cost of the lumber to inclose a field 32 rds. long by 16 rds. wide with a tight board fence 6 ft. high. Posts are placed 8 ft. apart and cost \$10 per hundred; and boards cost \$30 per M. The two rails on which to nail the boards are 2″ × 4″ × 16′ and cost \$26 per M.

21. What will it cost to build a picket fence 24 ft. long with pickets 3 in. wide and placed 3 in. apart, at 5 cents each, and 2 rails for near the top and bottom 12 ft. long. 2″ × 4″ at \$24 per M. and posts at 25 cents each, 8 ft. apart.

22. Find the cost of the lumber to build a fence 48 ft. long, and four boards high; the bottom board being 10 in. wide, and the other boards 6 in. wide and posts placed 8 ft. apart? The boards cost \$50 per M. and the posts cost 25 cents each.

23. Find the cost of the framework of a building at \$50 per M., if the following amounts of lumber are used: 120 running feet of 6″ × 8″ sills; 900 running feet of 10″ × 3″ joists; 120 running feet of 4″ × 8″ beams; 130 running feet of 5″ × 5″ posts; 650 running feet of 3″ × 6″ rafters.

TIMBER MEASURE

157. Find the number of cubic feet in a log 24 in. in mean diameter and 90 ft. long. If this log were a cylinder

of uniform diameter, 24 in. and 90 ft. long, the cubic contents would be area of the end, 3.14 ft. multiplied by 90, equals 282.60 cu. ft. If the log is to be squared and the number of cubic feet remaining be calculated, there must be an allowance made for the slabs. There are several rules for this kind of work, but none are accurate. Thus, find the number of cubic feet of square timber in a log 16 in. in mean diameter and 18 ft. long.

158. Rule: *Multiply four ninths of the square of the number of inches in the diameter by the number of feet in length and divide the product by 144. Or, use* $\frac{1}{2}$ *instead of* $\frac{4}{9}$.

$$\frac{\frac{4}{9}\times(16)^2\times 18}{144} = 14\tfrac{2}{9}.$$

Or, multiply the square of the diameter in inches by the length in feet, take $\frac{1}{3}$ *of the product and point off two figures.*

$$\tfrac{1}{3}\times(16)^2\times 18 = 15.36.$$

Find the number of board feet of inch boards that can be sawed from a log 24 in. in diameter, and 16 ft. long.

159. Doyle's Rule: *From the diameter in inches subtract 4, square the remainder, divide by 16, and multiply by the length in feet.*

$$(24 - 4)^2 = \frac{20\times 20\times 16}{16} = 400.$$

Find the number of board feet that can be sawed from a log 24 in. in diameter, and 12 ft. long.

$$(24 - 4)^2 = \frac{20\times 20\times 12}{16} = 300.$$

PROBLEMS

1. How many square feet of inch boards can be cut from a log 25 in. in diameter and 16 ft. long?

2. How many square feet of inch boards can be cut from a log 47 in. in diameter and 16 ft. long?

3. How many cubic feet of squared timber can be cut from a log 34 in. in diameter and 36 ft. long?

4. How many cubic feet of squared timber in a log 36 ft. long, and 20 in. in diameter, if $\frac{1}{2}$ is used for $\frac{4}{9}$ in the rule?

5. How many square feet of inch boards can be cut from a log 16 in. in diameter and 18 ft. long?

6. How many square feet of inch boards can be cut from a log 12 in. in diameter and 24 ft. long?

7. How many square feet of inch boards can be cut from a log 24 in. in diameter and 30 ft. long?

8. How many cubic feet in a log 40 in. in mean diameter and 18 ft. long?

9. How many cubic feet of squared timber in a log 32 in. in diameter and 30 ft. long?

10. How many square feet of inch boards in a log 32 ft. long and 16 in. in diameter?

11. How many square feet of inch boards can be cut from a log 12 ft. long and 8 in. in diameter?

12. How many cubic feet of squared timber can be cut from a log 14 ft. long and 12 in. in diameter?

13. How many square feet of inch board can be cut from a log 16 ft. long and $8\frac{1}{2}$ in. in diameter?

14. How many cubic feet in a log 18 ft. long and 6 in. in diameter?

15. How many board feet of lumber can be cut from 20 logs, each 14 ft. long and averaging 10 in. in diameter?

160. LAND AREA

144 sq. inches	= 1 sq. foot.		16 sq. rods	= 1 sq. chain.	
9 sq. feet	= 1 sq. yard.		10 sq. chains	= 1 acre.	
$30\frac{1}{4}$ sq. yards	= 1 sq. rod.		640 acres	= 1 sq. mile.	
160 sq. rods	= 1 acre.		7.92 inches	= 1 link.	
640 acres	= 1 sq. mile.		100 links	= 1 chain.	
36 sq. miles	= 1 township.		80 chains	= 1 mile.	

PROBLEMS

1. How many acres in a rectangular piece of land 40 rods by 80 rods?

2. How many acres in a piece of land 120 rods by 40 rods?

3. A road is ordinarily 4 rods wide. How many miles of it will make 40 acres?

4. How many square miles in Chicago, which contains 118,233 acres?

5. How many acres in the continental United States, which contains 3,026,789 square miles? How many cities the size of Chicago could it contain?

6. What will it cost to fence a square 40-acre piece of land with a 4-wire fence at $2.50 per hundred?

7. A rectangular piece of land measures 500 chains by 200 chains. How many acres does it contain?

8. How many acres in a piece of land 4 miles square?

9. How many pounds of water will be required to mature 20 acres of wheat, 40 bu. to the acre, if 400 lbs. of water is required to mature 1 lb. of wheat? (1 bu. of wheat = 60 lbs.)

10. How many pounds of water to mature 10 acres of corn, if 300 lbs. of water is required for 1 lb. of corn, and the field yields 80 bu. to the acre? (1 bu. of corn = 56 lbs.)

11. What is the number of acres in a rectangular field whose length is 2,640 ft. and whose width is 6,950 ft.?

12. How many acres in a field 80 chains long and 26.25 chains wide?

13. How many rows of corn 3 ft. 6 in. apart can be planted in a field 40 rods wide? How many hills of potatoes 3 ft. 6 in. apart will there be in the field, if it is 120 rods wide?

14. A cotton plantation of 500 acres yields 250 lbs. of cotton to the acre. How many 500-lb. bales are produced? What is this cotton worth at 10 cents per lb.?

15. A railroad right of way is 1,226.8 ft. long and 100 ft. wide. What is it worth at $40.00 per acre?

16. How many acres are there in a piece of land 4 miles wide by 4 miles long? How many rods in the distance around this piece of land?

17. In every township the Government sets aside one section, number 16, for school purposes. How many acres in this section? How many rods around this section? A section is how many miles wide? How many miles long?

18. How many acres in a tract of land 11,000 ft. long by 2,000 ft. wide?

11,000 × 2,000 sq. ft. = 22,000,000 sq. ft. 1 acre = 43,560 sq. ft. Divide 22,000,000 sq. ft. by 43,560 or multiply by .000,023. 22,000,000 × .000,023 = 506 acres.

19. How many acres in a township of land?

20. How many acres in a city lot whose frontage is 70 ft. and depth 120 ft.?

21. What will it cost to put a tile drain through the center of a city lot from back to front, if the lot is 120 ft. deep and the cost for digging is 55 cents per rod, and $23.00 a thousand for tile, each tile being a foot long?

22. The side of a rectangular lot is 40 rods, and the lot contains 16½ acres. How wide is it?

23. How many apple trees 20 ft. apart may be planted in a 4-acre tract in the form of a square?

24. What will it cost to inclose a 40-acre lot with woven wire fence at 40 cents per rod?

25. A field is 80 rods long and 40 rods wide. How many rods of fence will be required to divide the field into two equal squares? How many acres in the field?

PAPER

161. 24 sheets of paper = 1 quire. 20 quires = 1 ream, or 500 sheets = 1 ream. 480 sheets is the exact number in one ream.

162. Paper size.

14″ × 17″	called	Cap size.
16″ × 21″	"	Demy "
17″ × 22″	"	Folio "
17″ × 28″	"	Double Cap.
18″ × 23″	"	Medium size.
19″ × 24″	"	Royal "
20″ × 28″	"	Super Royal.
16″ × 42″	"	Double Demy, narrow.
21″ × 32″	"	Double Demy, broad.
18″ × 46″	"	Double Medium, narrow.
23″ × 36″	"	Double Medium, broad.
24″ × 38″	"	Double Royal.

Paper for book sizes, 24 × 36; 25 × 38; 28 × 42; 32 × 44.

Paper marked, 19″ × 24″ – 24 = Royal, weighing 24 lbs. to the ream.

Paper marked, 16″ × 21″ – 18, or 20 or 22 = Demy weighing 18 lbs. or 20 lbs. or 22 lbs. to the ream. Paper marked 17″ × 22″ – 20, or 22, or 24 = Folio, weighing 20, or 22, or 24 lbs. to the ream.

PROBLEMS

1. I wish to cut 20 lbs. of paper $8\frac{1}{2}''$ × 11″. What size of paper should I use? How many reams of 16 lb. paper will be necessary?

2. How many letterheads, each $8\frac{1}{2}''$ × 11″ can be cut from $\frac{1}{2}$ ream of folio paper?

3. From what size of paper can billheads $8\frac{1}{2}''$ × 4″ be cut with the least waste?

4. One ream of a medium weight 24″ × 36″ paper weighs 30 lbs. What would a ream of the same paper weigh, if it were 28″ × 42″? 32″ × 44″?

6. A book contains 250 pages 5″ × 7½″. What book paper will cut to the best advantage? How many reams will be required for 10,000 copies?

7. How many pieces of 4″ × 8½″ paper can be cut out of Royal size paper?

THE METRIC SYSTEM

163. Long Measure.

10 millimeters (mm.)	= 1 centimeter	(cm.)
10 centimeters	= 1 decimeter	(dm.)
10 decimeters	= 1 meter	(m.) = 39.37 inches.
10 meters	= 1 dekameter	(Dm.)
10 dekameters	= 1 hektometer	(Hm.)
10 hektometers	= 1 kilometer	(Km.)

164. Square Measure.

100 sq. millimeters (sq. mm.)	= 1 sq. centimeter (sq. cm.)
100 sq. centimeters	= 1 sq. decimeter (sq. dm.)
100 sq. decimeters	= 1 sq. meter (sq. m.)
100 sq. meters	= 1 sq. dekameter (sq. Dm.)
100 sq. dekameters	= 1 sq. hektometer (sq. Hm.)
100 sq. hektometers	= 1 sq. kilometer (sq. Km.)

165. Cubic Measure.

1000 cu. millimeters (cu. mm.)	= 1 cu. centimeter (cu. cm.)
1000 cu. centimeters	= 1 cu. decimeter (cu. dm.)
1000 cu. decimeters	= 1 cu. meter (cu. m.)
1000 cu. meters	= 1 cu. dekameter (cu. Dm.)
1000 cu. dekameters	= 1 cu. hektometer (cu.Hm.)
1000 cu. hektometers	= 1 cu. kilometer (cu. Km.)

166. Measure of Weight.

10 milligrams (mg.)	= 1 centigram (cg.)
10 centigrams	= 1 decigram (dg.)
10 decigrams	= 1 gram (g.)
10 grams	= 1 dekagram (Dg.)
10 dekagrams	= 1 hektogram (Hg.)
10 hektograms	= 1 kilogram (Kg.)

167. Measure of Capacity.

10 milliliters (mi.)	=	1 centiliter (cl.)
10 centiliters	=	1 deciliter (dl.)
10 deciliters	=	1 liter (l.)
10 liters	=	1 dekaliter (Dl.)
10 dekaliters	=	1 hekoliter (Hl.)
10 hekoliters	=	1 kiloliter (Kl.)

Deci = $\frac{1}{10}$ centi = $\frac{1}{100}$ milli = $\frac{1}{1000}$
deka = 10 hekto = 100 kilo = 1,000
myria = 10,000.

1 meter = 39.37 inches;

1 liter = 1.057 liquid quarts;

1 gram = 15.432 grains;

1,000 kilograms = 1 metric ton = the weight of 1 cu. m. of water;

1 hektare = 2.5 acres.

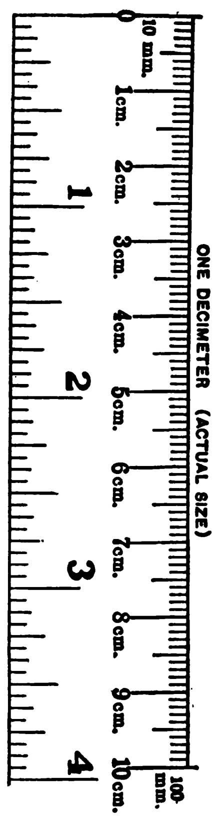

Figure 3

PROBLEMS

1. If a cubic meter of marble weighs 6 metric tons, what is the weight of a block 5.5 m. long, 3.5 m. wide and 4.2 m. thick?

2. How many cubic meters of earth are there in an excavation 8 m. long, 8 m. wide and 4 m. deep? One meter = how many lengths of Figure 1?

3. How many cubic meters are there in a bin 10 m. long, 2 m. high and 4 m. wide?

4. How many steres of wood are there in 2 cords of wood? (1 stere = 1.31 cu. yds.)

5. How many liters of water are there in a tank 4 dm. by 22 dm. by 8 dm.?

6. How many liters are there in 24 kl. of water?

7. A cubic decimeter of water weighs how many grams? (1 cu. cm. weighs 1 g.)

8. A cubic foot of water weighs 1,000 oz. How many grams does it weigh?

1 kilogram = 2.2 lbs.; 1 metric ton = 1.1 tons; 1 liter = .91 dry qt.

9. A cylindrical tank of radius .8 m. is filled with water to the depth of 4.5 m. How many liters of water does it contain?

10. A tank holding 10 cu. m. of water weighs how many metric tons?

11. What will it cost to make a cement sidewalk 1.245 km. long, 4 m. wide at 70 cents a square meter?

12. How many square meters are there in the floor of a room 8 m. long and 6 m. wide? How many lengths of Figure 3 = 8 m.?

13. What will it cost to calcimine the ceiling of a room $8\frac{1}{2}$ meters long by 4 meters wide at 40 cents a square meter?

14. A cu. dm. of water weighs 2.2 lbs. What is the weight of the water in a tank containing 6 cu. m.? How many cu. dm.?

RATIO

168. The quotient produced by dividing one number by another of the same kind is the ratio of the first number to the second; or, ratio is the relation of one number to another in regard to their magnitudes.

The ratio of the number 10 to 5 is 10 ÷ 5 = 2. Or, 2 is the ratio of 10 to 5. That is, 10 is 2 times 5. The ratio of 12 to 3 is 12 ÷ 3 = 4. Or, 4 is the ratio of 12 to 3. That

is, 12 is 4 times 3. The ratio of 3 to 6 is $3 \div 6 = \frac{1}{2}$. Or, $\frac{1}{2}$ is the ratio of 3 to 6. That is, 3 is $\frac{1}{2}$ of 6.

Ratio is expressed by the colon (:), or by the sign of division ($\div$). The ratio of 10 to 5 is expressed 10 : 5, or, $10 \div 5$; or, in the form of a fraction, $\frac{10}{5} = 2$.

The first term is called the antecedent and the second term is called the consequent. The number 10 is the antecedent and 5 is the consequent.

To find the ratio of 3 pecks to 6 bushels, or any numbers of different kinds, we must first make them of the same kind. In this example we make them both pecks or both bushels. Thus: 6×4 pecks = 24 pecks. The ratio of 3 pecks to 24 pecks is $3 : 24 = 3 \div 24 = \frac{3}{24} = \frac{1}{8}$.

PROBLEMS

1. One bin contains 20 bu. of potatoes and another 5 bu. What is the ratio of the number of bushels in the large bin to the number of bushels in the small bin?

2. A pound of steak costs 18 cents and a pound of pork costs 26 cents. What is the ratio of the price of steak to that of pork?

3. What is the ratio of 8 hours to one day?

4. A circle 2 ft. in diameter has a circumference of $6\frac{2}{7}$ ft. What is the ratio of the diameter to the circumference?

5. One door is 6 ft. 6 in. by 2 ft. 6 in., another door is 6 ft. 8 in. by 3 ft. What is the ratio of the area of the first to that of the second? (Find the areas and then the ratio of these results.)

6. One line is one rod long; another is $8\frac{1}{4}$ ft. long. What is the ratio?

7. Divide $20 between A and B in the ratio of 2: 3. When A receives $2, B receives $3, then they both receive $5, but $20 is 4 times $5, the sum. Therefore, 4 times each

of the numbers which go to make this sum, as $4 \times \$2 = \$8$ and $4 \times \$3 = \$12$ are the amounts respectively for A and B.

8. A board 18 ft. long is to be cut into two pieces in the ratio of 2 : 1. What will be the length of each piece?

9. Divide 2,700 in the ratio of 7 : 2.

10. Divide \$9,000 between A and B in the ratio of 4 : 5. What will each receive?

$4 + 5 = 9$. $\$9,000 \div 9 = \$1,000$. $4 \times 1,000 = \$4,000$, A's share. $5 \times 1,000 = \$5,000$, B's share.

What per cent will each receive?

$4:5 = \frac{4}{5} = \frac{80}{100} = .80$. $.80 + 1.00 = 1.80$. $\$9,000 \div 1.80 = \$5,000$. $.80 \times 5,000 = 4,000$, A's; $1.00 \times 5,000 = 5,000$, B's.

11. If bell metal contains 25 parts of copper to 11 parts of tin, what weight of each metal in a bell weighing 1,080 lbs.?

12. The heating surface of a locomotive boiler is 1,880 sq. ft. and the grate surface is 10 ft. by 8 ft. What is the ratio of the heating surface of the boiler to the grate surface?

PROPORTION

169. Proportion is the expression of equality between ratios; or, two equal ratios form a proportion.

Direct Proportion. The ratio of 3 : 9 is $\frac{3}{9} = \frac{1}{3}$. The ratio of 2 : 6 is $\frac{2}{6}$ which is also $\frac{1}{3}$. This is expressed $\frac{3}{9} = \frac{2}{6}$, or 3 : 9 :: 2 : 6. A double colon is used for the sign of equality. Thus, 3 : 9 :: 2 : 6, is read, three is to nine as two is to six. The first and the last terms are the *extremes* and the second and third terms are the *means*.

The product of the extremes equals the product of the means.

2 : 3 :: 10 : 15. $2 \times 15 = 3 \times 10$. $30 = 30$.

170. To find a missing term, *multiply together the two terms of either the extremes or the means, and divide by the one extreme or mean given.*

2 : 3 :: ? : 15. $2 \times 15 = 3 \times ?$. $2 \times 15 = 30$. $30 \div 3 = 10$.

A staff 10 ft. long casts a shadow 12 ft. long. What is the height of a steeple which casts a shadow 180 ft. long at the same time? The first and second terms are like terms,—staff and steeple, or shadows of them; and the third and fourth terms are like terms. We may begin with staff and steeple. We reason thus: the height of the staff: the height of the steeple :: the length of the shadow of the staff: the length of the shadow of the steeple. Or, begin with shadows; the length of the shadow of the staff : the length of the shadow of the steeple :: the height of the staff : the height of the steeple.

Any proportion may be stated in four ways; 3 : 8 :: 9 : 24. 8 : 3 :: 24 : 9. 24 : 9 :: 8 : 3. 9 : 24 :: 3 : 8.

If 50 lbs. of sugar cost $1.75, what will 200 lbs. cost? The more sugar, the more the cost.

$$50 \text{ lbs.} : 200 \text{ lbs.} :: \$1.75 : ?.$$

$$50 \times ? = 200 \times 1.75 = \frac{200 \times 1.75}{50} = \$7.$$

171. Inverse Proportion. If 17 men require 81 days to do a job of work, how long will it take 51 men to do it? The more men we have the less time they require to do it.

This is an inverse proportion, and we make the proportion thus:

$$17 \text{ men} : 51 \text{ men} :: ? : 81. \quad \frac{\overset{27}{\cancel{81}} \times \cancel{17}}{\underset{3}{\cancel{51}}} = 27 \text{ days}.$$

172. Partitive Proportion. Divide 270 into parts proportional to 2, 3 and 4. If the parts were 2, 3, and 4 the whole number would be $2 + 3 + 4 = 9$. The ratio of 9, the whole, to each would be 9 : 2, 9 : 3, and 9 : 4. Therefore, since 270 is the whole, we have 9 : 2 :: 270 : ? (1st.). 9 : 3 :: 270 : ? (2nd.). 9 : 4 :: 270 : ? (3rd.). The parts are 60, 90 and 120 respectively.

PROBLEMS

1. A locomotive goes 3 miles in 4 minutes. How far will it go in 60 minutes?

2. If it requires 40 yds. of carpet to cover a floor when the carpet is 3 ft. wide, how many yards will it require, if the carpet is $\frac{3}{4}$ yd. wide?

3. A beam $10' \times 8'' \times 6''$ weighs 950 lbs. What will another beam of the same material weigh, if it is $12' \times 12'' \times 8''$?

4. If it takes 9 men 4 days to do a piece of work, how many days will it take 18 men to do the same work?

$$\left.\begin{matrix} 9 \text{ men} : 18 \text{ men} \\ 4 \text{ days} : ? \text{ days} \end{matrix}\right\} :: 1 \text{ (the work)} : 1 \text{ (the work)}$$

$$1 \times 9 \times 4 = 36. \quad 1 \times 18 \times ? = 18 \times ?$$
$$18 \times ? = 36. \quad ? = 2.$$

5. A farmer had 42 acres in wheat. A six-acre field averaged 18 bu. per acre. If all produced at the same rate, how many bushels did he receive from the whole crop.

6. Gunpowder is composed of $\frac{1}{8}$ lb. of sulphur, $\frac{3}{4}$ lb. of niter, and $\frac{1}{8}$ lb. of charcoal in one pound of powder. How many pounds of each are contained in 200 lbs. of powder?

7. If a tree casts a shadow 60 ft. long when a post 5 ft. high casts a shadow 8 ft. long, how high is the tree?

8. If the weekly payroll of a manufacturing establishment is $1,280, how much does this firm pay out for labor in one year?

9. Pig iron is 93% pure iron, with small amounts of carbon, sulphur, silicon, phosphorus, etc. How many pounds of pure iron in 8,000 lbs. of pig iron?

10. A bin 9 ft. $\times$ 6 ft. $\times$ 9 ft. holds 320 bu. of wheat. How many bushels will a bin hold that is 16 ft. $\times$ 8 ft. $\times$ 10 ft.?

MISCELLANEOUS REVIEW PROBLEMS

1. A locomotive has a weight of 204,000 pounds, and 63% of the total weight is on the driving wheels. How many pounds weight is on the drivers?

2. A lathe operating on ordinary tool steel runs at 250 revolutions per minute. With high speed steel, the rate of cutting was increased 35%. How many R. P. M. was the lathe running after the change was made?

3. If a company received 580 castings and 29 of these were rejected as defective, what per cent were rejected?

4. An apprentice's rate is 20 cents per hour, which is 40% of a journeyman's rate. What is a journeyman's rate?

5. If a boiler were working at a pressure of 160 lbs. per sq. in., and the pressure were increased to 200 lbs. per sq. in., what would be the per cent of increase?

6. Find the ratio of the cylinder diameter to the rod diameter in the following instance: Diameter of cylinder 20.5″, diameter of rod 3.652″.

7. If a mixture has copper 4 parts, lead 3 parts, and tin 1 part, how many pounds of each will it take to make a 72-lb. casting?

8. If a worm runs 280 turns a minute and the worm wheel 14 turns a minute, what is the ratio of the reduction in speed?

9. The weight of a freight car consisting of the weight of car and trucks is 42,600 lbs. The weight of the freight carried is 108,000 lbs. What is the ratio of the weight of the load to the weight of the car and trucks?

10. On a 36″ planer, the ratio of the cutting speed to the return speed of the table is 1 to 2.94. With a cutting speed of 50 ft. per minute, what is the return speed per minute?

11. A T-rail is 30 ft. long and weighs 85 lbs. to the yard. What will each rail cost at $30.00 per ton of 2,240 lbs.?

12. A girder-rail is 60 ft. long and weighs 110 lbs. to the yard. What will it cost at $40.50 per ton of 2,240 lbs.?

13. If the weight of a bar of round iron 1″ in diameter and of a certain length is 13.35 lbs., what is the weight of a bar 2″ in diameter of the same length?

Similar solids are to each other as the cubes of their like dimensions.

14. An oil tank whose capacity is 60 gallons is being filled by a 1″ pipe at the rate of $1\frac{1}{3}$ cu. ft. per minute. How long will it take to fill the tank? (One gallon contains 231 cu. in.)

15. A ball 3 inches in diameter weighs 7 pounds, what is the weight of a ball 5 inches in diameter?

16. How many pounds of nickel must be added to 8,400 lbs. of steel to have the mixture contain 3.5% of nickel?

17. Nickel-chrome steel contains .45 per cent of chromium and 1.25 per cent of nickel. How much of each is contained in 10 tons of nickel-chromium?

18. If steel contains 1.2 per cent of carbon, how many pounds of carbon are contained in 1,000 lbs. of steel?

19. How many pounds of nickel are contained in 20 tons of nickel-steel, if 3.40 per cent is nickel?

20. A bar of round iron 2 in. in diameter and of a certain length weighs 42.5 lbs., what is the weight of a bar 3 in. in diameter and of the same length?

21. How many gallons of water are contained in a tank whose dimensions are 12 ft. by 24 ft. by 8 ft.?

22. What sum of money at 4% will produce a yearly income of $1,600?

23. Peach trees are planted 20 ft. apart each way. How many will it take to fill a square field of 10 acres?

24. Apple trees are planted 30 ft. apart each way. How many will it take to fill a square field of $22\frac{1}{2}$ acres?

25. How many strawberry plants can be set on $5\frac{3}{8}$ acres of land, if the plants are set in rows 4 ft. apart and 2 ft. apart in the rows?

26. A dairyman has 20 cows, each on an average giving 3 gal. of milk per day, weighing $8\frac{5}{8}$ lbs. per gallon, and the milk testing 3.5 per cent butter-fat. How much butter-fat does the dairyman procure in 20 days?

27. A cow gives 32 lbs. of milk per day, testing 3.2 per cent butter-fat. How much butter-fat does she produce per month of 30 days?

28. A cow gave in one year 12,500 lbs. of milk. This produced 800 lbs. of butter-fat. What was the per cent of butter-fat?

29. A cow gave in one year 26,000 lbs. of milk, containing 1,200 lbs. of butter-fat. If the butter was $1\frac{1}{8}$ times the weight of the butter-fat, how many pounds of butter were furnished in one year?

30. If 24 acres of cotton yield 18,000 lbs. of seed cotton, and 1,500 lbs. of seed cotton make a bale of lint, how many bales of lint do the 24 acres make?

31. What is the value of a load of seed cotton weighing 2,000 lbs. when lint is selling at 12 cents per pound, and seed at $18 per ton? 3 lbs. of seed cotton = 2 lbs. of seed and 1 lb. of lint.

32. A pound of cotton will make 170 spools of No. 40 sewing thread. If the thread sells at 4 cents per spool, how much is received from one bale of cotton of 470 lbs. per bale?

33. A coffee tree yields 6 lbs. of coffee per year. The life of the tree is 25 years. What is the production of 20 coffee trees during 25 years and the value of the coffee at 15 cents per pound?

EQUATIONS

173. An *equation* is an expression of equality between two quantities.

174. The *members* of an equation are the two parts, one on either side of the equality sign.

175. The *terms* of an equation are the parts of the members connected by the addition or subtraction sign.

176. In solving equations, ordinary numbers, as 1, 2, 3, etc., are called known numbers, and unknown numbers are represented by letters of the alphabet, usually from the latter part such as, x, y, z., etc.

177. In solving equations it is customary to get all of the known numbers on the right-hand side of the equality sign, i.e., into the *right-hand member* and the unknown numbers into the *left-hand member* of the equation. In order to do this, various mathematical processes are employed.

178. If you will think of an equation as representing a balance, the *right-hand member* being in one scale pan and the *left-hand member* being in the other scale pan with the sign of equality as a fulcrum, the truth of the following axioms will be apparent, the object being always to preserve the balance of the equation.

(1) The same number may be added to both members of an equation without destroying its equality.

(2) The same number may be subtracted from both members of an equation without destroying its equality.

(3) Both members of an equation may be multiplied by the same number without destroying its equality.

(4) Both members of an equation may be divided by the same number without destroying its equality.

179. In the equation, $x + 8 = 11$, point out the left-hand member, the right-hand member, the known numbers, the unknown numbers, the terms.

180. In the equation $x + 8 = 11$, what must be done in order to leave the x alone in the left-hand member, i. e., which axiom should we apply? If we subtract 8 from both members but merely indicate the subtraction in the right-hand member we have, $x = 11-8$. Notice that the 8 does not now appear in the left-hand member but appears in the right-hand member but with the sign $(-)$ instead of $(+)$ as in the original equation. Observe this change in sign in the following:

$$\begin{array}{lrcl} & x - 4 & = & 12 \\ \text{Add } 4 \ldots\ldots & +4 & & +4 \\ \hline & x & = & 12 + 4 \end{array}$$

181. The quickest way to get a term out of one member of an equation, then, is to change its sign and place it in the other member of the equation. This process is called *transposition*. This is equivalent to adding or subtracting this term to or from both members.

182. The steps to be taken in the solution of a simple equation are illustrated in the following:

Find the value of x in, $5x - 16 = 26 - x$.

Solution:

$$\begin{array}{lrcl} & 5x - 16 & = & 26 - x \\ \text{Transposing} \ldots\ldots\ldots\ldots\ldots & 5x + x & = & 26 + 16 \\ \text{Uniting terms in each member} & 6x & = & 42 \\ \text{Dividing both members by 6} & x & = & 7 \end{array}$$

183. Signs. The sign, whether $+$ or $-$ to be given to, the result, is the sign of the greater quantity. In the expression $(5x - 3x)$, the greater quantity is $5x$, and the sign is $+$, therefore; we have written $2x$. (When no sign is written $+$ is understood).

In multiplication and division, like signs give $+$ and unlike signs give $-$. $+y$ times $+z = +yz$. The two quantities y and z have like signs (both are $+$). The product takes the $+$ sign.

$-y$ times $-z = +yz$. The two quantities y and z

have like signs (both $-$) and the product takes the $+$ sign.

$+y$ times $-z = -yz$. The two quantities y and z have unlike signs (one $+$ and the other $-$). The product takes the $-$ sign.

Multiply $3x + 2$ by $2x - 3$.

$$
\begin{array}{rrr}
 & 3x & +\ 2 \\
 & 2x & -\ 3 \\
\hline
 & -\ 9x & -\ 6 \\
6x^2 & +\ 4x & \\
\hline
6x^2 & -\ 5x & -\ 6
\end{array}
$$

-3 times $+2$ is -6. (Unlike signs and take $-$).

-3 times $+3x$ is $-9x$. (Unlike signs and take $-$).

$+2x$ times $+2$ is $+4x$. (Like signs and take $+$).

$+2x$ times $+3x$ is $+6x^2$. (Like signs and take $+$).

In the ($2x$ times $3x$), the x times x is x^2, and $2x$ times $3x$ is $6x^2$. Add, and $6x^2$ is written. $-9x$ and $+4x$ is $-5x$, (the sign of the greater quantity $-9x$); hence the result is $-5x$. The final result is $6x^2 - 5x - 6$.

Multiply $2x - 4$ by $x + 3$.

$$
\begin{array}{rrr}
 & 2x & -\ 4 \\
 & x & +\ 3 \\
\hline
 & 6x & -12 \\
2x^2 & -\ 4x & \\
\hline
2x^2 & +\ 2x & -12
\end{array}
$$

$+3$ times -4 is -12. (Unlike signs give $-$).

$+3$ times $+2x$ is $+6x$. (Like signs give $+$).

$+x$ times -4 is $-4x$. (Unlike signs give $-$).

$+x$ times $+2x$ is $+2x^2$. (Like signs give $+$).

EXERCISES

Find the product of the following:

1. $(x - 6), (x - 7)$.
2. $(y - 8), (y + 6)$.
3. $(3x - 5), (4x + 6)$.
4. $(5x - 7), (5x - 6)$.
5. $(m + n), (m - n)$.
6. $(a + b), (a + b)$.
7. $(a - b), (a + b)$.
8. $(a^2 - b), (a^2 + b)$.
9. $(y^2 - 7), (y^2 + 6)$.
10. $(y^2 + y + 1), (y - 1)$.

184. Coefficients. In addition and subtraction the coefficients of quantities are added or subtracted. The coefficients are written before the quantities, as, $3x$, $5x$, etc. The numbers 3, 5, etc., are the coefficients and show how many of the quantities have been added or subtracted. $3x$ means $x + x + x$. $5x = x + x + x + x + x$. $3x$ and $5x$ added $= 3x + 5x = 8x$. $3x$ subtracted from $5x = 5x - 3x = 2x$.

185. Exponents. $x^2 = xx$, or x times x. $x^4 = xxxx$, or x times x times x times x.

$y^5 = yyyyy$. But x^2, x^4, y^5, etc., are used instead of repeating the letters.

The numbers 2, 4, 5 and such numbers written above and to the right of the letters x and y, etc., show that the letters have been taken 2, 4 and 5, etc. times as factors. These numbers are called *exponents*.

An exponent shows how many times a quantity has been used as a factor.

$$3y^2 = 3 \times y \times y. \quad (3y)^2 = 3y \times 3y = 9y^2.$$
$$5x^3 = 5 \times x \times x \times x.$$

When there is no exponent written, 1 is understood.

In multiplication exponents of like quantities are added. In division exponents of like quantities are subtracted, or, the exponent of the divisor is subtracted from the exponent of the dividend. Thus $x^2 \times x^3 = x^5$, which is x taken 5

times as a factor. And in division, $x^5 \div x^2 = x^{5-2} = x^3$, which is x times x times x times x times x − x times x; or x taken 5 times as a factor less x taken 2 times as a factor = x taken 3 times as a factor.

EXERCISES

Find the value of x in the following:

1. $10 - 2x = 30.$
2. $x - 7 = 17.$
3. $x - 4 - 6 = 7.$
4. $x + \frac{1}{2} = 4.$
5. $4x - 3x = 8.$
6. $2x + 3x = 45.$
7. $9x + 2x - 5x = 48.$
8. $6x - 2x - 3x = 16.$
9. $7x + 3x - x - 2x = 32 + 3.$
10. $8 + 2x + 4x - 5x = 12 - 4.$

Find the value of x when $\frac{2x}{3} = 8.$

Multiply both members by 3.

$$\frac{2x}{3} \times 3 = 2x. \quad 8 \times 3 = 24. \quad 2x = 24. \quad x = 12.$$

Find the value of x, when $\frac{2x}{3} + \frac{3x}{4} = \frac{1}{2} + \frac{8}{1}.$

We must multiply both numerator and denominator of each fraction by numbers that will make all the denominators alike, or reduce them all to the same denomination. We may reduce all these fractions to 12ths. Multiply both terms of the first fraction by 4, of the second by 3, of the third by 6, and of $\frac{8}{1}$, expressed as a fraction by 12. We have

$$\frac{2x \times 4}{3 \times 4} = \frac{8x}{12}. \quad \frac{3x \times 3}{4 \times 3} = \frac{9x}{12}. \quad \frac{1 \times 6}{2 \times 6} = \frac{6}{12}. \quad \frac{8 \times 12}{1 \times 12} = \frac{96}{12}.$$

$$\frac{8x}{12} + \frac{9x}{12} = \frac{17x}{12}. \quad \frac{6}{12} + \frac{96}{12} = \frac{102}{12}. \quad \frac{17x}{12} = \frac{102}{12}.$$

We drop the 12, or divide the denominators by 12.

$$17x = 102. \quad x = 6.$$

Find the value of the unknown quantity in the following:

1. $\frac{x}{3} + \frac{2x}{3} = 32.$

2. $\frac{y}{2} - \frac{y}{5} = 6.$

3. $\frac{4y}{5} - \frac{2y}{9} + \frac{3y}{4}$ 239.

4. $\frac{5z}{9} + \frac{2z}{3} - \frac{z}{2} = 52.$

5. $\frac{x}{2} + \frac{2x}{3} = 28.$

6. $\frac{2y}{3} + \frac{3y}{4} = 102.$

7. $\frac{4y}{5} - \frac{2y}{3} = 48.$

8. $y + 2y + \frac{3y}{7} = 24.$

9. $2\frac{3}{4}y + 2y - \frac{2y}{2} = ?$; if $y = 4$.

10. $3\frac{1}{2}x - 4\frac{3}{4}x + 8x = ?$; if $x = 24$.

PROBLEMS

1. If a number is increased by 24 the result is 38. What is the number?

$$\text{Let } x = \text{the number.} \quad \text{Then } x + 24 = 38.$$
$$x = 38 - 24.$$
$$x = 14.$$

2. Divide 40 into two parts one of which shall be 4 times the other. What are the parts?

$$\text{Let } x = \text{one part.} \quad \text{Then } x + 4x = 40.$$
$$5x = 40.$$
$$x = 8.$$
$$\text{Proof, } 8 + 32 = 40.$$

3. After giving away $\frac{3}{8}$ of his money and losing $\frac{1}{4}$ of it, John had \$15 left. How many dollars had he at first?

Let x = the number of dollars he had at first. $x - (\frac{3}{8}x + \frac{1}{4}x) = \$15$. To clear the equation of fractions, multiply each term by the least common denominator of the fraction. 8 is the least common denominator. 8 times x is $8x$: 8 times $\frac{3x}{8}$ is $3x$: 8 times $\frac{x}{4}$ is $2x$: 8 times 15 is 120. We

have $8x - (3x + 2x) = 120$. A minus sign before a parenthesis shows that the quantity in the parenthesis is to be subtracted. We subtract in Algebra by changing the sign of the subtrahend, and adding quantities with like signs and subtracting quantities with unlike signs. To remove a parenthesis or write the quantity without a parenthesis, change the signs of all the quantities within the parenthesis. This equation written without a parenthesis is $8x - 3x - 2x = 120$. Thus $3x = 120$. $x = 40$. He had \$40.00 at first. He gave away and lost \$15 and \$10, or \$25. \$40 − \$25 = \$15. He had left \$15.

4. The difference between $\frac{3}{4}$ of a number and $\frac{3}{5}$ of it is 15. What is the number?

5. Find two numbers whose difference is 4 and whose quotient is 96.

6. Eight times a number less 27 equals six times the number. What is the number?

7. A certain number is multiplied by 15; 30 is subtracted, and 8 times the number is then subtracted, and 8 is added. The result is 20. What is the number?

8. Three eighths of what number is 45 less than the number itself?

9. Find the value of x in $5x - 6\,(2x - 5) = -40$.

10. Find the value of x in $\dfrac{3x - 6}{2} - \dfrac{4x - 4}{5} = 62$.

(Clear fractions, $15x - 30 - (8x - 8) = 62$.)

11. Find the value of x in $x = 12\,\dfrac{(6 + 6)}{2} - 12$.

12. Find the value of x in $2x = 8\,\dfrac{(12 + 4)}{2} - (6 + 2)$.

186. Two unknown quantities.

1. $9x + 7y = 41$. When $x = 3$, find the value of y.

When $x = 3$, $9x$ will equal $9 \times 3 = 27$, and $27 + 7y = 41$. $7y = 41 - 27$. $7y = 14$; $y = 2$.

2. $12x + 6z = 78$. When $x = 4$, find the value of y.

EXERCISES

Find the value of the unknown quantity:

$$\begin{array}{rcll} 3x + 5y &=& 25 & (1) \\ 4x + 3y &=& 26 & (2) \\ \hline 12x + 20y &=& 100 & (3) \\ 12x + 9y &=& 78 & (4) \\ \hline 11y &=& 22 & \\ y &=& 2 & \\ 3x + 10 &=& 25 & \\ 3x &=& 15 & \\ x &=& 5 & \end{array}$$

We multiply one or both of these equations by some number or numbers to make the number of x's, or the number of y's the same in each equation. Multiplying (1) by 4, and (2) by 3, will give $12x$ in each. Subtracting (4) from (3) gives $11y = 22$; and $y = 2$. If $y = 2$, $5y$ in (1) will equal 10, and we have $3x + 10 = 25$, and $3x = 15$, and $x = 5$.

1. $2x + 7z = 22$.
   $x = 4$; $z = ?$
2. $8x - 20y = 20$.
   $x = 5$; $y = ?$
3. $\frac{3}{4}x + \frac{1}{4}y = 9$.
   $x = 8$; $y = ?$
4. $7y - 3z = 17$.
   $y = 5$; $z = ?$
5. $\frac{1}{2}x + \frac{1}{4}y = 7$.
   $x = 8$; $y = ?$
6. $3x + 2y = 22$.
   $5x + 3y = 35$.
7. $3\frac{1}{2}x + 4\frac{1}{2}y = 41$.
   $x = 4$; $y = ?$
8. $8x - 5y = 41$.
   $y = 3$; $x = ?$
9. $7x - 8y = 30$.
   $x = 10$; $y = ?$
10. $3x - (4y - 2) = 7$.
    $x = 7$; $y = ?$
11. $5x + (3y - 4) = 33$.
    $x = 5$; $y = ?$
12. $4x + 3y = 32$.
    $2x + 5y = 30$.

13. $3x + 3y = 27.$
$x + 2y = 13.$

14. $x + y = 18.$
$x - y = 4.$

15. $4x + 5y = 41.$
$6x - 4y = 4.$

16. $5x = 4y + 17.$ Transpose
$3x = 2y + 11.$

17. $4(x - y) = 32.$
$3x - 4y = 20.$

18. $2(3x - 2y) - 4(x - 3y) = -40.$ $2(3x - 3y) = 6.$

19. $\dfrac{2x + 4y}{x + 2y} = 2.$ $\dfrac{4x + 6y}{2x - y} = 6.$

PROBLEMS

The sum of two numbers is 12. Twice the first added to three times the second is 31. What are the numbers? Let x = the first, and y = the second.

$$x + y = 12. \quad (1) \qquad 2x + 3y = 31. \quad (2)$$

Multiply the (1) by 2, and subtract (1) from (2). $y = 7$. Substituting the value of y, which is 7, in (1), gives $x = 5$.

1. The difference between two numbers is 20. Four times the first less five times the second is 65. What are the numbers?

2. The sum of two numbers is 50. The sum of twice the first and three times the second less the sum of the first and second is 65. What are the numbers?

3. Separate the number 120 into three parts the second of which shall be twice the first and the third shall be three times the first.

4. Separate 200 into two parts, the smaller to be $\frac{1}{3}$ of the larger.

5. A school of 800 pupils has 200 more girls than boys. How many are there of each?

6. A box and its cover together weigh 27 ounces. The cover is $\frac{2}{7}$ as heavy as the box. What is the weight of each?

7. The sum of two numbers is 284; one of them is 41 more than the other. What are the numbers?

8. The sum of two numbers is 53. Four times the first is 20 more than two times the second. What are the numbers?

9. The cost of 10 apples and 8 peaches is 26 cents. The cost of 6 apples and 4 peaches is 14 cents. What is the price of each?

POWERS

187. A **power** of a number is the product arising from using it a number of times as a factor.

A number used twice as a factor gives its second power or square.

A number used three times as a factor gives its third power or its cube. Three used twice as a factor gives $3 \times 3 = 9$, the second power of 3 or the square of 3. Three used three times as a factor gives $3 \times 3 \times 3 = 27$, the third power of 3, or the cube of 3.

To show that a number is to be raised to a certain power or used a certain number of times as a factor, write the power to which it is to be raised to the right and a little above the number, as 3^2. This indicates that 3 is to be raised to the second power, or squared. This number is called an *exponent.*

An exponent shows how many times the number is to be used as a factor. Here 2 is an exponent, and shows that 3 is to be used two times as a factor.

To raise numbers to powers, is called *involution.* The cube or third power of 5 is $5 \times 5 \times 5 = 125$. $5^3 = 5 \times 5 \times 5 = 125$.

PROBLEMS

1. What is the second power of 24?
2. What is the third power of 33?
3. What is the fourth power of 8?
4. What is the third power of $\frac{5}{6}$?
5. Find the required power of the following: $(\frac{3}{4})^3$, $(\frac{5}{6})^5$, $(2\frac{1}{2})^4$, $(14\frac{1}{2})^2$.
6. $(25)^2 = ?$ $(48)^2 = ?$ $(25)^3 = ?$ $(40)^4 = ?$ $(10)^6 = ?$ $(200)^3 = ?$
7. $(.02)^2 = ?$ $(.02)^3 = ?$ $(.14)^2 = ?$ $(.001)^3 = ?$ $(.25)^2 = ?$ $(2.5)^2 = ?$
8. Expand the following: 10^3, 8^5, 15^3, 20^4.
9. Expand the following: $(\frac{2}{3})^4$, $(\frac{3}{4})^2$, $(\frac{7}{8})^3$.
10. Find the square of the following: 22, 30, $\frac{4}{5}$, .07, .15.
11. Find the cube of the following: $\frac{7}{9}$, $\frac{4}{7}$, $\frac{2}{5}$, $3\frac{1}{3}$.
12. Find the fifth power of the following: .05, $\frac{3}{5}$, $2\frac{1}{2}$, $3\frac{3}{4}$.

ROOTS

188. The *factor* or number which has been raised to a power is the *root* of the power. To find the repeated factor or root is the reverse of involution and is called *evolution*. In the expression, $3 \times 3 = 9$, or, $3^2 = 9$ the root of the power 9 is 3. The process of finding the root 3 is evolution. A root is indicated by the radical sign which is placed over the number whose root is to be found. $\sqrt{36}$ indicates that the square root of 36 is to be found. $\sqrt[3]{8}$ indicates that the cube root of 8 is to be found.

We know the square root of a number, if we know the two equal factors of that number. We know 7 is the square root of 49, because 7 used twice as a factor will make 49. $7 \times 7 = 49$.

We know the cube root of a number, if we know what number used three times as a factor will produce the number. We know 5 is the cube root of 125, as 5 used three times as a factor will make 125. $5 \times 5 \times 5 = 125$.

A factor used four times to produce a number is the 4th root of the number. $7 \times 7 \times 7 \times 7 = 2{,}401$. 7 is the 4th root of 2,401.

In the same manner, by factoring we can find any root of a number.

Find the square root of 625. The factors of 625 are four 5's. For the square root, separate all the factors into two groups. The product of the factors of one of the groups gives the square root. For the cube root separate the factors into three equal groups. The product of the factors of any one group is the cube root.

$$\begin{array}{r|r} 5 & 625 \\ 5 & 125 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

The factors are 5×5 in one group, and 5×5 in another group. Therefore, 5×5 or 25 is the square root of 625.

$$\begin{array}{r|r} 3 & 729 \\ 3 & 243 \\ 3 & 81 \\ 3 & 27 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

Find the cube root of 729. 3×3 or 9 is used three times; hence 9 is the cube root of 729.

PROBLEMS

1. Find the square root of 2,500.
2. Find the cube root of 216.
3. Find the cube root of 1,728.
4. Find the fourth root of 625.
5. Find the fifth root of 1,137.
6. Find the cube root of $\frac{64}{216}$.

7. Nine is the cube root of what number?
8. Seven is the cube root of what number?
9. Find the square root of 289.
10. Find the square root of 841.

The factors of some numbers cannot be easily found, and some have no factors. There is another method for extracting the square and cube root of any number.

SQUARE ROOT

189. Find the square root of 55,225.

```
         5 52 25 (235
         4
        ---------
  40)    152
  43)    129
  ------------
 460)     23 25
 465)     23 25
 -------------
```

1. Begin at the right and separate the number into periods of two figures each, because the square of any number having units place only will not occupy more than two places, and the square of any number having units and tens only will not occupy more than four places, etc. There will be as many places in the root as there are periods. In the example there are three periods in the number and there are three places in the root.

2. Find the greatest square in the first or left hand period, which is (4), and place its root 2 as the first figure of the root.

3. Subtract this square (4) from the first period and bring down the next period (52), and annex it to the first remainder (1). This gives a new dividend (152).

4. Take two times the first figure of the root (2), and place it at the left of the new dividend, and annex one

cipher. This is a trial divisor. Find how many times this trial divisor (40) is contained in the dividend (152). Place the result (3) as the second figure of the root. Also add this second figure of the root (3) to the trial divisor (40). This gives the complete divisor (43).

5. Multiply the complete divisor (43) by the second figure of the root (3), and subtract and bring down the next period (25) and proceed as before until the root is found.

When the number is not a perfect square, annex periods of ciphers and continue the root as a decimal. If the trial divisor is not contained in the dividend, annex a cipher to the divisor and also to the root and bring down the next period.

To extract the square root of decimals, begin at the decimal point and point off to the right periods of two figures each, and, if there is an integral part, also to the left periods of two figures each, and annex ciphers, if necessary, to make two figures in each period.

Find the square root of 97.8121.

	97.8121	(9.89
	81	
180)	1681	
188)	1504	
1960)	17721	
1969)	17721	

$9^2 = 81$. Subtract 81 from 97, and bring down 81. Take two times 9, and annex one cipher = 180, the first trial divisor. 180 is contained in 1681 eight times. Add 8 to 180 = 188; the complete divisor. Multiply the complete divisor 188 by 8 = 1504. Subtract 1504 and bring down the next two figures (21). This is a new dividend 17721. Take two times 98 and annex one cipher = 1960.

the second trial divisor. 1960 is contained in 17721 nine times. Add 9 to 1960 = 1969, the complete divisor. Multiply the complete divisor 1969 by 9 = 17721. For the two integral figures, 97, in the square, we have one integral figure, 9, in the root, and for the four decimal figures, .8121, in the square, we have the two decimal figures, .89, in the root.

PROBLEMS

1. Find the square root of 1,849.
2. Find the square root of 61,504.
3. Find the square root of 1,679,616.
4. Find the square root of .008836.
5. Find the square root of $\frac{847}{1183}$.
6. Find the square root of 1,900.96.
7. Find the square root of .001225.
8. Find the square root of $\frac{1764}{2809}$.
9. Find the square root of 47 to five decimal places.
10. Find the square root of .031 to four decimal places.
11. How many more rods of fence around a field in the form of a rectangle 135 rods long and 60 rods wide than around a square field of equal area?
12. How long is the side of a square farm which contains 576,000 square rods?

CUBE ROOT

190. A number consisting of two figures is the sum of its tens and its units. Thus, 36 is the sum of 30 and 6; 24 is the sum of 20 and 4, etc.

We may cube 36 by using 36 three times as a factor. Thus, 36 × 36 × 36 = 46,656, or, we may cube it by using the tens plus the units three times as a factor.

$$
\begin{array}{rr}
 & 30 + 6 \\
 & 30 + 6 \\
\hline
 & 180 + 36 \\
900 + 180 & \\
\hline
900 + 360 + 36 &
\end{array}
$$

$900 + 360 + 36 = 1{,}296 =$ the square of $(30 + 6) =$ tens^2 + two times the tens times the units + units^2. Let t = the tens and u = the units. Then $(t + u)^2 = t^2 + 2tu + u^2 = 900 + 360 + 36$. Continue the multiplication.

$$
\begin{array}{rrrr}
 & 900 + & 360 + & 36 \\
 & & 30 + & 6 \\
\hline
 & 5400 + & 2160 + & 216 \\
27000 + & 10800 + & 1080 & \\
\hline
\end{array}
$$

$27000 + 16200 + 3240 + 216 = t^3 + 3t^2u + 3tu^2 + u^3 = (30 + 6)^3 = 46{,}656.$

To extract the cube root of a number is the reverse of this process.

First, separate the number into periods of three figures each, beginning at the right. There will be as many periods as there are figures in the cube root. From the above example we see that the number is made up of the cube of the tens, plus three times the square of the tens times the units, plus three times the tens times the square of the units, plus the cube of the units.

Second, find the largest cube in the left hand period, and write its cube root for the first figure of the root. Subtract this cube from the left hand period, and bring down the next period.

Third, write at the left three times the square of the first figure of the root, and annex two ciphers. This is a trial divisor. Divide the dividend by this, and place the result as the second figure of the root.

Fourth, take three times the first figure of the root times the second figure and annex one cipher, as an addition to the trial divisor. Square the second figure of the root and add it as a second addition to the trial divisor. Multiply the sum by the second figure of the root.

Subtract and bring down the next period for a new dividend. If there are other periods remaining, the first two figures of the root are regarded as tens and the number to be found as the units figure, and we proceed as before.

Find the cube root of 46,656.

$$\begin{array}{rrr}
 & t^3 + 3t^2u + 3tu^2 + u^3 = & 46\ 656\ (36 \\
 & 30 \times 30 \times 30 = t^3 = & 27\ 000 \\ \hline
 & & 19\ 656 \\
3 \times 30 \times 30 = 3t^2 = & 2700 & \\
3 \times 30 \times 6 = 3tu = & 540 & \\
6 \times 6 = u^2 = & 36 & 19\ 656 \\ \hline
 & 3276 & 00\ 000
\end{array}$$

$$3t^2 + 3tu + u^2 = 2700 + 540 + 36 = 3276$$
$$t^3 + (3t^2 + 3tu + u^2) \times u = 27000 + 19656 = 46656$$

Find the cube root of 14,706,125.

$$\begin{array}{rr}
2^2 \times 3 + 00 = & 1200 \\
2 \times 3 \times 4 + 0 = & 240 \\
4^2 = & 16 \\ \hline
 & 1456 \\
24^2 \times 3 + 00 = & 172800 \\
24 \times 3 \times 5 + 0 = & 3600 \\
5^2 = & 25 \\ \hline
 & 176425
\end{array}
\qquad
\begin{array}{l}
14\ 706\ 125\ (245 \\
8 \\ \hline
6\ 706 \\
5\ 824 \\ \hline
\quad 882\ 125 \\
\quad 882\ 125 \\ \hline
\quad 000\ 000
\end{array}$$

EXERCISES

1. Find the cube root of 42,875.
2. Find the cube root of 34,328,125.
3. Find the cube root of 884,736.
4. Find the cube root of 1,860,967.
5. Find the cube root of 32,768.
6. Find the cube root of 39,304.
7. Find the cube root of 15,625.
8. Find the cube root of 21,952.
9. Find the cube root of .091125.
10. Find the cube root of 27.543608.

(In 9 and 10 begin at the decimal point and point off to the right.)

11. Find the cube root of $\frac{343}{729}$.
12. Find the cube root of 248.7.

(In 12, annex two ciphers to fill out a period of three figures.)

13. Find the cube root of 33,386,248.

$$t^3 + 3t^2u + 3tu^2 + u^3 = 33\ 386\ 248\ (322$$

$$30 \times 30 \times 30 = t^3 = 27\ 000$$

$$6\ 386$$

$$\begin{array}{lcrr}
3 \times 30 \times 30 = 3t^2 & = & 2700 & \\
3 \times 30 \times 2 = 3tu & = & 180 & \\
2 \times 2 = u^2 & = & 4 & 5\ 768 \\
\hline
\text{1st comp. divis.} & = & 2884 & 618\ 248 \\
 & & & \\
3 \times 320 \times 320 = 3t^2 & = & 307200 & \\
3 \times 320 \times 2 = 3tu & = & 1920 & \\
2 \times 2 = u^2 & = & 4 & 618\ 248 \\
\hline
 & & 309124 & 000\ 000
\end{array}$$

14. Find the cube root of 12.812904.

15. Find the cube root of 1,728.42875.

By factoring we can find any root of a number.

Find the square root of 625. The factors of 625 may be found as follows:

$$\begin{array}{r} 5)\underline{625} \\ 5)\underline{125} \\ 5)\underline{25} \\ 5)\underline{5} \\ 1 \end{array}$$

For the square root, separate all the factors into two groups, with the same figures in each group. The product of the figures in one of the groups gives the square root.

For the cube root, separate all the factors of the number into three groups. The product of the factors of any one group is the cube root of the number given.

Find the cube root of 729. The group 3 and 3 or 9 is used three times. Hence 9 is the cube root of 729.

$$\begin{array}{r} 3)729 \\ 3)\underline{243} \\ 3)81 \\ 3)\underline{27} \\ 3)9 \\ 3)\underline{3} \\ 1 \end{array}$$

PROBLEMS

1. A cubical box contains 64,000 cu. in. What is the length of each side? How many square inches in the surface of each side?

2. A cubical cistern is to be constructed that will contain 432 tons of water. What will be the length of each side, if 1 cu. ft. of water weighs 1,000 oz.?

3. What will it cost at 30 cents per square yard to plaster the bottom and sides of the cistern mentioned in the above problem?

4. What will be the length of a side of a cubical bin that will contain exactly 1,200 bu., if a bushel contains 2,150.4 cu. in.?

5. A gallon contains 231 cu. in. What is the length of the side of a cubical tank that will hold 53,361 gal.?

6. What are the dimensions of a cubical bin that will have the same capacity as a bin 32 ft. long, 16 ft. wide and 8 ft. deep?

7. What are the dimensions to tenths of an inch of a cubical tank that will hold 8,000 gal.?

8. A bar of iron 8 ft. long, 2 ft. wide and 1 ft. thick was cast into the form of a cube. What was the length of each edge? (Find to tenths of an inch.)

PLANE FIGURES

191. Plane figures have length and breadth. The most common ones are the rectangle, parallelogram, triangle, trapezoid and circle.

THE RECTANGLE

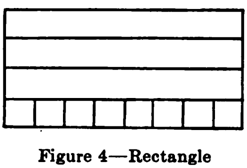

Figure 4—Rectangle

192. A **rectangle** is a plane surface bounded by four straight lines and having four right angles. When the four sides are equal, the figure is called a **square.**

There are four rows and eight square inches in each row in this rectangle; therefore:

193. Rule: *To find the area of a rectangle, multiply the number of square units in one row by the number of rows.* $Area = (B \times A)$. B = base. A = altitude.

THE PARALLELOGRAM

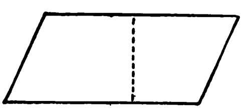

Figure 5—Parallelogram

194. A **parallelogram** is a quadrilateral whose opposite sides are parallel and equal.

Any parallelogram may be converted into a rectangle having an equal base, altitude and

11. A field is in the shape of a trapezoid. The parallel sides are 40 and 60 rods long. The distance between them is 54 rods. How many acres in the field?

12. If the parallel sides of a trapezoid are 24 and 36 feet, and the distance between them 18 feet, what would be the dimensions of a rectangle containing the same area?

THE RIGHT-ANGLED TRIANGLE

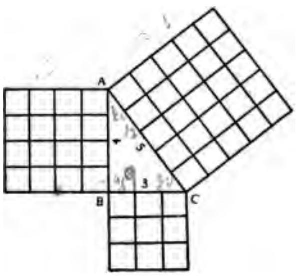

FIGURE 9

201. Count the number of squares on the hypotenuse of the right-angled triangle, the number of the base, and the number on the perpendicular. How does the number of squares on the hypotenuse compare with the number on the base and the hypotenuse together? If the hypotenuse is 5 inches long, each square is 1 in. long and 1 in. wide, and there are 25 squares, which equals 5^2. If the base is 3 in. long, each square is 1 in. long and 1 in. wide, and there are 9 squares, which equals 3^2. If the perpendicular is 4 in. long, each square is 1 in. long and 1 in. wide, and 16 squares, which equals 4^2. We have then, 5^2, or 25, $=$ 3^2, or $9 + 4^2$, or 16. The square on the hypotenuse, 25, $=$ the square on the base, 9 $+$ the square on the perpendicular, 16. $5^2 = 4^2 + 3^2$, or $25, = 16 + 9$. The square on the hypotenuse of a right-angled triangle $=$ the sum of the squares on the other two sides.

If one side of a right-angled triangle is 4 ft. long and the other side is 3 ft. long, how long is the hypotenuse? Square the 4 and the 3 and add, and we have the square on the hypotenuse, which is 25. Extract the square root.

The length of the hypotenuse is, therefore, 5 ft.

If the hypotenuse is 5 ft. long and the base is 3 ft. long, how long is the perpendicular? Square the base 3 ft. which gives 9 ft. Square the hypotenuse, which is 5 ft. and this gives 25 ft. Subtract the square of either side from the square of the hypotenuse, and we have the square of the other side. Subtracting 9 ft. from 25 ft. gives 16 ft. The square root of 16 ft. gives 4 ft., the perpendicular.

202. If, in a right-angled triangle, the side opposite one angle is 1 ft. long and the hypotenuse is 2 ft. long, then the other side is $\sqrt{3}$ ft. long.

If each of two of the angles is 45°, the sides opposite these angles are equal, and, if the sides are each 1 ft. long, the hypotenuse is the square root of 2 ft., or $\sqrt{2}$ ft.

The angle at the base of a right triangle is 30°, and the hypotenuse is 12 ft., what is the altitude? (The altitude is the side opposite the 30°. The proportion between the altitude and the hypotenuse is 1 to 2. Twelve feet is six times two feet; therefore, the altitude is six times one foot, or 6 ft.).

203. *Areas of similar figures are to each other as the squares of their like dimensions. Their like dimensions are to each other as the square roots of their areas.*

PROBLEMS

1. The base of a right triangle is 16 ft. and the altitude is 12 ft. What is the hypotenuse?

2. A ladder 20 ft. long just reaches the top of a house when the foot is pulled out 12 ft. from the house. How high is the house?

3. Two ropes, one 30 ft. long and one 50 ft. long are fastened to the top of a pole 30 ft. high, and are stretched to two stakes on opposite sides of the pole. How far apart are the two stakes?

4. The base of a right triangle is 10 ft. long and the angle at the base is 60°. What is the length of the hypotenuse? (The angle at the top is 30°.)

5. The area of a right triangle is 248 sq. ft. What is the area of one whose base is twice as long?.

THE CIRCLE

204. A **circle** may be formed by a large number of triangles, the sum of whose bases represents the circumference, and whose altitude is the radius, or one half the diameter. One half of these triangles may be placed as in Figure 10 and the other half placed points down in these triangles, thus making a rectangle, whose length is one half the circumference, and whose breadth equals the radius of the circle or one half the diameter of the circle. The circumference of a circle = the diameter times 3.1416. This number is represented by π. Therefore, the circumference C = the diameter, D, times π. Or, $C = \pi D$. Or, $C = 2\pi R$. R = the radius.

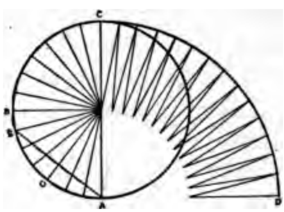

FIGURE 10

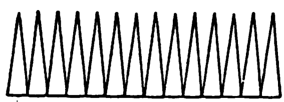

FIGURE 11

205. The area of a circle may be found from the rectangle, as shown above. Area = $\frac{1}{2}$ the circumference $\times$ $\frac{1}{2}$ the diameter; or, the radius = $\frac{1}{2}$ the diameter, and the area = $\frac{1}{2}$ circumference $\times R$. Multiply 2 R by πR and divide by 2 = area = $\frac{2\pi R \times R}{2} = \pi R^2$. Area of a circle = the square of the radius times the number 3.1416, or π. $A = \pi R^2$.

The circumference is $A\ B\ C\ D$ and the radius is $A\ O$ or $B\ O$. The diameter is $A\ C$.

Any one of the triangles in the Figure 10 is a sector of the circle.

206. The **area of a sector** $= \frac{1}{2}$ the radius times its arc. (The arc is the curved base of the triangle). We may use $\frac{22}{7}$, or $3\frac{1}{7}$ for 3.1416. Or π, which is the Greek letter pi.

With the formula $C = \pi D$, or $C = 2\pi R$; find the circumference of a circle whose diameter is 35 ft. Find the circumference of a circle whose R is 42 ft. 35 ft. $\times \frac{22}{7} =$ 110 ft. And $2 \times 42 \times \frac{22}{7} = 264$ ft. Find the area of a circle whose radius is 4 ft. $A = \pi R^2 = 16 \times \frac{22}{7} = 50\frac{2}{7}$ ft.

What is the area of a sector of a circle whose radius is 8 ft. and whose arc is 2 ft.? $\frac{1}{2} \times 8 \times 2 = 8$ ft.

207. A **segment** of a circle is that part of the circle contained between a chord of the circle and an arc of the circle. fig. 10 In Figure ~~9~~ the part of the circle between the chord $E\ A$, and the arc $E\ G\ A$ is the segment of the circle.

208. The area of the segment $E\ G\ A$ is equal to the difference between the area of the sector $O\ E\ G\ A$ and the area of the triangle $O\ E\ A$.

209. Rules: *The area of a circle equals the square of the radius multiplied by* π. *Or, the area of a circle equals one half the circumference times the radius.*

The area of a sector equals one half the part of the circumference, which is the arc, times the radius.

The area of the triangle is one half the altitude times the base. The difference between the area of the sector and the area of the triangle is the area of the segment.

The area of any triangle, given the sides, is the square root of one half the sum of the three sides multiplied by the three remainders obtained by subtracting each side separately from half the sum of the three sides. If S equals the sum of the three sides and a, b, c equal the sides,

the area $= \sqrt{\frac{1}{2}S \times (\frac{1}{2}S - a) \times (\frac{1}{2}S - b) \times (\frac{1}{2}S - c)}$.

If the sides of a triangle are 3, 4 and 5 ft., the area equals $\sqrt{6 \times (6 - 3) \times (6 - 4) \times (6 - 5)} = \sqrt{36}$ sq. ft. $= 6$ sq. ft.

8—

MISCELLANEOUS PROBLEMS

1. Find the area of a triangle whose sides are 13 ft., 14 ft., and 15 ft.

2. How many square feet in a triangle each of whose sides is 6 ft.?

3. Find the area of a trapezoid whose bases are 13 ft. and 7 ft., and whose altitude is 6 ft.

4. Find the area of a trapezoid whose bases are 7 ft. and 6 ft., and whose altitude is 9 ft.

5. Find the area of a trapezoid whose bases are 5 ft. and $7\frac{1}{2}$ ft., and whose altitude is 7 ft.

6. A piece of land in the form of a trapezoid contains 6 acres. The length of one of the parallel sides is 110 rods, and of the other 10 rods. How far apart are the sides?

7. How many square feet of surface will be covered by 1,000 hexagonal tiles, each 6 in. on a side?

8. Find the area of a circle whose radius is 5 ft.

9. Find the area of a circle whose diameter is 50 ft.

10. Find the circumference of a circle whose area is 78.54 sq. ft.

11. Find the area of a circle whose circumference is 157.08 ft.

12. The area of a circle is 1,017.8784 sq. ft. Find the radius and the circumference.

13. Two circles 5 in. and 12 in. in diameter, respectively, have the same center. What is the area of the ring between their circumferences?

14. What is the radius of a circle which contains a surface of $\frac{1}{2}$ acre?

15. What is the difference between the area of a circle and the area of a circumscribed square whose length is 12 ft.?

16. The diameter of a circle is 20 ft. What is the area of the inscribed square?

17. A circular park is two miles in circumference; what is its diameter?

18. What is the diameter of a car wheel which makes 420 revolutions per minute when the car is running at the rate of 45 miles per hour?

19. The diameter of a wagon wheel is 4 feet. How many times will the wheel revolve in going 5,280 ft.?

20. A piece of steel plate in the form of a right-angled triangle, whose base is 30 in. long, and whose altitude is 20 in., is $\frac{1}{4}$ inch thick, and weighs 10.20 lbs. per square foot. What is the area of this piece of steel in square inches? In square feet? What is the weight in pounds?

21. A piece of steel plate in the form of a triangle has a base of 60 in., an altitude of 30 in., and is $\frac{1}{4}$ in. thick. If it weighs 10.20 lbs. per square foot, what does the plate weigh? What is its area in square feet?

22. A circular sheet of steel plate is $\frac{1}{2}$ in. thick, and 16 in. in diameter. What is its area in square inches? What is its volume in cubic inches? If steel weighs .28 of a pound per cubic inch, what will this steel plate weigh?

23. A circular steel plate 12 in. in diameter has a circular hole 4 in. in diameter cut out of its center. The plate is $\frac{1}{2}$ in. thick and weighs .28 of a pound per cubic inch. What is the area of the plate before the hole is cut out? What is the area of the piece cut out? What is the area of the part left in square inches? What is the volume of the part cut out? What is the volume of the part left? (Answers in inches.) What is the weight of the part cut out? What is the weight of the part left?

24. A piece of land in the form of a triangle whose sides measure 9 rds., 12 rds., and 15 rds., respectively, contains how many square rods? What is it worth at $3 per square foot?

SURFACES OF SOLIDS

210. A **cube** is a solid bounded by six equal squares all of whose edges are equal.

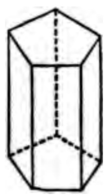

FIGURE 12. RIGHT PRISM

211. A **prism** is a solid having two faces which are equal and parallel polygons and whose other faces are parallelograms whose sides are parallel to one straight line. See Figure 12.

The parallel faces are called bases.

The altitude of a prism is the perpendicular distance between the two bases.

212. Rule: *The lateral surface of a right* prism is equal to the product of the altitude by the perimeter of the base.* $S = P \times A$. *P = perimeter. A = altitude.*

213. A **pyramid** has triangular surfaces which meet at a point, and a triangular or square base, or a base of any number of sides. Figure 13. If we cover a pyramid with paper exactly to fit, there will be as many triangular pieces of paper on the convex surface as there are sides to the pyramid. The paper on the base will be a polygon of as many sides as the pyramid has sides.

214. By cutting a regular pyramid parallel or inclined to the base, a smaller pyramid is cut off from the top, and what is left at the bottom is called a *frustum of a pyramid.* Figure 13 is a regular pyramid, and Figure 14 is a regular frustum of a pyramid.

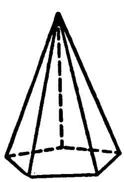

FIGURE 13.

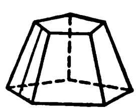

FIGURE 14.

If S = the convex surface of a frustum of a pyramid, L the slant height, and P and p the respective perimeters of the lower and upper bases, the

*Definitions and rules apply only to right solids.

formula for the convex surface of a regular pyramid is: $S = \frac{1}{2} L (P + p)$. Here, P = the perimeter of the lower base, and p = the perimeter of the upper base.

215. Rule: *The convex surface of a pyramid is one half the slant height times the perimeter of the base, or $\frac{1}{2} LP$. (The perimeter is the distance around the base.)*

$$S = \frac{P \times L}{2}. \quad L = \text{slant height.}$$

216. Rule: *The lateral surface of a frustum of a pyramid is equal to one half the sum of the perimeters of the bases multiplied by the slant height of the frustum.* $S = \frac{1}{2} L(P + p)$. P and p = Perimeters of lower and upper bases respectively, and L = slant height.

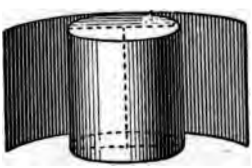

FIGURE 15.

217. A **right cylinder** has a curved surface and two equal parallel circular bases. Figure 15.

If we cover exactly a cylinder with paper, the paper on the two bases will be circular pieces, and the paper on the convex surface will be a rectangular piece.

218. Rule: *The area of one of the bases equals one half of the circumference times the radius.*

The convex area equals the area of the paper rectangle, or the circumference times the altitude of the cylinder.

219. Rule: *The lateral surface of a cylinder is equal to the product of the altitude by the perimeter of the base.* $S = P \times A$. P = *perimeter.* A = *altitude.*

220. A **cone** has a convex surface which tapers uniformly to a point, and a circular surface for a base. If we exactly cover a cone with paper, the paper on the base will be a circular piece, and the paper on the convex surface will be a triangular piece.

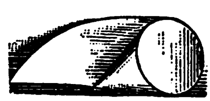

FIGURE 16.

221. By cutting a cone parallel to the base, a smaller cone is cut off from the top, and what is left at the bottom is called a *frustum of a cone.*

A washtub or a dish pan is a frustum of a cone. Figure 17 is a cone, and Figure 18 is a frustum of a cone. If $S =$ the convex surface of a frustum of a cone, and L the slant height, the formula for the convex surface of a frustum of a cone is: $S = \pi L(R + r)$. Here, $R =$ the radius of the lower base, and $r =$ the radius of the upper base. Or, $S = \frac{1}{2}\pi L(D + d)$. Where $D =$ the diameter of the lower base, and $d =$ the diameter of the upper base.

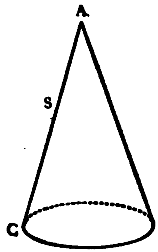

FIGURE 17.

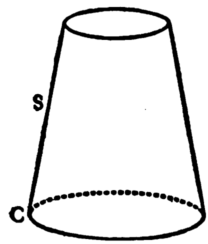

FIGURE 18.

The bottom of a dish pan is 10 in. in diameter, and the top is 14 in. in diameter and the slant height of the pan is 8 in. How many square inches of tin are required to make the sides of the pan? $S = \frac{1}{2}L(D + d)$. $S = 12 \times 8\pi =$ 301.7 inches.

222. Rule: *The convex surface of a cone is one half the slant height times the circumference of the base, or* $\frac{1}{2}\pi L D$.

223. Rule: *The lateral surface of a frustrum of a cone is equal to one half the sum of the perimeters of the bases multiplied by the slant height of the frustrum.* $S = \frac{1}{2} L(P + p)$. *P and p = perimeters of lower and upper bases respectively, and L = slant height.*

224. A sphere has a curved surface every point of which is equally distant from


FIGURE 19.

the center of the sphere. If we wind a hemisphere closely with a string, Figure 19, and then wind the same string closely on the base of the hemisphere, we shall find that it takes twice as much string to cover the convex surface as to cover the base.

225. Rule: *The area of a hemisphere is twice the area of the base. The area of a whole sphere would be four times the area of the base. The area of the base is the square of the radius of the sphere times* π. *And the area of the sphere is 4 times* πR^2. *Area of a sphere* $= 4\pi R^2$.

$$S = D \times C. \quad \text{Or, } S = 2\pi RD. \quad \text{Or, } S = 4\pi R^2.$$

226. *The areas of two similar surfaces are to each other as the squares of any two homologous lines.*

Homologous sides are those which have the same relative position.

PROBLEMS

1. What is the lateral surface of a triangular prism whose sides are 3 in., 4 in., and 5 in., and whose slant height is 20 in.?

2. What is the lateral surface of a square prism whose base is 4 in. on a side and altitude 9 in.?

3. What is the lateral surface of a square pyramid whose base is 12 in. on a side, and slant height is 20 in.?

4. What is the lateral area of a cylinder whose altitude is 8 ft., and whose radius is 8 in.?

5. What is the lateral area of a cone the radius of whose base is 12 in. and whose slant height is 16 in.?

6. What is the lateral area of the frustum of a cone the radii of whose bases are 6 ft. and 2 ft. and whose slant height is 8 ft.?

7. The bottom of a dish pan is 8 in. in diameter, the top is 12 in. in diameter and the slant height is 9 in. How many square inches of tin are required to make it?

8. How many square feet of tin will be required to cover a church steeple the base of which is an octagon 8 ft. on a side, and whose slant height is 100 ft.?

9. What is the area of the earth's surface, if the earth is a sphere whose radius is 4,000 miles? $S = 4\pi R^2$.

10. What is the area of the surface of a sphere whose diameter is 20 ft? $S = 4 \pi R^2$.

11. What is the cost of gilding a ball 22 in. in diameter at 40 cents a square foot?

12. How many square yards of silk are needed for a spherical balloon 40 ft. in diameter?

13. If the circumference of a great circle on a sphere is 12 in., what is the area of the surface of the sphere?

14. How many square inches of leather are required to cover a base ball whose diameter is $3\frac{1}{2}$ in.?

15. What is the radius of a sphere whose surface is 616 sq. in.?

VOLUME AND CAPACITY

227. Solids have length, breadth and thickness. The most frequent figures are cube, rectangle, prism, pyramid, cone and sphere.

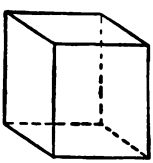

FIGURE 20

228. *The volume of a cube equals the product of the three equal dimensions or the cube of an edge.* Figure 20.

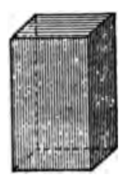

FIGURE 21

229. *The volume of a rectangular block equals the product of the three dimensions, length, breadth and thickness or height.* Figure 21.

230. *The volume of a prism equals the product of the area of its base and its altitude.*

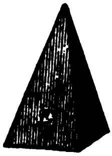
FIGURE 22

231. *The volume of a pyramid equals the product of the area of its base and one third of the altitude.*

$$V = \frac{B \times A}{3}. \quad \text{Figure 22.}$$

232. *The volume of a cylinder equals the product of its altitude and the area of its base.*

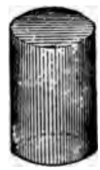
FIGURE 23

$$V = \pi R^2 A. \quad \text{Figure 23.}$$

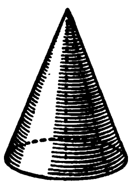
FIGURE 24

233. *The volume of a cone equals the product of the area of its base and one third of the altitude.*

$$V = \frac{\pi R^2 A}{3}. \quad \text{Figure 24.}$$

It takes exactly three cones each having the same base and altitude as a cylinder to be equal in volume to the cylinder. A cone is one third the volume of a cylinder of the same base and altitude.

234. *The volume of a sphere equals the product of the area of the sphere and one third of the radius.*

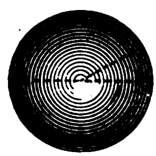
FIGURE 25

Or, $\frac{4}{3}\pi R^3$. Figure 25.

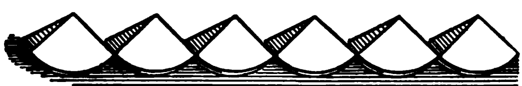
FIGURE 26.

By cutting many times through the center, a sphere may be cut into an indefinite number of solid pyramids, whose bases are the surface of the sphere and whose altitudes are

each equal to the radius of the sphere. When the pieces are many, the bases may be regarded as planes. Figure 26.

235. *The volume of each pyramid is equal to the base times one third the altitude. And the total volume of all the pyramids is the volume of the sphere, and is equal to the total surface of all the bases, which equals the surface of the sphere, multiplied by one third the altitude, which equals one third the radius.*

Surface of the sphere $= 4\pi R^2$. $V = 4\pi R^2 \times \frac{1}{3}R = \frac{4}{3}\pi R^3$.

236. *The volume of the frustum of a pyramid or cone is equal to the sum of the areas of the two bases plus the square root of the product of these bases multiplied by one third the altitude.*

$V = \frac{1}{3}A\ (B + b + \sqrt{Bb}.)$ Altitude $= A$. Bases $= B, b$.

237. *The volumes of two similar solids are to each other as the cubes of any two homologous lines. Their similar dimensions are to each other as the cube roots of their volumes.*

FORMULAS

238. The area of a parallelogram = base $\times$ altitude.

239. The area of a triangle = $\frac{1}{2}$ the base $\times$ altitude.

240. The area of a trapezoid = $\frac{1}{2}$ the sum of the two bases $\times$ altitude.

241. The area of a circle = the square of the radius $\times$ 3.1416. Or, $= \pi R^2$.

242. The circumference of a circle = the diameter $\times$ 3.1416. Or, πD. ($\pi = 3.1416$).

243. The surface of a sphere = the square of the diameter $\times$ 3.1416. Or, πD^2.

244. The surface of an ellipse = $\frac{1}{4}$ the product of the two diameters $\times \pi$.

245. The convex area of a cylinder = the altitude $\times$ the diameter $\times \pi$.

246. The entire area of a cylinder = the convex area + the area of the two end circles.

247. The convex area of a cone = $\frac{1}{2}$ the slant height × the circumference of the base, or, $\frac{1}{2}$ the slant height × D × π.

248. The convex area of a frustum of a cone is $\frac{1}{2}$ the sum of the upper and the lower diameters × the slant height × 3.1416.

249. The volume of a cylinder = the area of the base × the altitude.

250. The volume of a pyramid or of a cone = $\frac{1}{3}$ the area of the base × the altitude.

251. The volume of a sphere = $\frac{4}{3}$ × the cube of the radius × 3.1416.

252. The volumes of similar cylinders are to each other as the cubes of the altitudes or as the cubes of the radii of the bases.

253. The square on the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides.

254. The right angle of a right-angled triangle is 90°. If one of the other angles is 45°, the third is 45°. If one of the other angles is 30°, the third is 60°. The side opposite the angle 30° is $\frac{1}{2}$ the length of the hypotenuse.

PROBLEMS

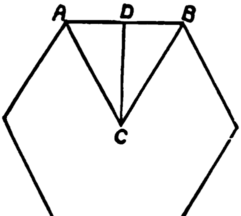

FIGURE 27.

In Fig. 27 the lines AC and BC are equal, being radii of a circle circumscribing the hexagon. CD is drawn from the center C at right angles to perpendicular AB. ADC and BDC are right-angled triangles. CD bisects AB. If the side DB is equal to 1, CD is equal to $\sqrt{(BC)^2 - (BD)^2}$. But $BC = AB$. $\therefore CD = \sqrt{2^2 - 1^2} = \sqrt{3} = 1.73+$.

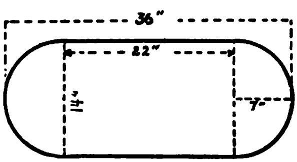

FIGURE 28.

radius of its circular ends 7 in. What is the volume? (The two semicircular ends taken together make a circle, 7 in. radius or 14 in. diameter. This circle is the base of a cylinder.) What figure remains after the cylinder is taken away? How is its volume found? What is the distance around this wash boiler? Figure 28.

24. There were 48,624 cu. ft. in an excavation for a building. How many loads?

(27 cu. ft. = 1 cu. yd. = 1 load.)

25. How many loads are taken from an excavation 20 ft. by 18 ft. by 8 ft., if earth expands $\frac{1}{10}$ in digging?

26. How many cubic yards of earth must be moved in digging a cellar 24 ft. by 16 ft. by 9 ft.?

27. How many bushels will a granary hold that is 24 ft. long by 8 ft. wide by 12 ft. high? (A standard bushel = $\frac{5}{4}$ cu. ft.)

28. How many bushels will a bin hold that is 36 ft. by 8 ft. by 10 ft. high?

29. How many bushels will a bin hold that is 24 ft. long by 6 ft. wide at bottom, 10 ft. wide at top, and 12 ft. high?

30. How many bushels of potatoes will a bin hold that is 12 ft. by 8 ft. by 6 ft.? (A heaped bushel = $\frac{3}{2}$ cu. ft.)

31. A man is to dig a circular cesspool 9 ft. in diameter and 24 ft. deep. How many loads of earth must he excavate, if earth expands $\frac{1}{10}$ in digging?

32. How many cubic yards of earth are removed in digging a sewer 2 miles long, 2 ft. wide and 6 ft. deep?

33. A silo 20 ft. in diameter is made to contain 260 tons of silage. What is the depth of the silo, if silage averages 40 lbs. per cubic foot?

34. A cow will eat 40 lbs. of silage per day. How long will the silage last a cow, if the silo is 36 ft. high and 20 ft. in diameter, and full?

35. A barn loft is 40 ft. by 30 ft. by 8 ft. high. How many tons of timothy hay will it hold, if 500 cu. ft. of timothy hay = 1 ton?

36. A barn loft is 36 ft. by 24 ft. by 8 ft. high. How many tons of clover hay will it contain, if 600 cu. ft. of clover hay weighs 1 ton.

37. A barn loft is 40 ft. long by 30 ft. wide by 8 ft. high to the plates and the ridge is 12 ft. above the level of the plates. If the loft is filled to the ridge with common hay, and common hay weighs 1 ton to 400 cu. ft., how many tons will the loft contain?

38. A wheat bin is 20 ft. by 12 ft. by 8 ft. and is filled with wheat. How many pounds of flour and bran and middlings, can be made from the wheat, if one bushel of wheat will make 40 lbs. of flour and 12 lbs. of bran and middlings, and 1 bu. = $\frac{5}{4}$ cu. ft.?

39. A corn crib is filled with ear corn, and is 32 ft. by 10 ft. by 8 ft. How many bushels will it contain if 1 bu. = $\frac{3}{2}$ cu. ft.? How many bushels of shelled corn will there be if 1 bu. of ear corn makes $\frac{1}{2}$ bu. of shelled corn? How many pounds of meal will there be if 1 bu. of shelled corn = 48 lbs. of meal?

40. How many loads of earth were contained in the excavation from between railroad embankments, if the area of the bottom of the excavation is 42 sq. rds. and the area at the top of the excavation is 68 sq. rds. and the perpendicular distance between the top and bottom is 48 ft.?

41. A wash boiler is 16 in. deep and has the base indicated in Figure 28. What is the capacity?

42. How many square inches of surface on the sides and bottom of the wash boiler shown in Figure 28?

CAPACITIES AND HEAPS

255. Table

1 bu. of wheat weighs 60 lbs.
1 bu. of corn weighs 56 lbs.
1 bu. = 2,150.4 cu. in. or, $1\frac{1}{4}$ cu. ft.
1 bu. heaped = 2,509 cu. in.
1 cu. ft. = .8 bu. of grain
1 cu. ft. = .68 bu. heaped
1 cu. ft. = 7.5 gal.
1 cu. ft. = .24 bbl.
1 cu. ft. of ice weighs 60 lbs.
1 cord of wood = 128 cu. ft.
1 ton of coal = 36 cu. ft.
1 ton of hay in mow or stack = 400 cu. ft.
1 cu. ft. anthracite coal = 56 lbs.
1 cu. ft. bituminous coal = 48 lbs.
1 ton of anthracite coal = 34.5 cu. ft.
1 ton of bituminous coal = 42 cu. ft.
1 gal. = 231 cu. in. $31\frac{1}{2}$ gal. = 1 bbl.
63 gal. = 1 hhd.

256. To find the capacity of barrels, *add the head and bung diameters and divide by 2. Use this mean diameter as in case of cylinders. Or, square the mean diameter in inches, and multiply by .0034 and by the length of the barrel in inches. This will give the number of gallons.*

PROBLEMS

1. The height of a cone-shaped heap of grain is 4 ft. and the diameter of the circle covered by the base is 8 ft. How many bushels in the heap?

2. The height of a cone-shaped pile of coal is 5 ft. and the circle covered by the base is 8 ft. in diameter. How many tons in the pile? (1 ton = 36 cu. ft.)

3. If 10 bu. of grain are in a heap on the floor in the form of a cone, and the highest point of the heap is 5 ft.,

what is the diameter of the circle on the floor covered by the heap?

4. A barrel has a head diameter of 21 in., a bung diameter of 25 in. and a length of 30 in. How many gallons will it hold?

5. The head diameter of a barrel is 18 in., the bung diameter is 21 in. and the length of the barrel is 30 in. How many gallons does it hold?

6. How many gallons in a barrel 19 in. in head diameter, 23 in. in bung diameter, and 31 in. long?

7. A bin is 4 ft. 6 in. by 6 ft. 4 in. How deep must it be to hold 30 bu. of grain?

8. A tank 8 ft. square and 6 ft. deep will hold how many barrels?

9. A steel cylinder is 8 in. in diameter, and 16 in. long. What is its weight? (A cubic inch of steel weighs 0.28 lb.)

10. What is the weight of water in a cylindrical tank whose diameter is 16 ft., and height 10 ft? (1 cu. ft. of water weighs $62\frac{1}{2}$ lbs.)

11. How many barrels will be required to hold a ton of coal, if they are 20 in. in head diameter, 24 in. in bung diameter, and 28 in. long?

12. How many gallons will a barrel contain whose mean diameter is 20 in. and depth 30 in.?

13. If the radius, R, of the outer circle of the cross-section of a water pipe is 3 in., and the radius r of the inner circle is $2\frac{3}{4}$ in., what is the area of the cross-section of the iron? $A \doteq \pi(R^2 - r^2)$.

14. What is the weight of a hollow steel cylinder, whose length is 10 ft., and the radii of whose outer and inner circles are, $R = 3$ ft. and $r = 2\frac{1}{2}$ ft.? (1 cu. ft. of steel = 484 lbs.)

15. A prism 6 in. high contains 40 cu. in. How many cubic inches will a similar prism 18 in. high contain?

16. A ball weighs 12 lbs. Find the weight of a similar ball, whose diameter is 4 times as great?

17. The altitudes of two similar cones are 4 ft. and 6 ft. respectively. If the volume of the first is 800 cu. ft., what is the volume of the second?

18. If it requires 6 hours for a pipe 2 in. in diameter to empty a cistern, how long will it require a pipe 1 in. in diameter to empty the same cistern? (The time is inversely proportional to the squares of the diameters.)

19. If 200 gal. of water flow through a pipe 2 in. in diameter in 4 hrs., how much water will flow through a pipe 6 in. in diameter in the same time? (The amounts of the liquids are to each other as the squares of the like dimensions.)

20. What is the volume of a frustum of a pyramid, if the area, A, of the bottom is 16 sq. in., the area, a, of the top is 4 sq. in., and the height, H, is 18 in.?

$$V = \tfrac{1}{3}H\,(A + a + \sqrt{Aa}).$$

21. What is the volume of a frustum of a cone, when the radius of the lower base, R, is 5 ft. and the radius of the upper base, r, is 3 ft. and the height, H, is 15 ft.?

$$V = \tfrac{1}{3}\pi H\,(R^2 + r^2 + R\,r).$$

22. How many cubic feet between railroad embankments, if the cross-section of the embankment is 200 ft. long by 10 ft. wide at the lower base, and 30 ft. wide at the upper base, and the embankment is 20 ft. high? Let $L =$ the length; $H =$ the height; $B =$ the lower base; $b =$ the upper base; $V =$ volume. $V = L\,H\,\tfrac{1}{2}\,(B + b)$.

23. A dish pan is 10 in. in diameter at the top, and 9 in. at the bottom and 5 in. deep. How many quarts will it contain?

24. How many tons of coal will a bin hold that is 12 ft. long, 8 ft. wide, and 6 ft. deep?

25. A pile of wood 20 ft. long, 8 ft. high and 4 ft. wide contains how many cords?

26. A pile of wood 80 ft. long, 12 ft. high and 4 ft. wide contains how many cords?

27. How many bushels of wheat will a bin contain, which is 5 ft. long, 3 ft. wide and $4\frac{1}{2}$ ft. high?

28. How many gallons of water will a tank hold that is 6 ft. long, 4 ft. wide and 2 ft. 3 in. deep?

29. How many bushels will a corncrib hold which is 20 ft. long, 12 ft. wide, and 7 ft. high?

30. How many tons in a stack of hay that is 10 ft. in diameter and 12 ft. high?

HEIGHTS AND DISTANCES

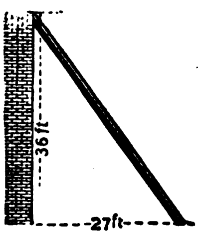

FIGURE 29.

257. In measuring heights and distances much use is made of the principles of the right-angled triangle.

If a ladder just reaches to the top of a wall 36 ft. high, when the foot of the ladder is 27 ft. from the bottom of the wall, how long is the ladder? See Figure 29. $\sqrt{36^2+27^2} = \sqrt{x^2}$. x = length of ladder.

258. Find the distance from the point E to the inaccessible object A. Figure 30. From E measure any distance, as $E\ D$, perpendicular to $E\ A$. Measure from E directly towards A, any distance, as $E\ B$. At B lay off a line $B\ C$ perpendicular to $E\ A$. Measure along the line $B\ C$, until a person at D can sight the staff at C, along the line $D\ A$. The two triangles D E A and

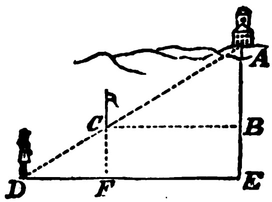

FIGURE 30.

$D\,F\,C$ are similar. Then $D\,E : E\,A :: D\,F : F\,C$; or, $D\,F : F\,C :: D\,E : E\,A$. Now $F\,C = E\,B$ and $D\,F = D\,E - C\,B$. If $D\,E = 36$ ft. and $E\,B = 9$ ft., then $F\,C = 9$ ft. If $B\,C = 24$ ft., then $E\,F = 24$ ft. and $D\,F = 12$ ft.

Therefore, $12 : 9 :: 36 : E\,A$. Or, $E\,A = \dfrac{9 \times 36}{12} = 27$.

259. Find the distance between two points A and B on opposite sides of a river. Figure 31. Measure the line $A\,D$ perpendicular to $A\,B$, say 600 ft. Measure $D\,E$ perpendicular to $A\,D$, say 50 ft. Sight a line from E to B. It intersects $A\,D$ at C, making $D\,C = 40$ ft.

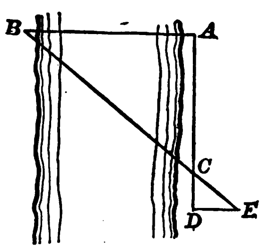

FIGURE 31

The two triangles $B\,A\,C$ and $C\,D\,E$ are similar and, therefore, the corresponding sides are proportional. Or, $D\,C : D\,E :: C\,A : A\,B$. $40 : 50 :: 600 : A\,B$. $A\,B = 750$.

Find the height of a steeple whose shadow measures 60 ft. if a post 16 ft. high casts a shadow 8 ft. long.

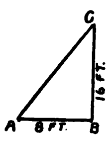

FIGURE 32

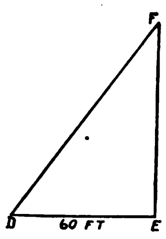

FIGURE 33

$A\,B : D\,E :: B\,C : E\,F$. Or, 8 ft. : 60 ft. :: 16 ft. : $E\,F$.

$$E\,F = \frac{60 \times 16}{8} = 120.$$

260. In Figure 30 find the height of the object A when the distance from a point D to a vertical line $A\,E'$ passing through the object is any number of feet, as 120 ft. Measure a convenient distance as 10 ft. from D to F toward $E\,A$. At F erect a perpendicular $F\,C$. Place the eye at D and sight to the object A. Mark the point on the staff $F\,C$ at which the line $D\,C$ cuts the staff $F\,C$. Measure $F\,C$. Let

it be 5 ft. $D\ F : D\ E :: F\ C : E\ A$. Or, $10 : 120 :: 5 : A\ E$.
$A\ E = \frac{120 \times 5}{10} = 60$, the number of feet.

If we use a staff distance $F\ C$ so the side $F\ C$ of the triangle $D\ F\ C$ is 2 times $D\ F$ then the height $E\ A$ is 2 times the distance $D\ E$ of the triangle $D\ E\ A$. Then we measure the distance $D\ E$ and know that the height of the object $A\ E$ is 2 times that distance.

Measure $D\ E = 30$ ft. $D\ F = 5$ ft. and $F\ C = 10$ ft. Then $D\ F : D\ E :: F\ C : E\ A$. Or, $5 : 30 :: 10 : E\ A$. And $E\ A = 60$ ft.

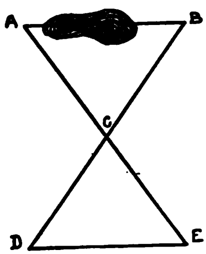

FIGURE 34.

261. To find the distance between two accessible objects, separated by an impassable barrier, *e. g.*, two trees separated by a pond, take a convenient station C, and measure $A\ C$ and $B\ C$. Extend $A\ C$ to E, making $C\ E = A\ C$. Extend $B\ C$ to D, making $D\ C = C\ B$. Then $A\ B = D\ E$. Measure $D\ E$ and we have the distance beween A and B

PROBLEMS

1. What is the height of a steeple which casts a shadow 300 ft. long, if a staff 10 ft. long casts a shadow at the same time, 20 ft. long?

2. A house casts a shadow 60 ft. long when a man 6 ft. tall casts a shadow 9 ft. long. How high is the house?

3. What is the height of a tree, if a line sighted from a rock 200 ft. from the tree cuts a perpendicular stake placed 180 ft. from the tree and 20 ft. from the rock, 4 ft. above the ground?

4. A pole 8 ft. high casts a shadow 5 ft. long. How high is a tree whose shadow at the same time is 84 ft. long?

5. A man 6 ft. tall stands 6 ft. from a lamp-post. If his shadow is 12 ft. long, what is the height of the lamp-post?

6. A man whose eye is $5\frac{1}{2}$ ft. above the ground, sights over the top of a 12-ft. pole and sees the top of a tree. He is 7 ft. from the pole and 63 ft. from the tree. How high is the tree?

7. A man holds a foot ruler vertically before the eye and sees the top of a building just in line with the top of the ruler, and the bottom of the building just in line with the bottom of the ruler. The distance of the bottom of the ruler from the eye is 15 in. and the distance of the bottom of the building from the eye is 100 ft. What is the height of the building?

8. What is the distance from a house A, on one side of a river, to a house B, on the other side of the river, if the perpendicular distance from A is 400 ft. to the point D, and the distance D C is 10 ft., and the distance DE is 8 ft. in Figure 31?

9. In Figure 31 what is the distance from A to B, if the distance from B to C is 80 rds., from C to E is 40 rds., from E to D is 25 rds., from D to C is 30 rds., and from C to A is 60 rds.?

10. A post 7 ft. high casts a shadow 4 ft. long. What is the height of a house that casts a shadow 150 ft. long at the same time of day?

11. In the Figure 30 how far is the building at A from the point E, if the distance E B is 10 ft. the distance B C is 30 ft. and D E is 40 ft.?

12. What is the distance from A to B across an impassible marsh, if the distance A C = E C, and D C = B C, and D E = 800 ft.? Figure 34.

PUBLIC LANDS.

262. A Base Line is laid out and then a **Principal Meridian** running north and south and cutting the Base Line at right angles. At intervals of 6 miles both north and south

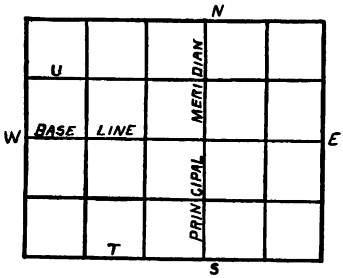

FIGURE 35.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

FIGURE 36.

of the base line and also east and west of the principal meridian, lines are drawn, thus forming Townships, or areas of 6 miles square, or 36 sq. miles. Fig. 36 represents a township divided into sections.

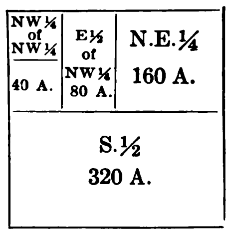

FIGURE 37.

Rows of townships running north and south are called Ranges. These Ranges are numbered 1, 2, 3, etc., east or west of the principal meridian.

If a township is in the third row or range west of the principal meridian, it is read R 3 W. This is called "range 3 west."

Townships are also located by numbering north or south of the Base Line. A township at T in the figure is designated T 2 S, R 2 W. And read "township 2 south, range 2 west." The township U is designated T 2 N, R 3 W.

Each township is divided into 36 sections of 1 square mile each, or 640 acres. These sections are numbered from

right to left and then from left to right, as shown in Figure 35. The numbering begins at the northeast corner, and ends at the southeast corner.

Each section is divided into quarter sections, which are often divided into halves or fourths.

PROBLEMS

1. How many acres are there in the SE$\frac{1}{4}$ of NE$\frac{1}{4}$ of section 8?

2. Write the description and find area of N$\frac{1}{2}$ of NW$\frac{1}{4}$ sec. 29, T 1 N, R 4 E.

3. Find the area of NE$\frac{1}{4}$ of N$\frac{1}{2}$ of W$\frac{1}{2}$ of SW$\frac{1}{4}$ sec. 24 T 1 N, R 3 W.

4. How many acres are there in the SW$\frac{1}{4}$ of NW$\frac{1}{4}$ of sec. 10?

5. How many acres are there in N$\frac{1}{2}$ of SE$\frac{1}{4}$ of sec. 34?

6. How many rods of fence are required to inclose NW$\frac{1}{4}$ of sec. 15, T 3 N, R 2 E?

7. How many rods of fence are required to inclose SE$\frac{1}{4}$ of sec. 18, T 2 S, R 3 W?

8. How many acres in S$\frac{1}{2}$ of W$\frac{1}{2}$ of SW$\frac{1}{4}$ sec. 22, T 5 N, R 3 W?

9. How many acres are there in $\frac{1}{2}$ of $\frac{1}{2}$ of a quarter section?

10. How many acres are there in $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{1}{2}$ of a quarter section?

FLOORING AND ROOFING

263. In estimating flooring and other matched lumber, an allowance must be made for the lumber cut away by the tongue and groove. This varies from $\frac{1}{4}$ to $\frac{3}{4}$ of an inch for each board. For boards from 2$\frac{1}{2}$ in. up to 5$\frac{1}{2}$ in. in width

the amount added for matching is about $\frac{1}{4}$ of the area. For boards of less width the amount is $\frac{1}{3}$ of the area.

For a floor 14 ft. by 12 ft. laid with $2\frac{1}{2}$ in. matched boards, the amount added is $\frac{1}{4}$ of $(14 \times 12 = 168)$. $\frac{1}{4}$ of 168 ft. = 42 ft., and 168 ft. + 42 ft. = 210 ft. of lumber.

For a floor 14 ft. by 12 ft. laid with $1\frac{1}{2}$ in. boards, the amount added is $\frac{1}{3}$ of $(14 \times 12 = 168)$. $\frac{1}{3}$ of $168 = 56$, and $168 + 56 = 224$ ft. of lumber.

PROBLEMS

1. What will it cost to lay a floor 18 ft. by 20 ft. with matched $2\frac{1}{2}$ in. flooring, at $70 per M.?

2. A floor is $12\frac{1}{2}$ ft. by $14\frac{1}{2}$ ft. What will it cost to lay the floor with matched lumber $1\frac{1}{2}$ in. wide, at $60 per M.?

3. A veranda on a corner of a house is 20 ft. 6 in. long across the front and 12 ft. 6 in. long on the side. If it is 8 ft. wide, and the flooring is $2\frac{1}{2}$ in. wide and we allow 3% for squaring, what will the lumber cost at $60 per M.?

4. At $60 per M. how much will it cost to ceil the four walls and ceiling of a room 12 ft. by 18 ft. and 10 ft. high with matched lumber 3 in. wide, and allow 25 ft. for squaring?

5. A room 12 ft. 6 in. wide and 16 ft. long is to be floored with flooring $\frac{7}{8}$ in. thick and $2\frac{1}{2}$ in. wide; it is also to be wainscoted 4 ft. up from the floor with 3 in. pine $\frac{7}{8}$ in. thick. What will the lumber cost for the floor and wainscoting at $80 per M.?

SHINGLES, SLATING AND TINNING

264. Shingles are generally 16 in. long and of various widths. They are packed in bundles, each being equivalent to 250 shingles 4 in. wide. When 16-in. shingles are exposed 5 in. to the weather, each bundle will cover $250 \times 4'' \times 5'' =$ 5,000 sq. in. The first course of shingles on a roof is usually

laid double. Shingles are sold by the thousand, consisting of four bundles. Roofing tin is generally 20 in. by 28 in. and is put up in boxes of 112 sheets. A box is estimated to cover 360 sq. ft. Slate is usually sold by the square. One square will cover 100 sq. ft. The average roofing slate is 18 in. by 9 in. and laid to expose 7 in. or $7\frac{1}{2}$ in. to the weather. Slate is generally laid so as to expose to the weather $1\frac{1}{2}$ in. less than one half of the length. Thus with 18 in. slate the length exposed is $\frac{1}{2}$ of 18 in. $-$ $1\frac{1}{2}$ in. $=$ $7\frac{1}{2}$ in. And for slate 18 in. long and 9 in. wide the surface exposed is $9 \times 7\frac{1}{2}$ in. $= 67\frac{1}{2}$ sq. in.

When shingles are laid 4 in. to the weather, 9 shingles are required per square foot. When they are exposed $4\frac{1}{2}$ in. to the weather, 8 shingles are required to the square foot. When they are exposed 5 in. to the weather, $7\frac{1}{5}$ are required to the square foot. To make due allowance for defects and waste, it is customary to count 1,000 shingles to a square of 100 sq. ft., when laid 4 in. to the weather. It would then take 10 shingles to the square foot.

PROBLEMS

1. Find the number of shingles for a roof, if the length of the ridge is 40 ft., the length of the rafters 15 ft. and the shingles are laid 4 in. to the weather.

15 ft. $\times$ 2 $\times$ 40 ft. $=$ 1,200 sq. ft. 1,200 $\times$ 10 $=$ 12,000, the number of required shingles $=$ 48 bundles.

2. If shingles are 5 in. wide and laid 6 in. to the weather, how many shingles are required for 200 sq. ft. of roof?

3. How many slates will be required for 1,000 sq. ft. of roofing if the slates are 24 in. long and 14 in. wide and $1\frac{1}{2}$ in. less than one half the length of each slate is exposed to the weather?

4. How many shingles will it take to cover a roof 38 ft. long by 30 ft. wide, if they are laid $4\frac{1}{2}$ in. to the weather?

5. How many boxes of tin will it take to cover a roof 80 ft. long by 24 ft. wide, if the tin is 20 in. by 28 in. and a box contains 112 sheets and covers 360 sq. ft.?

6. When shingles are $3 per M., how much will it cost to shingle a roof 40 ft. long by 20 ft. wide. if the shingles are laid $4\frac{1}{2}$ in. to the weather?

7. What will it cost at $4 per 1,000 to shingle a roof 16 ft. wide on a side, with ridge 25 ft. long, and shingles laid 5 in. to the weather?

8. What will it cost at $2.50 per 1,000 to shingle a hip roof of a house which is 24 ft. square, the rise of the roof being 9 ft., the eaves projection 18 in. and shingles laid 5 in. to the weather?

9. The dimensions of a house are 24 ft. by 22 ft. and the roof has a 7 ft. rise; what will it cost to shingle the two gables with shingles laid 5 in. to the weather, at $3 per thousand?

10. A round tower is 8 ft. in diameter, and 20 ft. high. How many shingles will it take to shingle the sides, if laid 4 in. to the weather?

11. At $1.50 per bundle of 250 shingles each, what will it cost to shingle a double roof, each half measuring 50 ft. by 26 ft., if each shingle covers 5 in. by $4\frac{1}{2}$ in.?

12. Find the cost of slating a roof 50 ft. by 40 ft. at $15 per square.

13. How many slates are required to slate a roof, each side of which is 50 ft. by 20 ft., if each slate covers 9 in. by 7 in.?

14. Estimating 1,000 shingles for $1\frac{1}{5}$ squares, how many bundles of shingles should be ordered for a roof containing 5,500 sq. ft.? (100 sq. ft. = 1 square.)

15. How many slate shingles, each 6 in. wide, would be required for a roof, each side of which is 36 ft. by 18 ft., allowing $4\frac{1}{2}$ in. to the weather?

16. How many shingles would be required for two sides of a roof, each 18 ft. by 30 ft., estimating 1,000 shingles for 120 sq. ft.? (120 sq. ft. = $1\frac{1}{5}$ squares.)

17. How many shingles will be required to cover a roof, each side of which is 42 ft. long by 18 ft. wide, allowing 800 shingles per 100 sq. ft.?

18. What will it cost to tin a roof, each side of which is 34 ft. long by 16 ft. wide at $8 per square (100 sq. ft.)?

19. How many shingles will be required to cover a roof having two sides each of which is 25 ft. by 20 ft., estimating 1,000 shingles to cover $1\frac{1}{4}$ squares? (125 sq. ft. = $1\frac{1}{4}$ squares.)

20. If 20 in. by 10 in. slate is used for roofing and 3 in. allowed for lap, how many slates will be required for a roof 40 ft. long by 25 ft. wide on each side?

STAIR BUILDING

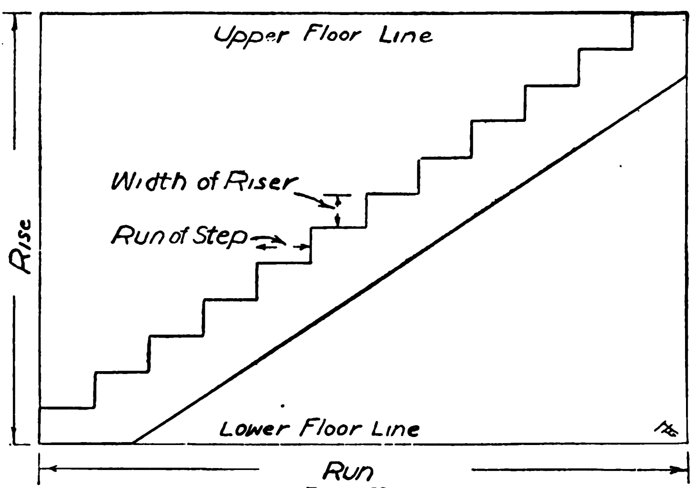

FIGURE 38.

265. The *rise* of a flight of stairs is the perpendicular distance between the bottom of the flight and the top, or between the two floor lines. Figure 38.

266. The *run* of a flight of stairs is the horizontal distance from the foot of the stairs to a point directly under the top of the stairs.

267. The *rise* of a step is the height of a step. The *tread* is the width of a step, or the part of the stairs we step on when going up or down.

268. The *nosing* is the front edge of a step, and is generally rounded and projects over the riser. A 7 to $7\frac{1}{2}$-in. rise, and 9 to 10-in. tread make a very easy stairway.

269. *Stringers* are boards on sides of stairs, cut for the insertion of the treads and risers.

270. Rule: *To find the number of steps for a flight of stairs, divide the rise of the flight by the number of inches in the desired height of the steps. The whole number obtained will be the number of steps. If you obtain a whole number and a fraction, disregard the fraction, and divide the rise again by this whole number. The result will be the width of each riser.*

A ceiling 9 ft. high, allowing 1 ft. for upper floor, joist and plastering, gives 10 ft. for height of stairs. If 7 in. is the desired height for each step, divide 10 ft. by 7 in. The result is $17\frac{1}{7}$. Now divide 10 ft. by 17, and the result is $7\frac{1}{17}$. The *whole* number 17 is the number of steps. The width of the risers is $7\frac{1}{17}$. Or the height of each step is $7\frac{1}{17}$ in.

271. Rule: *To find the width of the treads, divide the run of the stairs by the number of steps, and add the amount allowed for the nosing.*

If the *run* is 12 ft. 9 in., and the number of steps is 17, divide 12 ft. 9 in., or 153 in., by 17, and 9 in. is the result. To this 9 in. add $1\frac{1}{2}$ in. for nosing. This gives $10\frac{1}{2}$ in., the width of the steps, or the *width* of the treads.

PROBLEMS

1. The rise of a flight of stairs is 15 ft. If 7 in. is the desired height for each step, how many steps will there be? If the run is 18 ft. 9 in., how wide will the steps be?

2. The risers of a flight of stairs are $7\frac{1}{2}$ in. wide. How many steps will there be, if the rise of the stairs is 7 ft.?

3. The rise of a flight of stairs is 9 ft. and the run is 8 ft. 6 in. If there are 13 steps, what will be the width of the treads, and what the width of the risers?

4. What is the length of the stringer of a pair of stairs, if the rise is 10 ft., and the run is 9 ft. 4 in.?

5. How many feet of lumber are required for the two stringers of the stairs in example 4, if the stringers are 10 in. wide, 1 in. thick, and $1\frac{1}{2}$ ft. on the length of each stringer is allowed for cutting on a slant?

6. The distance from the floor to the ceiling of a room is 8 ft. How many steps will be required for a pair of stairs to the upper floor, if the width of the treads is 11 in. and width of risers 7 in.?

7. The distance from a lower floor to the surface of an upper floor is 15 ft. The run is 20 ft, and the desired height of each step is 7 in. How many steps will there be, and what will be the width of the steps?

8. The distance between floors in a house is 15 ft. The stairs must go part way to a platform, and then turn at right angles for the rest of the way. The height of the platform above the first floor is $7\frac{1}{2}$ ft. The riser of the steps is 7 in., and the treads 11 in. with projection of $1\frac{1}{2}$ in. nosing. How many steps will each flight require? If 1-in. lumber is used for risers and 2-in. lumber is used for treads and length of steps is 5 ft. 6 in., how many feet of lumber will be required for treads and risers for the steps from floor to floor?

9. How many feet of lumber will be required for the steps (treads and risers) of a back stoop the height of which is 6 ft. 2 in., the run 6 ft. 4 in., and the length 5 ft. 8 in., the risers to be of $\frac{7}{8}$-in. stock, and the treads of $1\frac{1}{4}$-in. stock, and to project over the risers $1\frac{1}{2}$ in.?

FRAMEWORK

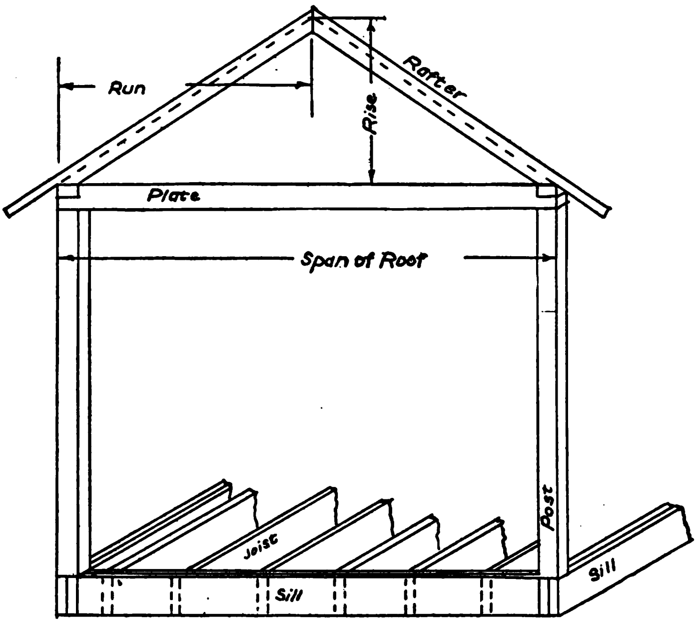

FIGURE 39

272. Formerly, frame houses were of the braced-frame type; that is, they were fastened together with mortise and tenon joints and braced. Houses at the present time are generally of the balloon-frame type. Figure 39 is a balloon-framed house. The braced-frame houses were built of much heavier and larger timbers than the balloon-framed houses of to-day. The sills of these old houses were 8″ × 8″, 8″ × 6″, or 6″ × 6″. The posts were 4″ × 4″ or 4″ × 6″ and mortised into the sills at the corners, and also mortised into heavy plates above, and all corners braced by mortise and tenon joints. In the balloon-frame house the sills are 2″ × 4″ or 6″ timbers, and the posts are generally 2″ × 4″ timbers, spiked together. All parts

are spiked together only. There are no mortise and tenon joints. The names of some of the parts of the frame are given in the figure.

PROBLEMS

1. If Figure 39 represents a house 24′ × 32′, with posts 15 ft. long, sills 6 in. sq., posts 4″ × 4″, plates doubled 2″ × 4″, and splices on sills take up 1 ft. on each piece; how many feet of lumber will be required for sills, posts and plates?

2. How many feet of lumber will be required for the sills of a house, 23′ × 38′, the timber to be 8″ × 8″ × 16′ long, allowing for two splices on each long side and one splice on each short side, one foot in length for each splice?

3. Find the amount of lumber required for the sills, floor joists and one girder to support the floor joists of a building 16′ × 24′. The sills are from 8″ × 8″ × 16′ long pieces. The joists are from 2″ × 10″ × 12′ long pieces. The girder is 8″ × 8″ and 16 ft. long. The joists are placed 16 in. apart, from center to center and 8 in. from the outer walls of the building.

4. How many pieces of studding will be required for the sides of a building 24 ft. long, allowing two pieces at each end, and studding placed 16 in. from center to center? (24 ft. = 288 in. 288 ÷ 16 = 18 spaces. Not counting the ends, the number of studding is 1 less than the number of spaces.)

5. How many pieces of studding are required for four sides of a house (16 in. on centers) 16 ft. by 24 ft., allowing double studding at the corners?

6. How much will 2″ × 4″ studding cost at $40 per thousand for the walls of a house, 24′ × 36′ and height of building from sills to top of plates 18 ft.?

7. How many floor joists will be required for a house 18 ft. wide by 26 ft. long, and placed 16 in. from center to center, and the first joist placed 12 in. from the sill?

8. Find the number of feet of lumber required for sills, studs, and plates of a building 18′ × 24′ and height from sills to plates 16 ft. The sills are 8″ × 8″ and the studs are 2″ × 4″ lumber and the plates also.

9. How much will the floor joists and the floor cost for the house in example 7, if the joists are 2″ × 8″ and 18′ long, and the flooring is $1\frac{1}{2}$″ wide and $\frac{7}{8}$″ thick at 40c. per M.? Add $\frac{1}{3}$ for matching.

10. Find the cost at $40 per M. for the studding and plates of a building 22′ × 24′ and plates 18′ above the sills. The plates are 2″ × 4″ doubled and miter-jointed. The studs are 2″ × 4″ and doubled at the corners of the building.

RAFTERS

273. Figure 40 represents a house frame 16 × 24 ft. How many feet of lumber must be bought for sill, if 4″×8″×18′ long lumber is to be used and splices are 6″ long?

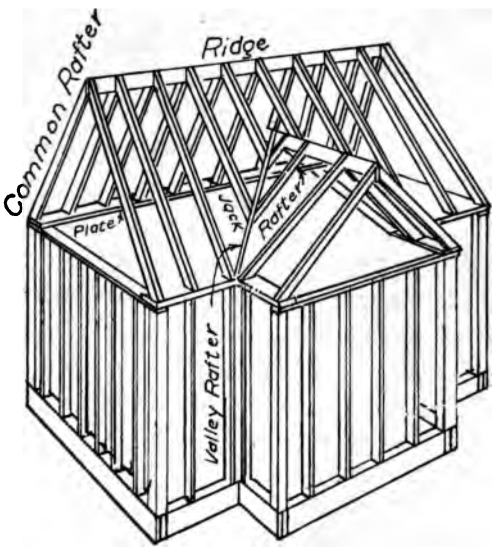

FIGURE 40

If the plate in Figure 40 is 16 ft. above the sill, how many feet of lumber must be bought for studding and posts, if 2″ × 4″ × 18′ lumber is used and the studding is placed 16″ from center to center with double studding at the corners for posts?

How many feet of lumber must be bought for the plate, if 2″ × 6″ × 18′ lumber is used, and one 6-in. splice on each long side is made?

274. The rise of the rafters is the height of the ridge above the level of the plate. A $\frac{1}{2}$ pitch roof is when the height of the ridge is $\frac{1}{2}$ the width of the building. If in Figure 40 the width of the building is 24 ft. and the rise of the rafters, or the height of the ridge above the plate, is 12 ft. or $\frac{1}{2}$ the width of the building, the pitch is $\frac{1}{2}$.

A $\frac{1}{3}$ pitch roof is when the height of the ridge is $\frac{1}{3}$ the width of the building. A $\frac{1}{4}$ pitch is when the height of the ridge is $\frac{1}{4}$ the width of the building.

The height of the ridge is the perpendicular of a right-angled triangle, and $\frac{1}{2}$ the width of the building is the base of a right-angled triangle.

The length of the rafter is the hypotenuse of the right-angled triangle.

For a building 32 ft. wide, $\frac{1}{2}$ of 32, or 16 ft., is the base; and for a $\frac{1}{4}$ pitch roof, $\frac{1}{4}$ of 32, or 8 ft., is the perpendicular. This is a $\frac{1}{4}$ pitch roof.

The length of the rafters is the square root of the sum of the base 16 ft. and the perpendicular 8 ft. which is 17.8 ft. (256 + 64 = 320 ft. The square root of 320 ft. is 17.8 ft.) To this add the projection, and we have the length of the rafters.

275. The steel square is used much in finding the length of rafters. If a building is 24 ft. wide, the $\frac{1}{2}$ width, or run, is 12 ft. For a $\frac{1}{2}$ pitch roof mark the 12 in. point on the tongue of the steel square, and the 12 in. point on the blade of the square. ($\frac{1}{2}$ of

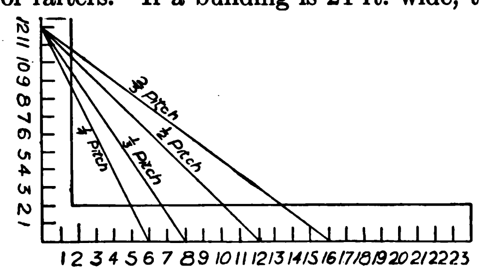

FIGURE 41

24 = 12.) Measure the distance between these points. It is the hypothenuse of a right-angled triangle, and is the length of the rafters, less the projection. Add the projection.

For a $\frac{2}{3}$ pitch roof, mark the 12 in. point on the tongue of the square, and the 16 in. point on the blade. (Width is 24 ft., and $\frac{2}{3}$ of 24 ft. = 16 ft.)

Measure the shortest distance between these points. This is the length of the rafters, without the projection, in feet, for a house 24 ft. wide, with a $\frac{2}{3}$ pitch.

For a $\frac{1}{4}$ pitch roof on a 24 ft. wide building, mark the 12-in. point on the tongue of the square and the 6-in. point on the blade. ($\frac{1}{4}$ of 24 = 6.)

The distance between these points is $13\frac{15}{32}$ ft. (Inches on the square are feet on the rafters.)

The carpenter usually calculates the length of rafters and other dimensions from scale drawings, by use of the triangular scale or some other suitable scale.

To find the length of rafters for buildings of any other width by use of the steel square, use the square for a 24 ft. wide building, and take the fractional part of the length between the points, that the width is of a 24-ft. wide building. For a 32-ft. wide building take $\frac{32}{24}$ of the length for a 24 ft. wide building. For a 22-ft. wide building take $\frac{22}{24}$ of the length for a 24-ft. wide building.

Find the length of rafters for a 36-ft. wide house at $\frac{2}{3}$ pitch.

For $\frac{2}{3}$ pitch on a 24-ft. wide house ($\frac{2}{3}$ of 24 ft. = 16 ft.). Mark the 12-in. point on the tongue of the square and the 16-in. point on the blade.

Measure the distance between these points. This is $20\frac{1}{16}$ in. on the square, and $20\frac{1}{16}$ ft. on the rafters for a building 24 ft. wide. For a building 36 ft. wide, take $\frac{36}{24}$ of $20\frac{1}{16}$ ft. = $30\frac{3}{32}$ ft. or 30 ft. and $1\frac{1}{8}$ in.

Find the length of rafters for a 36-ft. wide house at $\frac{1}{4}$ pitch.

For $\frac{1}{4}$ pitch on a 24-ft. wide house ($\frac{1}{4}$ of 24 = 6) mark the 12-in. point on the tongue of the square and the 6-in. point on the blade. Measure the distance between these points. This is $13\frac{15}{32}$ in. on the square and $13\frac{15}{32}$ ft. on the rafters. $\frac{36}{24}$ of $13\frac{15}{32}$ ft. = 20 ft. $2\frac{7}{16}$ in.

PROBLEMS

1. What is the length of the rafters with an 18-in. projection, for a $\frac{1}{4}$-pitch roof, on a house 36 ft. wide?

2. What is the length of the rafters with a 2-ft. projection, for a $\frac{3}{4}$ pitch roof, on a building 26 ft. wide?

3. The rise of a roof is 4 ft. and the width of the building is 40 ft. What is the length of the rafters with a 2-ft. projection?

4. What is the pitch of a roof, if the width of the building is 24 ft. and the rise of the roof is 16 ft.? What is the length of the rafters, if they project 18 in. over the side of the building?

5. How many feet of lumber will be required for the rafters of a building 30′ × 40′; roof $\frac{1}{2}$ pitch and 2″×6″ rafters placed 2 ft. from center to center, and eaves projecting 18 in.?

6. A house is 36 ft. wide and has a $\frac{3}{4}$ pitch. How high is the ridge above the plate?

7. How many and what length rafters will be required for a house 24 ft. wide by 30 ft. long, with a $\frac{3}{4}$ pitch, and rafters placed 2 ft. from center to center?

8. How many feet of lumber in 4 hip rafters, 2″ × 8″ if the building is 20 ft. square and the rise of the roof is 10 ft., allowing 18 in. for eaves projection?

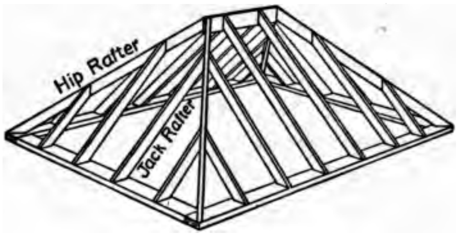

FIGURE 42.

9. The rise of a hip rafter* is 10 ft. and the run is 18 ft. What is its length?

10. A building is 24 ft. square and the rise of the roof is 12 ft. What is the length of the hip rafter?

11. The diameter of a circular tower is 16 ft. and its roof pitch is $\frac{2}{3}$. If the rafters are $2'' \times 6''$ and are placed 30 in. from center to center on the plate, how many feet of lumber will be required for the rafters, and how long will they be, allowing 18 in. for roof projection?

12. A house is built in the form of an L. The width of each part is 20 ft., the rise of the rafters on both parts is the same, with a $\frac{1}{2}$-pitch roof. What is the length of the common rafters? What is the length of the valley rafters? (Runs are sides of squares. The valley run is the diagonal. $\text{Run}^2 + \text{Run}^2 = \text{Run}^2$ of Valley Rafter. Run^2 of Valley Rafter $+ \text{Rise}^2 = \text{Valley Rafter}^2$. $\text{Run}^2 + \text{Rise}^2 = \text{Common Rafter}^2$.)

13. A house is built in the form of an L. The width of each part is 18 ft., and height of ridge is the same in both with a $\frac{2}{3}$ pitch of roof. What is the length of valley rafters?

14. A square building $20' \times 20'$ has a $\frac{3}{4}$ pitch roof. What is the height of the highest point of roof? What is the length of hip rafters? ($\text{Run}^2 + \text{Run}^2 = \text{Run}^2$ of hip rafter.) (Run^2 of hip $+ \text{Rise}^2 = \text{Hip}^2$.)

15. Find the lengths of the jack rafters on a building 20 ft. by 20 ft. with $\frac{1}{2}$-pitch roof, and jack rafters spaced 2 ft. apart. (Find common rafter from $\text{Run}^2 + \text{Rise}^2 =$

*A hip rafter extends from the wall plate to the ridge and forms the angle of a hip roof, or one having sloping ends and sloping sides.

Common rafter². Common rafter = 14.14. Then by proportion base of triangle is to altitude as base of smaller triangle is to its altitude. 10 : 14.14 :: 8 : ? 10 : 14.14 :: 6 : ? 10 : 14.14 :: 4 : ? 10 : 14.14 :: 2 : ?)

Jack rafters are the short rafters used in a hip roof.

16. Find the length of the common rafters, and the lengths of the jack rafters on a building 12′ × 24′ with a roof of ½ pitch.

PAINTING, PLASTERING AND LATHING

276. One pound of paint covers about 4 sq. yds. of surface for first coat, and from four to six yards for each additional coat. Colored paint will cover about ⅓ more than white paint. One gallon of prepared paint will cover about 300 sq. ft. of wood surface two coats.

Wooden laths are 4 ft. long, and generally 1½ in. wide, and for lime plaster are laid ⅜ in. apart. They are put up in bundles of 50 or 100 lath to a bundle. About 14 or 15 laths are required to the square yard, and 1,400 or 1,500 laths are required for 100 sq. yds.

PROBLEMS

1. How many laths will it take to cover both sides of a partition 8 ft. high and 14 ft. long, if 50 laths cover 25 sq. ft.?

2. A room is 10 ft. high, 20 ft. and 6 in. wide, and 30 ft. long. How many laths and how many pounds of lath nails will it require for the overhead and side walls? (9 lbs. of 3-penny nails are required for the laths for 100 sq. yds. of plastering.)

3. How much mortar is required to cover the walls and ceiling of a room that is 12 ft. wide, 13 ft. long, and 10 ft. high, if one bushel of mortar covers 3 sq. yds.?

4. How many bundles of laths, 100 laths to the bundle, are required for 4 rooms, each room 18′ × 12′ × 10′ high,

deducting 28 sq. ft. for openings in each room, and allowing 15 laths to the sq. yd.?

5. What will it cost to paint the floors of these rooms at 27 cents a square yard, and kalsomine the four walls and ceiling of each at 18 cents per square yard?

6. If it costs 40 cents per square yard for plastering, what will it cost to plaster the walls and ceiling of a room 20 ft. long and 14 ft. wide, and 9 ft. high?

7 What will it cost at 50 cents a square yard to plaster a room $16' \times 12'$ and 9 ft. high? What will it cost to lath this room at 50 cents per bundle of 50 laths?

8. How much mortar will it take to plaster a room $18' \times 14' \times 9'$ high, if one bushel covers 3 sq. yds.?

9. Find the cost of plastering a room $24' \times 28' \times 11'$ high, allowing for a door $6' \times 8'$ and 2 windows $8' \times 7\frac{1}{2}'$ each, at 22 cents per square yard.

10. At 70 cents per bundle put on, what will it cost to lath a room $48' \times 30' \times 12'$ high? (A bundle of lath covers 6 sq. yds.).

11. How many gallons of paint will be required to give two coats to a board fence 200 ft. long by 12 ft. high, if 1 gallon covers 300 sq. ft.?

12. What is the cost of painting three floors in a house at 10 cents per square yard, if one floor is $15' \times 20'$, another $12\frac{1}{2}' \times 14\frac{1}{2}'$ and the third is $22' \times 18'$?

MASONRY AND EXCAVATING

277. A perch of stone or masonry is $24\frac{3}{4}$ cubic feet. A bag of cement is 1 cubic foot.

PROBLEMS

1. The walls of a cellar are $30' \times 20' \times 9'$ high, and 2 ft. thick, outside measure. How many cubic feet of masonry do they contain?

$2 \times (20 + 30) \times 9 \times 2 = 1{,}800$ = the estimated number of cubic feet. $4 \times 4 \times 9 = 144$ the number of cubic feet to be deducted for corners. $1{,}800 - 144 = 1{,}656$ cu. ft.

2. How many cubic yards of stone in the walls of a cellar $26' \times 18' \times 9'$ high, and 18 in. thick, measured on the inside? Add the corners.

3. What will it cost at $5 per perch, to build a cellar wall $30' \times 20'$ outside measurement, the wall to be 10 ft. high and 2 ft. thick?

4. What will it cost to excavate for a cellar, the outside measurement of the wall of which is 36 ft. long, 32 ft. wide, and 8 ft. deep, below the surface, at 50 cents per cubic yard?

5. Find the number of bricks required to build the four walls of a house 40 ft. long, 36 ft. wide, outside measurement, and 20 ft. high, the walls being 2 bricks thick.

6. Find the cost of building concrete cellar walls for a house 36 ft. long, 24 ft. wide, 9 ft. high, and 9 in. thick, at $7.50 a cubic yard.

7. How many bags of cement are required for a wall 36 ft. long, 8 ft. high and 2 ft. thick? The concrete is made by mixing 1 part of cement with 2 parts of sand and 4 parts of stone.

8. A load of earth contains 1 cu. yd., how many loads of earth in the cellar of a house 36 ft. long, 28 ft. wide and 9 ft. deep?

BRICK WORK

278. The common dimensions of brick are $8'' \times 4'' \times 2'' = 64$ cu. in. $1{,}728 \div 64 = 27$, the number of brick to a cubic foot. Bricks of these dimensions when laid in mortar, courses of mortar being $\frac{1}{4}$ in. thick, run 24 bricks to the cubic foot. It is generally assumed that 22 bricks with mortar

will make 1 cu. ft. Seven bricks are required for each superficial foot of wall when the wall is 4 in. thick, or 1 brick thick; 14 bricks, if the wall is two bricks thick; and 28 bricks, if the wall is 4 bricks thick.

The number of bricks required to build a wall 10 ft. long, 12 ft. high, and 1 brick thick is $10 \times 12 \times 7 = 840$ bricks.

279. To find the number of thousands of bricks for a wall, *multiply the number of square feet of wall surface by the number of bricks wide in the thickness of the wall, and divide by 160. This quotient will give the number of thousands of bricks.*

This rule is based on the fact that a 4-inch wall, or a wall one brick wide in thickness, contains about 1,000 bricks to each 160 square feet of wall surface, if the mortar joints are $\frac{1}{4}$ inch wide.

Example: A wall 12 inches thick, and 80 feet long by 20 feet high will contain 30,000 bricks. $(80 \times 20 \times 3) \div 160 = 30{,}000$.

A 12-in. wall is 40 ft. long and 12 ft. high. How many bricks will it contain?

$$\frac{40 \times 12 \times 3}{160} = 9{,}000.$$

Or, find the number of cubic feet in the entire wall and multiply by 27, which is the number of common bricks to the cubic foot. One brick $= \frac{1}{27}$ cu. ft.

Or, for a 4-in. wall of common brick, (width one brick) multiply the square feet surface by $7\frac{1}{2}$ = the number of bricks.

For a 9-in. wall (width 2 bricks + mortar), multiply square feet surface by 15 = the number of bricks.

For a 13-in. wall, (width of 3 bricks + mortar), multiply square feet surface by $22\frac{1}{2}$ = the number of bricks.

For a 18-in. wall, (width 4 bricks + mortar), multiply square feet surface by 30 = the number of bricks.

For a 22-in. wall, (width 5 bricks + mortar), multiply square feet surface by 38 = the number of bricks.

For a 24-in. wall, (width 6 bricks + mortar), multiply by 45 = the number of bricks. Allowing for mortar, one brick = $\frac{1}{22}$ cu. ft. and 22 bricks = 1 cu. ft.

PROBLEMS

1. A 16-in. wall is 30 ft. long and 36 ft. high. How many common bricks does it contain?

2. A 20-in. wall is 20 ft. long and 40 ft. high. How many bricks does it contain?

3. A 12-in. wall is 36 ft. long and 20 ft. high. How many bricks does it contain?

4. A 4-in. wall is 36 ft. long and 20 ft. high. How many bricks does it contain?

5. An 8-in. wall is 40 ft. long and 30 ft. high. How many bricks does it contain?

6. The length of a foundation wall around a house is 120 ft., the height is 2 ft. and the width of the wall is 22 in. How many bricks does it contain?

7. The basement wall of house in example 6 is 13 in. thick and 8 ft. high to top of first floor. How many bricks does it contain?

8. From the top of first floor to roof plates, in this house in example 6, is 13 ft. The wall is a 9 in. wall. There are 4 windows 3′ × 3′ 6″ each, and 1 door 3′ × 7′. How many bricks does this part of the wall contain?

9. A plain brick building is 18′ × 24′ × 15′ high, with a half pitch roof, and an 8-in. wall; two 3′ × 5′ windows on each side, and one door 3′ × 7′. How many bricks will be required?

10. How many bricks will be required to build 4 piers, each 3′ × 4′ and 8 ft. high?

11. A brick garden wall is 50 ft. long and 30 ft. wide. It is 10 ft. high and has one opening for a gate 6 ft. wide and 10 ft. high. How many bricks will be required to build the wall 3 bricks thick?

12. How many bricks will it take for 2 16-in. square brick piers 7 ft. high?

13. How many bricks will be required for 4 brick piers, each 3′ × 4′ and 8 ft. high, if 17 bricks will build one cubic foot?

280. The number of bricks required for a chimney.

Bricks are generally 8″ × 4″ × 2″. For a common chimney with a flue 4″ × 8″, and taking into account the thickness of the mortar, 5 courses of 5 bricks in a course, or 25 bricks, will be necessary for 1 ft. of height of a chimney. Or, 5 courses of brick laid in a chimney will make 1 ft. of height.

For a flue 8″ × 8″ 6 bricks are necessary for a course, and for 5 courses or 1 ft. of height, 30 bricks will be needed.

For a flue 8″ × 12″ 7 bricks are needed for a course, and for 5 courses, or 1 ft. in height, 35 bricks will be needed.

For a flue 12″ × 12″ 8 bricks will be needed for a course, and for 5 courses or 1 ft. in height, 40 bricks will be needed.

281. To find the number of bricks needed for a chimney *of any height, count the number of bricks required for a single course, and 5 times this number will give the number for 1 ft. in height. This number multiplied by the height of the chimney gives the number of bricks required.*

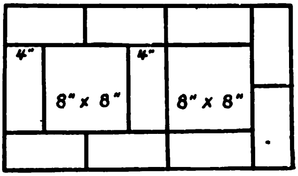

FIGURE 43

What number of bricks will be needed for a chimney 30 ft. high, with 2 flues 8″ × 8″ each? In Figure 43, we find that 10 bricks are needed for the course. Then 10 × 5 = 50 bricks are needed for 1 ft. high. And, therefore, for 30 ft. high 30 × 50 = 1,500, the number of bricks needed.

PROBLEMS

1. How many bricks will it require for a chimney with three flues each 8″ × 12″ and 40 ft. high?

2. How many bricks will it require for a chimney 25 ft. high with one flue 8″ × 8″ and another flue 8″ × 12″?

3. How many bricks will it require for a chimney 30 ft. high with 3 flues of 8″ × 8″, 8″ × 12″ and 12″ × 12″?

4. How many bricks will it require for a chimney 25 ft. high and two flues 12″ × 12″ each?

CONCRETE WORK

282. To find the number of concrete blocks *required for a wall, divide the external surface of the wall by the external surface of one block.*

Each block of the following groups of sizes contains the number of square feet of external surface given opposite the group. A block whose dimensions are 8″ × 4″ × 16″ has 8″ × 16″ = 128″ of external surface. $\frac{128''}{144''} = \frac{8}{9}$ sq. ft. of external surface.

$$\left.\begin{array}{l} 8'' \times 4'' \times 16'' \\ 8'' \times 8'' \times 16'' \\ 8'' \times 10'' \times 16'' \\ 8'' \times 12'' \times 16'' \end{array}\right\} = \tfrac{8}{9} \text{ sq. ft. of external surface.}$$

$$\left.\begin{array}{l}4'' \times 8'' \times 16'' \\ 4'' \times 10'' \times 16'' \\ 4'' \times 12'' \times 16''\end{array}\right\} = \tfrac{4}{9} \text{ sq. ft. of external surface.}$$

$$\left.\begin{array}{l}8'' \times 4'' \times 24'' \\ 8'' \times 8'' \times 24'' \\ 8'' \times 10'' \times 24'' \\ 8'' \times 12'' \times 24''\end{array}\right\} = \tfrac{4}{3} \text{ sq. ft. of external surface.}$$

$$\left.\begin{array}{l}4'' \times 8'' \times 24'' \\ 4'' \times 10'' \times 24'' \\ 4'' \times 12'' \times 24''\end{array}\right\} = \tfrac{2}{3} \text{ sq. ft. of external surface.}$$

Thus a wall 320 ft. long and 10 ft. high contains 3,200 sq. ft. And 3,200 sq. ft. ÷ $\frac{8}{9}$ sq. ft., any one of the sizes from the first group = 3,600 blocks, the number of blocks required for the wall.

Or, if we use blocks from the second group, 3,200 sq. ft. ÷ $\frac{4}{9}$ = 7,200 blocks. From the third group, 3,200 sq. ft. ÷ $\frac{4}{3}$ = 2,400 blocks.

If we use 1 barrel ($3\frac{3}{4}$ cu. ft.) of cement; 3 barrels ($11\frac{1}{4}$ cu. ft.) of sand and 6 barrels ($22\frac{1}{2}$ cu. ft.) of gravel, we have a 1 : 3 : 6 mixture. Or, an ordinary mixture.

When one barrel of cement is used, these quantities give $37\frac{1}{2}$ cu. ft. of material. One block 8″ × 4″ × 16″ = $\frac{8}{27}$ cu. ft.

And $37\frac{1}{2}$ cu. ft. ÷ $\frac{8}{27}$ cu. ft. = 126, the number of blocks to be made from this amount of the material.

With a 1 : 5 mixture, 1 barrel ($3\frac{3}{4}$ cu. ft.) cement, and 5 barrels ($18\frac{3}{4}$ cu. ft.) sand, we have $22\frac{1}{2}$ cu. ft. of material. If we divide $22\frac{1}{2}$ cu. ft. by the number of cu. ft. in any solid cement block we have the number of blocks that can be made from this amount of material. $22\frac{1}{2}$ cu. ft. ÷ $\frac{8}{27}$ (8 × 4 × 16) block = 76 nearly. In hollow blocks, the hollow space is about 30% of the whole block. So, only about 70% of as much material would be needed for hollow blocks as for solid blocks of the same size. With the same block

70% × $\frac{8}{27}$ cu. ft. = $\frac{28}{135}$ cu. ft. And $22\frac{1}{2}$ cu. ft. ÷ $\frac{28}{135}$ = 108.5 blocks. Or, 76 blocks ÷ .70 = 108.5 blocks when hollow.

CONCRETE REINFORCEMENT

283. The amount of steel needed is often specified as a percentage of the volume of the concrete to be reinforced. If the reinforcement is 1% of the volume of concrete which is to be reinforced, $\frac{1}{100}$ of 1 cu. ft. of steel needed for each cubic foot of cement is $\frac{1}{100}$ cu. ft. One cubic foot of steel weighs about 490 lbs. and $\frac{1}{100}$ of one cubic foot would weigh $\frac{1}{100}$ × 490 lbs. = 4.90 lbs. for 1 cu. ft. of concrete. If $1\frac{1}{4}$% of steel is needed, then $1\frac{1}{4}$ × 4.90 lbs. = 6.125 lbs. for each cu. ft. of cement will be needed. If $\frac{1}{4}$% of steel is needed, then $\frac{1}{4}$ of 4.90 lbs. = 1.225 lbs. will be needed for each cubic foot of cement.

PROBLEMS

1. How many concrete blocks will it require to build a concrete block basement wall, if length is 24′ × 36′ × 8′ high, and wall is one block thick, and 8″ × 12″ × 24″ blocks are used?

2. If the above wall is made of concrete, how many cubic feet of concrete are required, and how many barrels of cement, if the concrete is a 1 : 3 : 6 mixture?

3. If a cellar floor is laid in this basement of concrete 3 in. thick, how many cubic feet of concrete and how many barrels of cement will be required?

4. If a $\frac{3}{4}$-in. wearing coat is placed on the above floor, how many cubic feet of a 1 : 2 mixture, and how many barrels of cement will be required?

5. Four piers 2′ × 2′ × 6′ high are made of concrete, by using a 1 : 2 : 4 mixture. How many cubic feet of concrete are required? How many barrels of cement are required?

6. How many cubic feet of concrete are required for a chimney 20 ft. high; outside dimensions 16″ × 20″, with 8″ × 12″ flue and walls 4 in. thick?

7. It is required to reinforce 1,280 cu. ft. of cement by a 1% reinforcement. How many pounds of steel are required?

STRENGTH OF MATERIALS

284. The relative breaking, or tensile, strength of some metals is about as follows: Lead 1; silver 14; copper 20; iron 30. A silver wire will support 14 times as much weight as a lead wire of the same size. An iron wire will support $1\frac{1}{2}$ times as much weight as a copper wire of the same size.

The tensile strength of steel is about 100,000 lbs. per square inch. A steel rod of cross-section area of one square inch will, therefore, support about 100,000 lbs.

If a 10-lb. load is attached to a wire whose cross-section area is .002″, then the stress per square inch is 10 lbs. ÷ .002 = 5,000 lbs. The strain that this wire gets with 10 lbs. is the same as the strain that a wire of 1 sq. in. section area gets with a load of 5,000 lbs.

The amount of lengthening or shortening of a piece under stress is found by the following formula: $e = \frac{P\ L}{S\ C}$.

P = stress in pounds; L = length in inches; S = area in square inches; C = coefficient of elasticity; e = the number of inches of lengthening or shortening of a piece under stress.

PROBLEMS

1. How much will a round bar of steel 100 ft. long and 2 in. in diameter be lengthened by a pull of 60,000 lbs.? The coefficient of elasticity of steel is 30,000,000.

$$e = \frac{P\ L}{S\ C} = \frac{60{,}000 \times 100 \times 12}{D^2 \times .7854 \times 30{,}000{,}000} = .763.$$

The bar would, therefore, be lengthened .763 inch.

2. By how much would a round bar of cast-iron 50 ft. long, and 2 in. in diameter be lengthened by a pull of 20 tons? The coefficient of elasticity for cast-iron is 16,000,000.

3. By how much will a round bar of wrought iron be shortened, if its length is 20 ft., its diameter 4 in., and it is supporting a weight of 100 tons? (C = 29,000,000 for wrought iron.)

4. How much would a round bar of steel lengthen with a pull of 67,200 tons, if the length is 120 ft. and the diameter is 2 in.?

5. How much will a rod of brass lengthen with a pull of 2 tons, if its length is 150 ft. and its diameter is .5 in.? (C = 14,000,000 for brass.)

6. How much would a wrought iron bar 48 ft. long elongate under a pull of 10 tons, if the diameter were $1\frac{1}{2}$ in.?

7. If a 2-lb. weight stretches a wire .04 of an inch in diameter .02 of an inch longer, how much longer will a 16 lb. weight stretch a wire .08 of an inch in diameter? (Areas are to each other as the squares of their like dimensions). .08″ diameter is 4 times .02″ diameter, and 4^2 = 16. The same weight would stretch the larger one $\frac{1}{16}$ as much as the smaller one.

16 lbs. is 8 times 2 lbs. And 8 times $\frac{1}{16} = \frac{1}{2}$ as far as the shorter.

8. How many times greater must the diameter of one wire be than that of another of the same material, to have 4 times the tensile strength?

9. If 20 lbs. stretch a wire of a given length and diameter .5 of an inch, what weight will be required to stretch a wire of the same material and length, but of 4 times the diameter, through 1.5 in.?

The tensile strengths of wires of the same material are proportional to their cross-sections, or, proportional to the squares of their diameters. If the tensile strength of iron

is $\frac{77}{51}$ times as much as that of copper, copper must have a cross-section 1.51 times as great as that of iron to sustain the same weight. $(77 \div 51 = 1.51)$. And the copper wire must have a diameter of $\sqrt{1.51} = 1.23$ times as great as the diameter of the iron wire.

10. If a force of 50 lbs. stretches a wire 1 in., what weight will stretch a wire of the same material and length, but of twice the diameter, 3 in.?

The cross-section is 4 times as great; so it will take 4 times 50 lbs. or 200 lbs. to stretch the wire 1 in.; and to stretch it 3 in. will take 3 times 200 lbs. or 600 lbs.

11. If 10 lbs. stretch a wire $\frac{1}{4}$ in. in diameter $\frac{1}{8}$ of an inch, how far will 70 lbs. stretch a wire of the same length and material but of twice the diameter?

12. If 100 lbs. stretch a wire $\frac{3}{4}$ in. in diameter $\frac{1}{8}$ of an inch, how long will the same kind of wire become, if stretched by 400 lbs. and if it is 1 in. in diameter?

285. Loads. In designing structures, the load on beams should be much less than the ultimate strength of the beams. Therefore, we divide the ultimate strength of the beam by a safety factor for the material used for beams, and take the quotient as the load that the material can safely stand.

286. The safety factor for wood is 8; for stone and brick is 15; for wrought iron is 4; for cast iron is 6; for steel is 5.

287. Ways of loading and supporting beams.

A beam may be fixed at one end and loaded at the free end.

A beam may be fixed at one end and loaded on the whole length.

A beam may be supported at both ends and loaded in the middle.

A beam may be supported at both ends and loaded on the whole length.

Doubling the breadth of a beam doubles its strength and also its stiffness.

Doubling its depth gives four times the strength, and eight times the stiffness.

288. To find the breaking load, *or the strength of a beam, multiply the breadth in inches by the square of the depth in inches by the strength "s" in Table I, and by "S" in Table II, and divide by the length in feet.*

$W = \frac{b \times d^2 \times s \times S}{L}$. W = the weight or load to break the beam; b = the breadth in inches; d = the depth in inches; L = the length in feet.

What is the strength of an oak beam 20 ft. long, 10 in. wide, 12 in. deep, supported at both ends, and loaded in the middle?

$W = \frac{b \times d^2 \times s \times S}{L} = \frac{10 \times 144 \times 600 \times 1}{20} = 43{,}200$, the number of pounds representing the strength.

Table I

Strength = s

Oak	600
Pine	550
Cast Iron	2,540
Wrought iron	3,470
Steel	6,400

Table II

Strength = S

Fixed at one end and loaded at the other	$\frac{1}{4}$
Fixed at one end and loaded all over	$\frac{1}{2}$
Fixed at both ends and loaded in the middle	1
Fixed at both ends and loaded all over	2

PROBLEMS

1. What is the strength of a steel beam 12 ft. long, 8 in. wide, and 4 in. deep, fixed at one end and loaded at the other?

2. What is the strength of a wrought iron beam 12 ft. long, 8 in. wide and 6 in. deep, fixed at one end and uniformly loaded?

3. What is the strength of a steel beam 20 ft. long, 10 in. wide and 8 in. deep, fixed at both ends and loaded in the middle?

4. What is the strength of a cast-iron beam 30 ft. long, 10 in. wide 8 in. deep, fixed at both ends and loaded in the middle?

5. What is the strength of a steel beam 30 ft. long, 4 in. wide, 10 in. deep, fixed at both ends and uniformly loaded?

6. If the breaking load of a steel beam 12 ft. long and 8 in. wide, fixed at both ends and loaded in the middle, is 20,000 lbs., what should be the depth of the beam?

7. If the strength of a cast-iron beam 20 ft. long, 5 in. deep, fixed at both ends and loaded in the middle, is 12,700 lbs., what should be the width?

8. If the breaking load of a pine beam 20 ft. long, 8 in. wide, fixed at both ends and loaded in the middle, is 11,000 lbs., what should be the depth?

PULLEYS

SPEED

289. A driving pulley, or driver, is a pulley that imparts motion to a belt.

A driven pulley, or follower, is a pulley that is moved by the belt.

R. P. M. as applied to pulleys means the number of revolutions per minute that a pulley makes.

The number of revolutions per minute of pulleys is inversely proportional to their diameters.

290. **Rule** for speed of pulleys: *The diameter of the driver multiplied by its speed is equal to the diameter of the follower multiplied by its speed. Or,* $D \times N = d \times n$.

D = the diameter of the driver.
d = the diameter of the follower.
N = the R. P. M. of the driver.
l = distance between centers of pulleys.
n = the R. P. M. of the follower.

The diameter of a driving pulley is 20 in. and makes 180 R. P. M. What is the speed of the follower whose diameter is 8 in.?

$D \times N = d \times n$. Or, $20 \times 180 = 8 \times n = 450$. $d = 8''$.

PROBLEMS

1. The diameter of a driver is 12 in. and makes 400 R. P. M. What is the speed of the follower whose diameter is 3 in.?

2. A driving pulley on a shaft is 38 in. in diameter and makes 20 revolutions per minute. How many revolutions will the driven pulley make, if its diameter is 19 in.?

3. The diameter of a driving pulley is 8 in., and its R. P. M. is 1,200. What is the R. P. M. of the driven pulley, if its diameter is 5 in.?

4. The speed of a pulley 16 in. in diameter is 360 R. P. M. What is the speed of the driven shaft which has a pulley 12 in. in diameter belted to the driver?

5. The surface speed of a piece of work is 3,000 ft. per minute. Its diameter is 6 in. What is its R. P. M.?

6. A 40 in. pulley making 160 R. P. M. is belted to another making 400 R. P. M. Find the diameter of the latter.

7. What must be the speed of the driver, 12 in. in diameter, if the follower making 1,200 R. P. M. has a diameter of 6 in.?

LENGTH OF BELTING

164. To find the length of open belting required to connect two pulleys having nearly the same diameter, *take π times one half the sum of the diameters of the two pulleys, plus twice the distance between the centers of the pulleys. The formula is,*

$$L = \frac{\pi(D + d)}{2} + 2l.$$

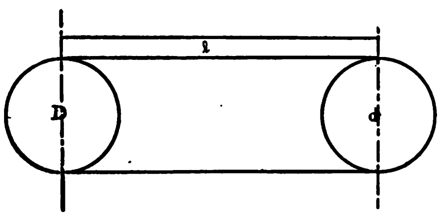

FIGURE 44

Where they have the same diameter, $L = \pi D + 2l$. Figure 44.

L = length of belting.

l = distance between centers of pulleys.

D and d = diameters of large and small pulleys respectively.

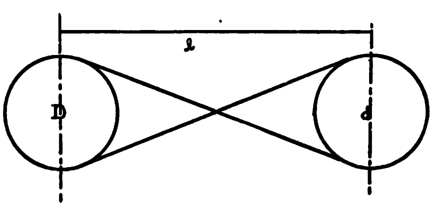

FIGURE 45

292. To find the approximate length of crossed belts when pulleys are of approximately equal diameters, *take twice the square root of the square of the diameter of the pulley plus the square of the distance between the centers of the two pulleys, plus π times the diameter of the pulley.*

$$L = 2\sqrt{D^2 + l^2} + \pi D.$$

PROBLEMS

1. Each of two pulleys has a diameter of 18 in. If the belt is 30 ft. long, what is the distance between the pulleys?

2. The distance between the centers of two pulleys of equal diameters is 16 ft., and the length of the belt is 40 ft. What is the diameter of the pulleys?

3. A line shaft runs 320 R. P. M., and its pulley is 15 in. in diameter. This is driven by a main linc shaft running 200 R. P. M. What is the diameter of the pulley on the main line shaft? The R. P. M. of one shaft multiplied by the diameter of its pulley equals the R. P. M. of the second shaft multiplied by the diameter of its pulley.

$$\text{R. P. M.} \times D = r.\,p.\,m. \times d.$$

4. What length of belting is required to connect two pulleys whose diameters are 20 and 16 in. respectively, and the distance between the centers 12 ft.?

5. A belt runs from a motor pulley to a circular saw pulley. If the length of the belt is 24 ft. and the diameters of the two pulleys are 12 in. and 6 in. respectively, what is the distance between the centers of the pulleys?

6. How long must a connecting belt be, if the distance between the centers of two pulleys is 9 ft., the diameter of the driven pulley is 24 in. and that of the driving or motor pulley is 4 in.?

7. What is the diameter of a driving pulley of a motor, if the diameter of the driven pulley of the lathe is 4 in., the distance between the centers of the two pulleys is 24 in. and the length of the connecting belt is 66 in.?

8. Two pulleys run with crossbelt; the diameters of each is 18 in. and the distance between their centers is 12 ft. What is the length of the belt?

9. Two pulleys have a crossbelt and run in opposite directions; the diameter of the driver is 12 in. and that of

the driven is 24 in. What length of belt is required if the distance between centers is 15 ft.?

10. How many square feet of leather will it take for a belt 6 in. wide to connect two pulleys whose diameters are 16 in. and 18 in. respectively and distance between centers 10 ft.?

11. The distance between centers of two pulleys is 18 ft., and the smaller of the pulleys is 12 in. in diameter. What is the length of the belt, if the diameters of the pulleys are to each other as 2 to 3?

HORSEPOWER OF BELTING

293. To find the horsepower of a belt we must know the velocity of the belt in feet per minute and the effective pull per inch of width of the belt. We can measure the R. P. M. or speed of the driving pulley, and its diameter. Then the velocity of the belt in feet per minute equals the circumference of the pulley in feet multiplied by the R. P. M. (revolutions per minute). Let V = the velocity of the belt in feet per minute and W = the width of the belt in inches. Let C = the circumference of the driving pulley in feet. Then $V = C \times R.\ P.\ M.$ The velocity of a belt should be about 2,000 feet per minute. One H. P. = 33,000 foot-pounds per minute. One foot-pound = the work to lift 1 pound 1 foot high per minute.

The velocity of the belt multiplied by the effective pull per inch of width of belt gives the number of foot-pounds one inch width of belt will do per minute. Multiplying this by the width of the belt in inches gives the power of the belt in foot-pounds of work. Dividing by 33,000 gives the horsepower. Let K = the effective pull of the belt per inch of width. Then $H.\,P. = \dfrac{V \times K \times W}{33{,}000}$.

A belt is 5.4 in. wide; the effective pull per inch of width is 31 lbs., and the velocity is 2,000 ft. per minute. How many H. P. will it transmit?

$$H.P. = \frac{2{,}000 \times 31 \times 5.4}{33{,}000}. \quad \text{H. P.} = 10+.$$

The effective pull or force tending to turn the pulley is the difference between the tensions on the slack and on the driving sides. This is about 33 lbs. per inch of width for single belts and about 80 lbs. per inch of width for double belts.

How many H. P. will a single belt 4 in. wide, running over a pulley 6 in. in diameter, transmit, if the pulley makes 1,130 R. P. M.?

The circumference of the pulley in feet is $\frac{\pi \times 6}{12} = 1.57$.

And $V = C \times R.P.M.$ = a velocity of 1,774+ feet per minute.

$$H.P. = \frac{V \times K \times W}{33{,}000} = \frac{1{,}774 \times 33 \times 4}{33{,}000} = 7 + \text{ horsepower.}$$

This formula may be used for single belts where the effective pull is 33 lbs. per inch of width.

$$H.P. = \frac{V \times W}{1{,}000}.$$

What width of belt will be required to transmit 5 H. P. over a pulley 6 in. in diameter, making 1,000 R. P. M.? The circumference of the pulley in feet is $\frac{\pi \times 6}{12} = 1.57$. $V = C \times R.P.M.$ = 1,774 + feet per minute. If this belt is a single belt and has an effective pull of 33 lbs. per inch of width, use the formula, $H.P. = \frac{V \times W}{1{,}000}$.

Transpose this equation, and

$$W = \frac{1{,}000 \times H.P.}{V} = \frac{1{,}000 \times 5}{1{,}774} = 2.8 \text{ in.}$$

PROBLEMS

1. What linear velocity will be given a belt connecting two pulleys, each 15 in. in diameter, and making 1,000 R. P. M.?

2. A belt running 1,200 ft. per minute has a total effective pull of 34 lbs. What horsepower is it transmitting?

3. What is the effective pull of a belt which transmits 6 H. P. when running 1,000 ft. per minute?

4. A driver is 12 in. in diameter, and is running at 800 R. P. M. The belt is 8 in. wide with an effective pull of 40 lbs. per inch. What horsepower is transmitted?

5. What width of belt is required to transmit 15 H. P., the speed of the belt being 1,500 ft. per minute and the allowable effective pull 50 lbs. per inch width?

6. A belt with a speed of 1,100 ft. per minute has an effective pull of 33 lbs. What H. P. is it transmitting?

7. What width of belt is required to transmit 10 H. P. with a belt velocity of 1,800 ft. per minute?

8. What H. P. will be transmitted by a 4-in.-wide, double belt, with an effective pull of 80 lbs. per inch of width, if it transmits from a 6 in. pulley running at 1,130 R. P. M.?

CUTTING SPEED AND FEED

294. The **surface speed,** or cutting speed, of a tool used in a lathe is the distance that the tool advances or cuts per minute. If a tool cuts 20 ft. measured around the work in one minute, the cutting speed is 20 ft. per minute.

If the circumference of a piece of work that is being turned is 15 in., this distance around the work passes under the tool point once with each revolution of the work. If the work makes 200 R. P. M., the cutting speed in feet will be 15 in., or $\frac{5}{4}$ of a foot $\times$ 200 = 250 ft. per minute.

If Ct = the cutting speed, C = the circumference of the work, R. P. M. the number of revolutions per minute of the work, all expressed in feet, then $Ct = C \times R.\ P.\ M.$ This is the same formula as is found under the Horsepower of Belting, where $V = C \times R.\ P.\ M.$ Cutting speed = surface speed.

What is the surface speed of an emery wheel in feet per minute, if its R. P. M. is 600 and its diameter is 20 in.?

$$Ct = \frac{20 \times \pi}{12} \times 600 = 3{,}141.6, \text{ number of feet per minute.}$$

295. The **feed** is the distance that the tool advances in inches along the work for each revolution of the work, and is the width of the chip cut.

If a lathe spindle revolves 60 times while the carriage advances 1 in., then the feed is $\frac{1}{60}$ of an inch.

If F = the feed, $R.\ P.\ M.$ = the number of revolutions per minute, d = the distance the tool advances along the work, then,

$$F = \frac{d}{R.\ P.\ M.} \quad \text{And } d = F\,(R.\ P.\ M.)$$

The time to traverse the whole length of the work is the whole length of the work to be cut divided by the feed, or width of the cut, in one minute.

If the width or feed is $1\frac{1}{2}$ in., and the whole length is 8 in., then $8 \div 1\frac{1}{2} = 5\frac{1}{3}$ minutes. Let T = time for one cut of the tool along the whole length of the work; F = the feed; L = the whole length of the cut along the work. Then,

$$T = \frac{L}{F \times R.\ P.\ M.}$$

PROBLEMS

1. A brass rod 2 in. in diameter is being turned. The spindle of the lathe makes 400 R. P. M. What is the cutting speed?

2. The diameter of a cutter is 4 in. and it makes 40 R. P. M. What is the cutting speed?

3. The surface speed of a turning piece of work is 2,000 ft. per minute. What is the R. P. M. if the diameter is 5 in.?

4. What is the diameter of a cylindrical piece of work, if its R. P. M. is 1,250 and its surface speed is 750?

5. In turning a flat disk 10 in. in diameter, if the turning chisel is placed 3 in. from the center point, and the R. P. M. is 1,800, what is the cutting speed?

6. A cutting speed of 60 ft. per minute is required in boring a hole 6 in. in diameter with a boring tool. What is the R. P. M. of the boring tool?

7. How many R. P. M. are required to get a cutting speed of 80 ft. per minute on work whose circumference is 31.4 in.?

8. At what speed per minute does a belt travel through the air, if it connects two pulleys each 20 in. in diameter, which are revolving at the rate of 382 R. P. M.?

9. A grindstone has a surface speed of 1,000 ft. per minute. What is its R. P. M. if its diameter is 5 ft.?

10. What is the cutting speed of a band saw which encircles two wheels each 28 in. in diameter, and has a R. P. M. of 630?

11. The surface speed of a wheel is 6,000 ft. per minute, and its R. P. M. is 400. What is its diameter?

12. A 40-in. pulley making 160 R. P. M. is belted to another making 400 R. P. M. Find the diameter of the latter.

13. A piece of work revolves 80 times while the tool moves along the work $\frac{7}{8}$ of an inch. What is the feed?

14. A milling cutter revolves 150 times while the table moves $1\frac{1}{2}$ in., what is the feed?

15. A $\frac{7}{8}$ in. drill has a R. P. M. of 300, and is fed $\frac{1}{64}$ in. per revolution. How many minutes will it take to drill through the piece of iron 2 in. thick?

16. A milling cutter has a R. P. M. of 200, and a feed of .012 in. per revolution. What is the feed per minute?

17. If the feed per revolution of a $\frac{3}{8}$ in. drill is .005, how many revolutions will the drill make in passing through a piece of iron $4\frac{3}{4}$ in. thick?

18. How long will it take to drill through a $3\frac{1}{4}$ in. piece of iron, if the feed is .005 in. and the speed is 280 R. P. M.?

19. If the R. P. M. of a drill is 366, what is the feed required to drill through a $\frac{7}{8}$ in. piece of iron in $\frac{1}{2}$ minute?

20. How long will it take to drill through a piece of iron $4\frac{1}{4}$ in. thick, if the feed is .005 in., and the speed is 400 R. P. M.?

TRAINS OF GEARING

296. In a train of cog wheels the power multiplied by the continued product of the number of teeth in each wheel, or the circumference of each wheel, equals the load multiplied by the continued product of number of teeth in each pinion or small wheel, or the circumference of each pinion or small wheel.

In Figure 46, if A, B and C represent the number of teeth in the large wheels respectively, or the circumferences of these wheels, and a, b and c represent the same in the pinions or small wheels, then P (the power) multiplied by A by B by C = (the load) L multiplied by a by b by c.

$$L\,(a \times b \times c) = P\,(A \times B \times C) \text{ or}$$

$$L = \frac{P \times A \times B \times C}{a \times b \times c}.$$

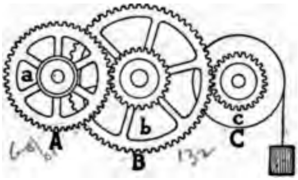

FIGURE 46

If a wheel A has 60 teeth, a pinion b, 24 teeth, a wheel B, 132, a pinion c, 18; what load will a 100-lb. power support applied to this train of wheels?

$$L = \frac{100 \times 60 \times 132}{24 \times 18} = 1,833\tfrac{1}{3}, \text{ the number of pounds.}$$

If A makes one revolution per minute, how many will C make? N = number of revolutions.

$$N = \frac{1 \times 60 \times 132}{24 \times 18} = 18\tfrac{1}{3}.$$

In a train of wheels, Figure 47, 4 men are exerting a force of 30 lbs. each on two crank handles each of whose lengths is equal to the diameter of a drum on which a rope is coiled supporting a weight. The small wheel on the first axis, the driver, has 20 teeth, the follower 100. The driver on the second axis has 40 teeth, the follower 120. The driver on the third axis has 30 teeth, the follower 150. What weight can be raised?

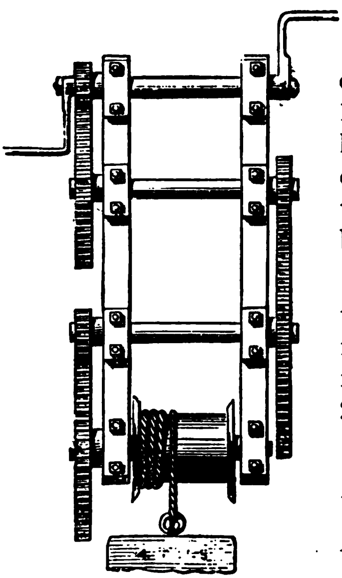

FIGURE 47

The ratio of the crank handle to the drum is 2. The power is, therefore, multiplied by 2. The power is 120 (4×30) lbs. 2×120 lbs. = 240 lbs.

$$L = \frac{240 \times 100 \times 120 \times 150}{20 \times 40 \times 30} = 18,000,$$

the number of pounds.

Let $r.\ p.\ m.$ = the revolutions per minute of the driven.

The $R.\ P.\ M.\ \times$ the number of teeth of a driver = the $r.\ p.\ m.\ \times$ the number of teeth of the driven.

Let N = the number of teeth, or the circumference, of the driver, and n = the number of teeth, or the circumference, of the driven.

Then $R.\ P.\ M.\ \times N = r.\ p.\ m.\ \times n.$

If a driving gear of 24 teeth revolves 240 times per minute, what is the speed of a driven gear which has 192 teeth?

$$r.\ p.\ m. = \frac{R.\ P.\ M.\ \times N}{n} = \frac{240 \times 24}{192} = 30.$$

297. Ratio of Gearing. If a gear on one shaft has 40 teeth and a gear on another shaft has 160, each time the small gear turns around once it engages 40 teeth on the large gear, and to engage 160 teeth on the large gear it must turn around 4 times. Therefore the ratio of the gearing is 1 to 4. If the ratio is 1 to 5, one gear has 5 times as many teeth as the other has. The one that has 5 times the number of teeth revolves $\frac{1}{5}$ as rapidly. If one gear has 20 teeth and another 80, the gear with 20 teeth revolves 4 times while the other revolves once. If we wish one gear to revolve 5 times as rapidly as another, we give it $\frac{1}{5}$ the number of teeth.

298. Idlers, or intermediate gears, have no effect on the ratio of speed between two gears. If two gears have a ratio of 4 to 1 and we place between them another gear to mesh with them, the two given gears will still have the same ratio, 4 to 1.

If we have given the number of teeth in one gear, and the speed ratio of the two gears, we can find the number of teeth in the other gear.

If the ratio between two gears is 2 to 3, and the number of teeth in the first is 60, the number of teeth in the second is $\frac{2}{3}$ of 60 = 40.

If R and r are the speed ratios of the driving gear and the driven gear respectively, and N and n are the number of teeth in each, respectively, then $R : r :: N : n$. The *R. P. M.* of driving shaft : *r. p. m.* of driven shaft :: No. teeth of driving gear : No. teeth of driven gear.

The same ratios or proportions for number of teeth or circumferences are also true for diameters.

PROBLEMS

1. Gear A has 60 teeth and gear B has 40; how many revolutions per minute does gear B make, if gear A makes 120 ?

2. If the R. P. M. of a gear containing 84 teeth is 40, how many teeth must another gear contain to work in contact and make 60 r. p. m. in the same time?

3. A gear of 42 teeth drives one of 12 teeth, on the shaft of which is a pulley of 21 in. diameter, which drives another pulley of 6 in. diameter. What is the R. P. M. of the last pulley to one R. P. M. of the first pulley?

4. A gear has 125 teeth and a pinion has 20 teeth; how many revolutions will the pinion make to one of the gear?

5. We have two gears A and B, and wish to transmit motion from A to D by means of two other gears B and C. The R. P. M. of A is 130 and of D is 360. What diameters of pulleys can we use for B and C?

6. If a gear has 75 teeth and a speed of 80 R. P. M., what is the r. p. m. of a gear which has 25 teeth?

7. A gear A has 72 teeth and makes 36 R. P. M. and a gear C has 25 teeth and makes 250 R. P. M. How many teeth should the intermediate gear B have?

8. In a train of three drivers and three followers the R. P. M. of the first driver is 300, and the number of teeth of the drivers is 20, 40 and 30 respectively. What is the

R. P. M. of the last driven wheel, if the number of teeth in the driven wheels is 100, 120 and 140 respectively?

9. A gear of 72 teeth revolves 140 times per minute. What is the speed of a driven gear which has 24 teeth?

10. A gear has 200 teeth and a pinion has 40. How many R. P. M. will the pinion make to one of the gear?

GEAR WHEELS

299. Pitch is the distance from the center of one tooth to the center of the next. Pitch circle is a circle concentric

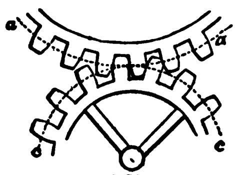

FIGURE 48

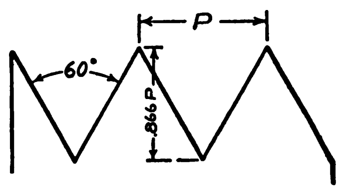

FIGURE 49

with the circumference of a toothed wheel, and is about half way between the points of the teeth and the roots. It touches the pitch circle of the gear working with it.

The pitch circle is the circle *c c* or the circle *a a* in Figure 48.

A pitch diameter is the diameter of the pitch circle.

The diametral pitch is the number of teeth per inch of the diameter of the pitch circle. If we multiply the number of inches in the diameter of the pitch circle by the number of teeth per inch of this diameter, or the diametral pitch we have the number of teeth in the wheel.

If the diameter of a pitch circle, or pitch diameter, is $2\frac{1}{2}$ in. long, and the number of teeth for each of these $2\frac{1}{2}$ in., or the diametral pitch, is 12, then the whole number of

teeth in the wheel is 30. ($2\frac{1}{2}$ in. $\times$ 12 = 30). Such gear is called a 12-pitch gear.

The depth of a 60° V thread is .866 of the pitch = .866 P., Figure 49. Or, the depth is .866 divided by the number of threads to the inch. The double depth is 1.732 P. or 1.732 in. $\div$ by the number of threads to the inch.

The lead of a screw = the distance a thread advances in one complete turn. In a single thread screw, the lead = the distance occupied by one thread. In a double thread screw, the lead = the distance occupied by two threads, etc.

The number of turns to the inch, or the number of turns a screw makes to advance 1 in., = 1 $\div$ the lead. 1 in. $\div$ $\frac{1}{8}$ in. = 8 turns to 1 in.

1 in. $\div$ $\frac{3}{8}$ in. = $2\frac{2}{3}$ turns to an inch. A screw that turns 96 times in 6.002 in. turns 96 $\div$ 6.002 in. = 15.99 turns in 1 in. 1 in. $\div$ the lead = turns per inch. $\frac{7}{16}$ in. lead = $\frac{16}{7}$ turns to an inch, or $2\frac{2}{7}$ turns to an inch.

To thread a screw to a $\frac{7}{16}$ in. lead in a lathe having a lead screw 8 turns to an inch, we divide the turns of the lead screw by the turns to be threaded. Thus we have 8 $\div$ $\frac{16}{7} = \frac{56}{16}$. Multiplying by $\frac{2}{2}$ to get available gears gives $\frac{112}{32}$. A 112 gear will go on the spindle and a 32 gear will go on the screw.

Let Pd = the diametral pitch, D = the pitch circle diameter, N = the number of teeth in the wheel, pc = circular pitch.

$$pc = \frac{3.1416}{Pd}. \quad Pd = \frac{3.1416}{pc}. \quad Pd = \frac{N}{D}.$$

$$D = \frac{N\,pc}{3.1416}. \quad O \text{ (outside diam.)} = D + \frac{2}{Pd}. \quad O = D + 2a.$$

$$W \text{ (working depth)} = \frac{2}{Pd}.$$

The diametral pitch is 4, and the diameter of the pitch circle is 6 in. How many teeth?

$$Pd = \frac{N}{D}. \quad 4 = \frac{N}{6}. \quad N = 24, \text{ the number of teeth.}$$

PROBLEMS

1. The pitch diameter is 7 in., and the diametral pitch is 10 in. What is the number of teeth?

2. The diametral pitch of a gear is 12. What is the circular pitch?

3. A 12-pitch gear has 36 teeth. What is the diameter of the pitch circle?

4. A 4-pitch gear has 20 teeth. What is the diameter of the pitch circle?

5. In a gear wheel the number of teeth is 24, the diametral pitch is 8. What is the diameter of the pitch circle?

6. A gear has 48 teeth and is $4\frac{1}{8}$ in. outside diameter. Find the diametral pitch.

7. What is the working depth of a tooth of 6 diametral pitch?

8. What is the thickness at the pitch line of a tooth of 6 diametral pitch?

9. What is the clearance at the bottom of a tooth of 4 diametral pitch?

10. The diametral pitch of a gear is 8, and the outside diameter is $9\frac{1}{4}$ in. What is the pitch diameter?

11. What is the diametral pitch when the circular pitch is $1\frac{1}{4}$ in.?

12. What is the circular pitch, when the diametral pitch is $2\frac{3}{4}$ in.?

13. A gear has 50 teeth and is 20-pitch. What is the working depth?

14. A gear has 60 teeth, and the pitch diameter is 6 in. What is the diametral pitch?

15. A gear blank is $6\frac{3}{4}$ outside diameter and is to be cut with 12 diameter pitch gear cutter. How many teeth must it have?

16. A gear has 50 teeth, and its diametral pitch is 12. What is its pitch diameter?

17. A gear has 3.222-in. pitch diameter, and a pitch of 9. What is the outside diameter?

18. A gear has a pitch diameter of $6\frac{5}{8}$ in. and 82 teeth. What is the outside diameter?

19. There are 60 teeth in a gear of 15 diametral pitch. What is the pitch diameter?

20. The diametral pitch is 14, and the pitch diameter is 4 in. What is the depth of the cut?

21. Given 70 teeth in a gear of 10 diametral pitch. Find the pitch diameter.

22. A gear has 24 teeth and a diametral pitch of 8. What is the pitch diameter?

23. One gear has 70 teeth and another has 60 teeth, and both have a diametral pitch of 10. What is the distance between their centers?

LEVERS

300. There are three classes of levers. In a lever of the first class the fulcrum is between the power and the weight. Figure 55.

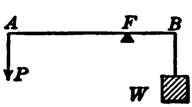

FIGURE 55

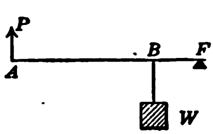

FIGURE 56

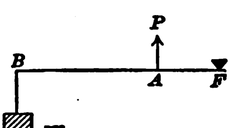

FIGURE 57

In a lever of the second class the weight is between the fulcrum and the power. Figure 56.

In a lever of the third class the power is between the fulcrum and the weight. Figure 57.

301. The law of levers. *The power times the length of the power arm equals the weight times the length of the weight arm.*

$P \times Pa = W \times Wa$. Pa = power arm. Wa = weight arm. P = power. W = weight.

The safety valve shown in Figure 58 is a lever of the third class.

The steam pressure, or power, is between the fulcrum and the weight.

The weight arm is the distance from the fulcrum F to B, or to a line drawn through the line of direction of the weight.

The power arm is the distance from the fulcrum F to A, or to a line drawn through the line of direction of power.

PROBLEMS

1. A lever is 16 ft. long. At one end is a weight of 100 pounds. What force must be applied at the other end, 4 feet from the fulcrum, to balance the given weight?

2. A plank 12 ft. long is used as a seesaw by two girls who weigh 96 lbs. and 100 lbs., respectively. How far from the lighter girl must the prop, or fulcrum, be placed?

3. The handles of a wheelbarrow extend 5 ft. from the axle of the wheel. A load of 300 lbs. is placed 18 inches from the axle of the wheel. How much force must be exerted to raise the handles of the wheelbarrow?

4. The force arm of a pump handle is 3 ft. long and the weight arm is $\frac{3}{4}$ ft. long. What lift will be given to the piston by a 20-lb. force?

5. The length of the handles of a large pair of shears is 12 inches long from the rivet. The cloth to be cut is $1\frac{1}{2}$ inches from the rivet. What force must be applied to exert 80 lbs. of pressure on the cloth?

6. The radius of the drum of a windlass is 6 inches and the crank arm is 18 inches long. What force at the crank will just balance a 200-lb. weight?

7. The radius of the roller of a clothes wringer is 2 inches and the handle is 12 inches long. If the hand exerts a 10-lb. force, what is the pressure to drive the clothes through the rollers?

8. The handles of a nutcracker are 8 inches long and the nut is placed $\frac{1}{2}$ inch from the fulcrum. A force of 5 lbs. gives what pressure on the nut?

9 A man and boy are carrying 400 lbs. on a pole 8 ft. long. Where must the load be placed, if the boy is to bear 50 lbs. of the load?

10. A tack puller has a weight arm of $\frac{3}{4}$ inch and a force arm of 14 inches. With a force of 20 lbs. on the handle, what weight or pull is given the tack?

HORSEPOWER OF AN ENGINE

302. The volume of the cylinder in cubic inches multiplied by the average working pressure in pounds per square inch and divided by 12 gives the number of foot-pounds per stroke of the piston. This last number multiplied by the number of strokes of the piston gives the number of foot-pounds per minute. The number of foot-pounds per minute divided by 33,000 gives the horsepower of the engine. If there are two cylinders, the horsepower is twice as much.

The average pressure is found roughly by taking one half the difference between the boiler pressure and the condenser pressure, as per the steam gauges. The volume of

the cylinder in cubic inches is the area of the piston in square inches multiplied by the length of the stroke. Indicator cards are used for finding the average effective pressure. The number of times steam pressure is applied to the piston per minute, or in steam engines twice the number of revolutions per minute, is the number of strokes of the piston.

The formula for the horsepower of engines follows:

$$H.\,P. = \frac{P\,L\,A\,N}{33{,}000}.$$

Let $H.\,P$ = horsepower.

P = average effective pressure in pounds per sq. in.

A = area of the piston in square inches.

L = the length of the stroke in feet.

N = the number of strokes of the piston per minute.

What is the horsepower of a steam engine whose average pressure by the indicator card is 64.96 lbs.; length of stroke $2\frac{1}{2}$ ft.; piston diameter 18 in. and R. P. M. 100? The area is $\pi R^2 = 254.47$ sq. in. The number of strokes of piston is $100 \times 2 = 200$.

$$H.\,P. = \frac{64.96 \times 254.47 \times 2\frac{1}{2} \text{ ft.} \times 200}{33{,}000} = 241+$$

This is the indicated H. P. which is larger than the brake horsepower found by the Prony Brake, which is called the actual brake or delivered horsepower. The horsepower of steam turbine engines much used in ocean steamship service and of the gas engine is not taken by indicator cards, but is the brake horsepower.

What is the horsepower of an engine that can throw 12,000 lbs. of water every minute 100 ft. high?

$$H.\,P. = \frac{\text{foot-pounds}}{33{,}000} = \frac{12{,}000 \times 100}{33{,}000} = 36.3+$$

The piston of a force pump has an area of 40 sq. in. and a stroke of 20 in. If it works at a pressure of 50 lbs. per

square inch, and the number of strokes per minute is 200, what is the H. P.?

$$H.P. = \frac{P\,L\,A\,N}{33{,}000} = \frac{50 \times 1\tfrac{2}{3} \times 40 \times 200}{33{,}000} = 20.2.$$

Water is flowing into a mine 300 ft. deep at the rate of 200 cu. ft. per minute. What horsepower will keep it dry?

200 cu. ft. per min. × 62.5 lbs. per cubic foot = the number of pounds of water per minute. The number of pounds of water per minute multiplied by the depth of the mine (300) = the foot-pounds per minute. The foot-pounds per minute divided by 33,000 gives the horsepower.

$$H.P. = \frac{\text{foot-pounds}}{33{,}000} = \frac{62.5 \times 200 \times 300}{33{,}000} = 113.6.$$

A steam crane lifts 100 tons of coal 20 ft. high in 20 minutes, neglecting friction. What is the H. P. of the engine? This formula is $H.P. = \dfrac{W \times S}{T \times 33{,}000}$

W = weight; S = space or height; T = time in minutes.

$$\text{H. P.} = \frac{2{,}000 \times 100 \times 20}{20 \times 33{,}000}$$

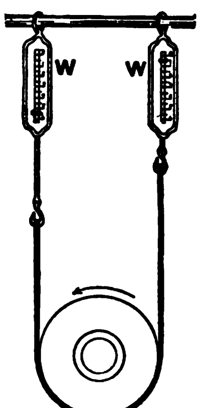

FIGURE 50

303. To find the brake horsepower. Let r = the radius of the pulley in feet, $W - w$ = the difference of the two scale readings in pounds; and N = the number of revolutions per minute. Figure 50. For the strap brake, a piece of leather belting and two spring balances are all that is necessary.

The formula is,

$$H.P. = \frac{2\pi R \times N \times (W - w)}{33{,}000}.$$

The strap brake horsepower of a motor = No. lbs. pull on brake × circumference of pulley in feet × No. R.P.S. divided by 550 ft.-lbs.

PROBLEMS

1. The difference in the readings of the two balances of a strap brake is 4 pounds, the diameter of the pulley is $\frac{1}{3}$ foot, and the motor makes 1,200 R.P.S. (revolutions per second). What is the horsepower of the motor? The work done by the motor in foot-pounds per second is:

Force $\times$ distance $\times$ No. R.P.S. = 4 lbs. $\times$ ($\frac{1}{3}$ ft. $\times$ π = 1.05 ft.) $\times$ 1,200 = 55.6 ft. 550 ft.-lbs. per second = 1 H.P. Therefore, 55.6 divided by 550 = .1, or $\frac{1}{10}$ H.P. The force is the difference between the two balance readings. The distance is the circumference of the pulley. The No. R.P.S. is the number of revolutions per second of the pulley.

2. The difference in the readings of the two balances of a strap brake, when testing for the horsepower of a motor, is 5.2 lbs., the diameter of the pulley is $\frac{1}{2}$ ft., and the motor makes 1,500 R.P.S. What is the horsepower of the motor?

3. In testing for the horsepower of a motor with a strap brake the balance readings were 5.3 lbs. and 5.8 lbs. respectively, the radius of the pulley was 3 inches and the number of revolutions of the pulley per second was 1,600. What was the horsepower of the motor?

4. The strap brake horsepower of a motor is $\frac{1}{10}$ H.P., the diameter of the pulley is $\frac{1}{3}$ foot. What is the motor speed or the number of revolutions per second of the pulley?

5. The number of pounds pull on the brake of a strap brake when testing to find the horsepower of a motor, was 5.5 lbs., the pulley diameter was 6 inches, and the R.P.S. was 2,000. What was the horsepower of the motor?

6. What is the horsepower of a motor, if the pull is 6.5 lbs., the diameter of the pulley is 2.3 ft. and the R.P.S. is 2,000?

7. The radius of the pulley of a water motor is 4 inches, the difference in pounds pull on the balances is 5.3 lbs., the R.P.S. is 1,500. What is the H.P.?

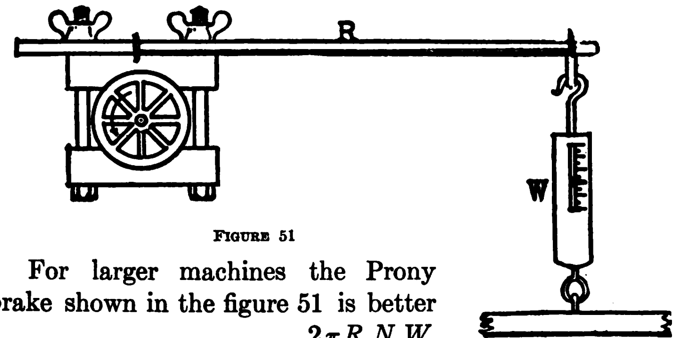

FIGURE 51

For larger machines the Prony brake shown in the figure 51 is better and the formula is $H.P. = \frac{2\pi R\ N\ W}{33{,}000}$.

R = the perpendicular distance from the center of the pulley to the line of action of the scale in feet; W= the scale reading in pounds; N = the number of revolutions per minute.

Two pieces of timber are fitted to the pulley of an engine or on a shafting, with a long lever attached with a spring balance at the end, Figure 51. The pulley is revolved in the direction of the arrow, and the weight is indicated on the scale.

A Prony brake with a 5-ft. lever was placed on a gas engine. The pulley made 300 R. P. M. and the brake balanced with a 12-lb. pull. What was the horsepower?

$$H.P. = \frac{2\pi R N W}{33{,}000} = \frac{2 \times 5 \times 3.1416 \times 12 \times 300}{33{,}000} = 3.4.$$

PROBLEMS

1. A small motor is tested with a Prony brake. The brake arm is 2 ft. long. The indicated weight is 5 lbs. and the motor is running at 900 R. P. M. What is the brake horsepower?

2. An engine makes 1,100 R. P. M. and balances a 25-pound weight at the end of a lever 4 ft. long. What is the H. P.?

3. An engine makes 200 R. P. M. and will support 500 lbs. at the end of a 6-ft. lever. What is the H. P.?

4. The brake arm of a Prony brake is 30 in. long and in testing a motor which was making 900 R. P. M. the brake balanced with 3 lbs. What was the horsepower?

5. In testing a gasoline engine with a Prony brake, it was found that the brake balanced with 500 lbs. at the end of a 6 ft. lever. If the engine was making 200 R. P. M., what was the horsepower?

6. What is the horsepower of an engine which is running at 1,100 R. P. M. and balances 40 lbs. at end of a 4-ft. lever?

7. A motor is tested with a Prony brake. The brake arm is 60 in. long, the weight is 5 lbs. and the motor running at 900 R. P. M. What is the brake horsepower?

8. An engine makes 1,100 R. P. M. and balances 40 pounds at the end of a 5-ft. lever. What is the brake horsepower?

9. An engine makes 300 R. P. M. and supports 400 lbs. at the end of a 6-ft. lever. What is the brake H. P.?

10. In testing with a Prony brake an engine running at 80 R. P. M. the lever arm was 10.5 ft. long, and the pressure at the end was 1,600 lbs. What was the H. P.?

11. An engine that makes 300 R. P. M. will support 600 lbs. at the end of a 7-ft. lever. What is the H. P.?

12. A gasoline engine makes 1,200 R. P. M. and will balance 30 lbs. at the end of a 5-ft. lever. What is the H. P.?

304. Indicator Cards. Steam from the boiler is admitted to the steam chest and the cylinder through the pipe *a* and port *b* at the left of the piston, Figure 52. The exhaust steam is driven out through the port *e* and the exhaust pipe *c*. The eccentric now closes the port *b*, and opens the port at the right end of the cylinder, and the steam rushes in to push the piston back. Admitting the live steam is called admission; closing it is called cut off. Opening the exhaust port is called the re-

lease; and closing it is closing the exhaust. One cubic foot of water when changed to steam occupies nearly 1,700 cu. ft. at the normal atmospheric pressure. The total pressure tending to move the piston is equal to the area in square inches of the end of the piston multiplied by the difference between the pressure per square inch on the admission side and the back pressure per square inch on the exhaust side. Steam is not admitted from the boiler during the whole stroke, but for part of it, from one third to one half of the length of stroke. Then the valve cuts off, and the expanding steam continues to push the piston along the rest of the distance.

305. The pressure in pounds per square inch multiplied by the volume of the cylinder in cubic inches, and divided

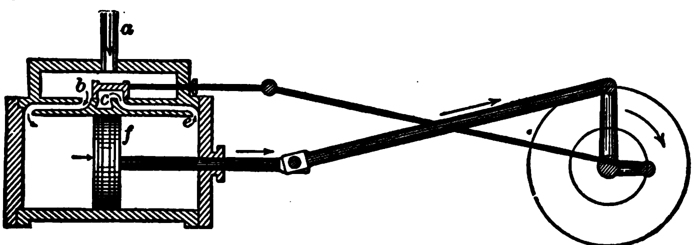

FIGURE 52

by 12 gives the number of foot-pounds per stroke of the piston. This number multiplied by the number of single strokes per minute, and divided by 33,000, gives the horsepower. To get the volume of the cylinder we multiply the length of the stroke by the cross-section area of the cylinder.

$$H.P. = \frac{P \times L \times A \times N}{33{,}000 \times 12},$$

where P = the pressure in pounds per square inch, L = the length of the stroke in inches, A = area of piston in square inches and N = the number of single strokes.

This is for one cylinder. If there are two cylinders, we take two times the number found by this formula. The

pressure is found by the "steam indicator" and is then called the indicator horsepower, or is found by the brake test, and is called the actual horsepower, which is a little less than the indicated horsepower, on account of friction in the moving parts of the engine. Or we may take by the steam gauges the boiler pressure and the condenser pressure and then one half the difference between these two. This will give the approximate horsepower.

In the steam engine the piston travels four times the length of the crank in one revolution. If the average steam pressure is 80 lbs. per square inch, and the area of the piston is 150 sq. in., then the total force is 12,000 lbs. If the crank is .8 of a foot long and makes 80 R. P. M. then the piston travels $4 \times .8 \times 80 = 256$ ft. per minute. The work done in one minute is $12{,}000 \times 256 = 3{,}072{,}000$ ft.-lbs.

$3{,}072{,}000 \div 33{,}000 = 93\frac{1}{11}$, the number of H. P.

The springs commonly used for the indicator are made so that the figure is drawn to a scale of 1 in. to 40 lbs., 50 lbs., 80 lbs., 100 lbs. or 1 in. to 120 lbs. of steam pressure.

306. The indicated pressure *of the engine is found by taking the average altitude of the vertical lines and multiplying by the scale of the springs.*

If the indicator card in Figure 53 for a double acting engine is divided into 10 spaces, and the sum of the lengths of the vertical lines is 18.22 in., the average height is $18.22 \div 10 = 1.822$ inches. If a No. 40 spring is used, every inch measured vertically on the card equals 40 lbs. per sq. in. and $1.822 \times 40 = 72.88$ lbs. per square inch for the indicated pressure on the piston.

On a steam indicator card the average height is .94; the scale of spring used is 80; the R. P. M. is 100; the length of stroke is 30 in.; the piston diameter is 15 in. What is the indicated H. P.? ($.94 \times 80 = 75.20$ lbs. pressure.)

$$H.\,P. = \frac{75.20 \times 30 \times 176.7 \times 100}{33{,}000 \times 12}$$

PROBLEMS

1. In an indicator card the average height is .66 in., spring scale 60, the R. P. M. is 200; piston diameter 10 in. and stroke 15 in. What is the indicated H. P.?

2. The average height of an indicator card is .48 in. with a No. 60 spring. The diameter of piston is 18 in., stroke 30 in., the R. P. M. 300. What is the H. P.?

3. What is the H. P. of a double acting engine, the diameter of whose piston is 26 in., length of stroke 30 in., R. P. M. 80, total lengths of indicator lines 22 in., indicating spring No. 40?

4. What is the H. P. of a steamboat engine, if the diameter of cylinder is 4 ft., the stroke is 6 ft., the shaft R. P. M. is 40 and the mean effective pressure is 30 lbs. per square inch?

5. What is the H. P. of a marine engine, if diameter of cylinder is 5 ft., stroke 6 ft., R. P. M. 20, and pressure 30 lbs. per square inch?

6. The cylinder of a 55 H. P. engine has a diameter of 12 inches, length 28 inches and pressure 60 lbs. per square inch. What is the R. P. M.?

7. What is the H. P. of an engine, the diameter of whose cylinder is 20 in., length of stroke 4 ft., the R. P. M. 60, and mean effective pressure is 60 lbs.?

8. What is the H. P. of a locomotive engine, if the pressure is 90 lbs. per square inch, each of the two cylinders 16 in. in diameter, and 24 in. long and the driving wheels make 130 R. P. M.?

9. What is the diameter of the cylinder of a 500 H. P. engine, stroke 5 ft., R. P. M. 150, mean effective pressure 25 lbs. per square inch?

10. What is the indicated horsepower of a two-cylinder locomotive, when the pressure is 54 lbs. per square inch,

the length of stroke 24 in., area of piston 284 sq. in., and the R. P. M. 260?

11. What is the horsepower of an engine whose piston area is 100 sq. in. and stroke 16 in., if it works under an average pressure of 60 lbs. per square inch and makes 400 strokes per minute?

12. What is the horsepower of an engine whose cylinder diameter is 28 in., stroke 48 in., and makes 124 strokes per minute, and pressure is 36 lbs. per square inch?

13. What is the indicated horsepower of a locomotive, when the pressure is 140 lbs. per square inch, the length of stroke 48 in., area of piston 380 sq. in., and the R. P. M. 48?

INDICATED HORSEPOWER

307. To find the horsepower of a steam engine by means of the steam indicator.

An indicator is a device for making a diagram of what takes place in an engine cylinder while working. It is shown in Figure 53. It consists of a cylinder which is connected with one end of the engine cylinder by a steam pipe.

This indicator cylinder contains a piston above which is a coil spring of such strength that a given pressure per square inch of steam upon the lower side of a piston will compress the spring a definite amount. These springs vary in strength. A 40-lb. spring is of such strength that 40 lbs. per square inch of steam pressure acting beneath the piston will raise the pencil point 1 in.

A pencil point is attached to a stem which extends through the cap of the cylinder and with levers such that the point moves up and down over a paper wound on a drum. This drum rotates back and forth on a verticle stem, and its motion is produced by a string attached to the crosshead of the engine. The paper on which the pencil marks is wound

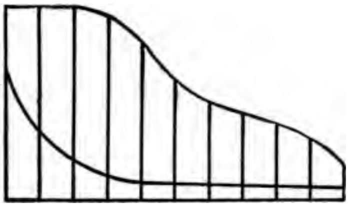

FIGURE 53 SINGLE INDICATOR CARD

on the drum. An indicator card taken in this way is shown in Figure 53. The area of this card may be measured by means of a planimeter, and the area divided by its length, which will give the length of an average vertical line, or an average ordinate. Multiplying the average ordinate by the number of the spring will give the mean effective pressure in pounds per square inch, the M.E.P. in pounds per square inch. This gives P in the formula for horsepower:

$$H.\ P. = \frac{P\ L\ A\ N}{33,000}.$$

The cards I and II, Figure 54, are taken at the same time for the two ends of the cylinder and should be of the same dimensions. The area of the crank end is less than the area of the head end by the area of the piston rod. We substitute the area for A in the formula. If a planimeter is not used, we generally divide the indicator card taken into ten equal parts as in the Figure 53, and measure with an ordinary rule the heights of the vertical lines. Add these

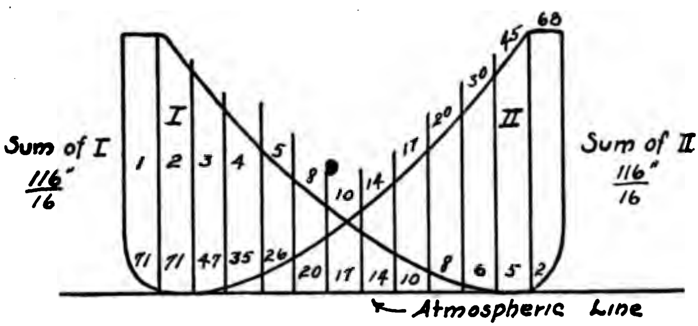

FIGURE 54 DOUBLE INDICATOR CARD

lengths in card I and measure and add those in card II, and see that they are the same or nearly so. Divide either of these amounts by the number of spaces, and multiply by the number of the spring. This gives the M. E. P. in pounds per square inch.

In the figure given, Figure 54, the sum of No. 1 card is $\frac{116}{16}$ in. and the same for No. II card. We use either one. Divide $\frac{116}{16}$, or $7\frac{1}{4}$, by 13 (spaces) $= \frac{29}{52}$. $\frac{29}{52}$ divided by 40, scale number, $=$ 22.3, the M. E. P.

The cylinder area is $16^2 \times .7854 = 201$ sq. in.

The length of stroke is $\frac{42 \times 2}{12} = 7$, length of stroke in feet (double).

Revolutions $= N = 68$.

$$H.P. = \frac{P\,L\,A\,N}{33{,}000} = \frac{22.3 \times 7 \times 201 \times 68}{33{,}000} = 64.6.$$

To make the problem simple we have not considered the piston rod. If we were to consider the piston rod we would have the area of the piston A and the area of the rod a, and then $\frac{2A - a}{2}$ = area of piston.

308. These indicator diagrams were taken from a Rice & Sargent Reciprocating Engine, rated at 1,500 H. P. and 118 R. P. M. The cylinder was of the tandem compound type. Diameter of high pressure cylinder was $24\frac{1}{4}$ in., diameter of low pressure cylinder 44 in., and stroke 42 in. High pressure piston rod diameter $4\frac{1}{2}$ in., low pressure piston rod diameter 6 in.

A Crosby Steam Indicator containing an 80-lb. spring was connected to the high pressure cylinder of the engine and one containing a 24-lb. spring was connected to the low pres-

sure cylinder. Indicator cards were taken at frequent intervals during the test. One card shows diagram from the high pressure engine and the other from the low pressure engine.

309. The indicated horsepower is obtained from the following formula:

$$I.\ H.\ P. = \frac{P\ L\ A\ N}{33{,}000}$$

P = mean effective pressure, which is the average effective pressure behind the piston in pounds.

L = the length of the engine stroke in feet.

A = the effective area of the piston head on which the steam acts in square inches.

N = the number of revolutions per minute of the engine.

310. The Indicated Diagrams were integrated with a planimeter and the area determined. The scale of the planimeter was such as to read directly the mean effective pressure indicated by the diagram for a 40-lb. spring.

In this case, since an 80-lb. spring was used on the high pressure cylinder, the planimeter readings were multiplied by 2, and since a 24-lb. spring was used on the low pressure cylinder, the planimeter readings were multiplied by .6, to give the mean effective pressure.

Steam Engine High Pressure Indicator Card

311. No. 1 Engine; No. H. P. Cylinder; Card No. 8.

Diameter of cylinder, $\frac{24.3''}{44''}$; diameter of rod, $\frac{4.5''}{6''}$; stroke, 42 in.; clearance, 00 in.; date, Oct., 1912; time, 11:10; end of cylinder, both; scale of spring, 80; boiler gauge, 155 lbs.; Vacuum gauge 00 lbs.; revolutions per minute, 118.

Steam Low Pressure Indicator Card

312. No. 1 engine; No. L. P cylinder; card No. 8.

Diameter of cylinder, $\frac{24.3''}{44''}$; diameter of rod, $\frac{4.5''}{6''}$; stroke

42 in.; clearance, 00 in.; date, Oct. 1912; time, 11:10; end of cylinder, both; scale of spring, 24; boiler gauge, 155 lbs; vacuum gauge, 00 lbs.; revolutions per minute, 118.

313. Area of piston head head, high pressure cylinder, head end. Since the steam pressure is exerted over the entire piston head area, we take the diameter of the high pressure cylinder, which is 24.3 in. This gives a cross-section area of 462 sq. in. (Area $= \frac{1}{4} \pi D^2$).

314. Area of piston head, high pressure cylinder, crank end. Steam pressure is exerted on total piston head area minus area of high pressure piston rod, which is $4\frac{1}{2}$ in. in diameter. Therefore, area $= 462 - \frac{1}{4} \pi (4\frac{1}{2})^2 = 446.1$.

315. Area of piston head, cylinder and head end. Here pressure is exerted on the piston head minus the area of the piston rod. The diameter of the piston head is 44 in., and the diameter of the piston rod is $4\frac{1}{2}$ in.; hence the area of the head end is: area $= \frac{1}{4} \pi (44)^2 - \frac{1}{4} \pi (4\frac{1}{2})^2 = 1{,}506.1$, the number of square inches.

316. Area of piston head, cylinder and crank end. Here pressure is exerted on entire piston head area. Area $= \frac{1}{4} \pi(44)^2 = 1{,}583.36$, number of square inches.

The spring used should be of such a strength or number that the diagram will not be over $1\frac{3}{4}$ in. high. The proper spring may be found by dividing the boiler pressure in pounds by the height of the diagram in inches. If the boiler pressure is 70 lbs., then $\frac{70}{1\frac{3}{4}} = 40$, the number of the spring to be used. It is generally more convenient to make the diagram $2\frac{1}{2}$ in. to $3\frac{1}{2}$ in. long, and the height $1\frac{1}{2}$ in. to $1\frac{3}{4}$ in. above the atmospheric line.

The planimeter by being run over the diagram shows the number of square inches in it. If we divide this number of square inches by the length of the diagram, we have its *average* height. The average height multiplied by the num-

ber of the spring used gives the mean effective pressure in pounds per square inch on the piston of the engine.

317. **Proof of Actual Indicator Card**

Taken........Sept. 22, 1915 **at**......Menomonie, Wis.
Which Cylinder............. **Which End**..........Both
Boiler Pres.......9) lbs. **Vac.**......**ins.** **Revs.**......68
Scale....40 **Stroke**....42 in. **Diam. of cylinder**....16
Diam. of Rod......3

$16^2 \times .7854 = 201$ piston area.

$\frac{42 \times 2}{12} = 7$, length of stroke in feet.

$$H.P. = \frac{P\,L\,A\,N}{33{,}000} = \frac{22.3 \times 7 \times 201 \times 68}{33{,}000} = 64.6.$$

Revolutions for double stroke.

L. P. = Low Pressure. *H. P.* = High Pressure.

Result of Calculations

	Planimeter read	*P*	*L*	*A*	*N*	*IHP*
H. P. Head End	29.2	58.4	3.5	462	118	338
Crank End	31.8	63.6	3.5	446.1	118	354
L. P. Head End	28.3	16.9	3.5	1,506.1	118	318
Crank End	28.5	15.1	3.5	1,495	118	283
						1,293

318. The planimeter readings in the first column H. P. are multiplied by 2, because the planimeter was to read directly, for a 40-lb. spring, and an 80-lb. spring was used. The planimeter readings in the first column, L. P., is multiplied by .6 because the planimeter was to read directly for a 40-lb. and a 24-lb. spring was used.

319. To find the H. P. of an engine, it is necessary to ascertain separately three factors, and then find the product

of the three. The first is the area of the two ends of the piston, the head end and the crank end; the second is the total travel of the piston in feet per minute; and the third is the mean effective pressure, M. E. P. of the steam urging the piston forward. The total travel of the piston in feet per minute is twice the length of the stroke in feet multiplied by the number of revolutions of the crank shaft per minute. The piston area at the back end is the same as the area of a cross-section of the cylinder; at the head end it is the same less the area of the cross-section of the piston rod.

The M. E. P computed from an indicator card taken from an air cylinder compressor is 30.6 lbs. per square inch; diameter of cylinder is 28 in.; stroke is 48 in.; number of strokes per minute is 108. What is the H. P.?

$$v = \frac{28^2 \times .7854 \times 48 \times 108}{1,728} \text{ cu. ft. per minute.}$$

$H.P. = \dfrac{144\,v\,p}{33,000}$. v = volume of the fluid used or discharged in cubic feet per minute; p = average pressure in pounds per square inch.

$$\frac{144\,v\,p}{33,000} = \frac{144 \times 28^2 \times .7854 \times 48 \times 108 \times 30.6}{1.728 \times 33,000} = 246.66 \text{ H. P.}$$

A ventilating fan delivers 5,000 cu. ft. of air per minute at a pressure of .56 lb. above the atmospheric pressure; what is the theoretical H. P. to drive the fan?

$$H.P = \frac{144\,v\,p}{33,000} = \frac{144 \times 5,000 \times .56}{33,000} = 12.218.$$

PROBLEMS

1. A double acting engine has a cylinder 26 in. in diameter, and a 30-in. stroke. It makes 80 R. P. M. The indicator card being divided up gives a length of mean ordinate 2.5 in. A No. 40 spring is used and gives 2.5 × 40 = 100 lbs. per square inch as the M. E. P. on the *piston*. What is the indicated horsepower of the engine?

2. A non-condensing engine has a head end and crank end of cylinder the same 16 in. in diameter. The diameter of piston rod is 3 in. and there is a 30-in. stroke. It runs at a speed of 150 R. P. M. The card is taken with a 60-lb. spring. The card measured with a planimeter shows a M. E. P. of 48 lbs. What is the I. H. P.? The average area = (.7854 × 16² − 7 sq. in., area of piston rod) = 198 sq. in.

$$I.H.P. = \frac{P\,L\,A\,N}{33{,}000} = \frac{48 \times 2.5 \times 198 \times 300}{33{,}000}$$

N = number of strokes per minute = number of revolutions × 2 = 150 × 2 = 300.

SAFETY VALVE

320. In the equation for the safety valve, three weights are given: the weight of the ball, the lever, and the valve. Three lengths are given: the length of the lever, the distance from the center of gravity to the fulcrum, and the distance from the valve to the fulcrum. Find the center of gravity of the lever, by balancing the valve and lever over some sharp edge and measure the distance from this point to the fulcrum.

The equation is the following: $S \times b = V \times b + L \times h + W \times y$.

S = the pressure indicated by the steam gauge × the area of the valve.

b = the distance from the valve to the fulcrum in inches.

V = the weight of the valve in pounds.

L = the weight of the lever in pounds.

h = the distance from the center of gravity to the fulcrum in inches.

W = the weight of the ball in pounds.

y = the distance from the ball to the fulcrum in inches.

$S \times b$ = power $\times$ power arm.
$W \times y$ = weight $\times$ weight arm.
$V \times b$ = weight of valve $\times$ power arm.
$L \times h$ = weight of lever $\times$ weight arm.

What weight ball must be put on a 3-in. safety valve that it may blow off at 100 lbs.? The weight of the valve is 4

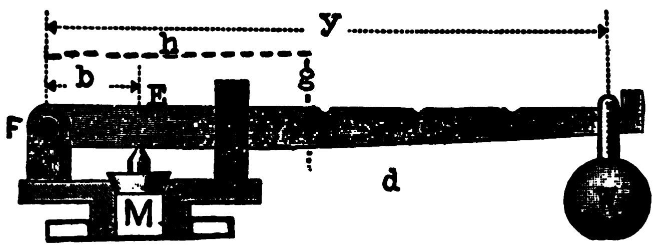

FIGURE 58

lbs.; the weight of the lever is 20 lbs.; the distance from the ball to the fulcrum is 34 in.; the distance from the center of gravity to the fulcrum is 12 in.; the distance from the valve to the fulcrum is 4 in. $3^2 \times .7854 \times 100$ lbs. = 707 lbs. = S. $S \times b = 4 \times 707$ lbs. = 2,828 lbs. $V \times b = 4 \times 4$ lbs. = 16 lbs. $L \times h = 12 \times 20$ lbs. = 240 lbs. $W \times y = W \times 34$. 16 + 240 + 34 $\times$ W. $W \times 34 + 16 + 240 = 2{,}828$. $34 \times W =$ 2,572 lbs. W = 75.64 lbs.

PROBLEMS

1. If the area of a safety valve is 7 sq. in., the weight of lever is 7 lbs., the weight of valve is 4 lbs., the weight of ball is 50 lbs., the center of gravity is 7 in. from fulcrum, the valve is 2 in. from fulcrum, the pressure on the valves is 80 lbs. per square in., where must W be placed to blow off at 80 lbs.?

2. With a 2-in. safety valve where must the 35-lb. weight be placed to blow off at 85 lbs., if the weight of lever is 8 lbs., the weight of valve is 4 lbs., the distance from the

center of gravity to the fulcrum is 12 in., the distance from the valve to the fulcrum is 3 in., the distance from the ball to the fulcrum is 30 in.?

3. The diameter of a safety valve is 4 in.; the weight on the lever is 90 lbs.; the weight of the lever is 12 lbs.; the weight of the valve is 4 lbs.; the distance from the valve to the fulcrum is 2.5 in.; the length of the lever is 30 in.: the distance from the center of gravity to the fulcrum is $11\frac{1}{2}$ in. How far from the fulcrum must the weight be placed so as to blow off at 80 lbs.?

4. A safety valve is $2\frac{1}{2}$ in. in diameter, and weighs 3 lbs. The distance from the valve to the fulcrum is 5 in.; the distance from the center of gravity to the fulcrum is 10 in.; the distance from the ball to the fulcrum is 24 in.; the weight of the lever is 15 lbs. What weight must be used to blow off at 90 lbs.?

5. What weight valve, whose diameter is 2 in., must be used to blow off at 90 lbs., if the ball weighs 60 lbs., the lever weighs 8 lbs.; the distance from the ball to the fulcrum is 12.7 in., the distance from the center of gravity to the fulcrum is 10 in. and the distance from the valve to the fulcrum is 3 in.?

6. How far must a valve, whose diameter is 4 in., be placed from the fulcrum, if the valve weighs 8 lbs., the ball weighs 64 lbs. and blows off at 90 lbs., the lever weighs 24 lbs., the distance from the ball to the fulcrum is 38 in. and the distance from the center of gravity to the fulcrum is 16 in.?

PUMPS

321. Let P = the pressure in pounds per square inch on the piston to lift the piston. Let H = the height in feet through which the water is lifted. Let L = the length of the stroke of the piston in feet. Let D = the diameter of the piston in inches. Let Q = the number of cubic feet of water dis-

charged per minute. Let N = the number of strokes per minute. Then Q = the area of the piston times the length of the stroke in feet times the number of strokes per minute ÷ by 144.

$$\text{Or, } Q = \frac{D^2 \times .7854 \times L \times N}{144}.$$

Work in feet per minute = $Q \times 62.5 \times H$. One cubic foot of water weighs 62.5 lbs. H = the height in feet to which the water is lifted.

H. P. = foot-pounds per minute ÷ 33,000.

$$\text{Or, } H.P. = \frac{Q \times 62.5 \times H}{33{,}000}.$$

Work done per stroke in foot-pounds = $D^2 \times .7854 \times L \times P$.

The work done per stroke by a pump whose piston area is 40 sq. in., stroke 2 ft., and piston pressure is 40 lbs. per square inch is $D^2 \times .7854$ = 40 sq. in., piston area. L = 2 ft. P = 40 lbs. per square inch. $40 \times 2 \times 40 =$ 3,200, the number of foot-pounds.

PROBLEMS

1. What is the H. P. of a pump whose plunger is 10 in. in diameter; stroke of piston 2 ft., pressure 60 lbs. per sq. in.; and number of strokes per minute 40?

2. How many gallons will be delivered per minute by a pump whose plunger diameter is 4 in., length of stroke 12 in., and number of strokes per minute 24? (1 cu. ft. = 7.48 gal.)

3. How many cubic feet of water will be delivered per minute by a pump whose plunger diameter is 6 in., length of stroke 20 in., and number of strokes per minute 36?

4. How many cubic feet of water will be delivered per minute by a pump whose plunger diameter is 5 in., length of stroke 16 in., and number of strokes 40?

PRESSURE OF WATER AT DEPTH

322. The pressure of water at any depth with a free upper surface, is the weight of the overlying water, and is 1 lb. per square inch for every 2.3 ft. of depth. A cubic foot of water weighs 62.5 lbs. A cubic inch of water weighs .03617 lb. A column of water 12 in. long and 1 sq. in. in cross-section weighs (.03617 × 12) .434 lb. Let $P =$ the pressure in pounds per square inch; $H =$ the head of water in feet; $W =$ the weight of a column of water 1 ft. long and 1 sq. in. in cross section. Then $P = WH$, or, $P = .434\ H$.

If the depth of water in a standpipe is 100 ft., the pressure of water in pounds per inch on the bottom of the pipe is $P = .434 \times H = .434 \times 100 = 43.4$ lbs. per square inch. If it is required to find the head of water for a given pressure per square inch., $H = \frac{P}{W}$. Or, $H = \frac{P}{.434}$. To find the head of water in a standpipe to give a pressure of 100 lbs. per square inch, $H = \frac{100}{.434} = 230.4$, the head in feet.

PROBLEMS

1. If the difference in pressure in pounds per square inch, as found by a pressure gauge, between two faucets in a building is 23 lbs., what is the difference in level between the two points in feet? $23 \div 2.3 = 10$, the difference in feet.

2. A submarine boat goes 50 ft. below the surface. What is the pressure in pounds per square inch on the surface? $P = .434 \times H = .434 \times 50 = 21.70$, the number of pounds per square inch. What is the pressure on a part of the surface of the boat 2 ft. by 2 ft.?

3. What is the pressure per square foot on the surface of a diver who is working 40 ft. below the surface of a lake?

4. What is the pressure per square inch corresponding to a head of 350 ft.?

5. What is the head of water which will give a pressure of 90 lbs. per square inch?

6. What is the pressure of water per square inch striking the blades of a turbine wheel when the head of water is 96 ft.?

TURBINES

323. High Pressure Turbines: They have elevated heads and revolve at high speed.

324. Low Pressure Turbines: They have large volumes of water at low head and revolve at low speed.

325. Reaction Turbines: All parts are filled with moving water and only a part of the energy of the water is converted into velocity. A greater part is in the form of pressure due to the reaction of the moving water as it issues from the vanes of the wheel.

326. Impulse Turbines: These are only partly filled with water, which strikes the turbine blades in the form of jets. The water energy is converted into velocity before it acts upon the moving parts of the turbine. The acting force is the pressure due to the impulse of the jets of water issuing from the portals.

327. The weight of water used *per second in an impulse wheel is equal to the cross-section area of the jet in square feet multiplied by the velocity of the jet in feet per second multiplied by 62.5.*

$$W = \frac{D^2 \times .7854 \times V \times 62.5}{144}.$$

W = the weight in pounds. V = the velocity in feet per second. 62.5 = the weight in pounds of 1 cu. ft. of water.

$$H.\,P. = \frac{W \times H}{550}.$$

H = the height of the head of water above the jet. The velocity of a body falling from a height, at end of fall, is, $V = 8.02\sqrt{H}$ ft. per second.

An impulse turbine wheel is working to the best advantage when the velocity of the rim of the wheel is one half the velocity of the jet.

328. To find the diameter of a turbine wheel which works to the best advantage with a dynamo. Let D = the diameter and V = the velocity of a turbine wheel.

$$V = D \times 3.1416 \times R.\ P.\ M. \quad D = \frac{V}{3.1416 \times R.\ P.\ M.}$$

329. Rule: *The diameter of a turbine wheel equals the velocity of the rim of the turbine wheel in feet per minute divided by 3.1416 × R. P. M of the dynamo.*

What should be the diameter (in feet) of a turbine wheel connected to the shaft of a dynamo, if the head of water is 400 ft. and the dynamo makes 900 R. P. M.? $V = 8.02\sqrt{H}$, or $V = 8.02\sqrt{400} = 160.40$ ft. per second. The velocity of the rim of the turbine should be one half of this. Or, $\frac{1}{2}$ of $160.4 = 80.2$, the number of feet per second. And $60 \times 80.2 = 4{,}812$, the number of feet per per minute = the velocity of the rim of the turbine wheel in feet per minute.

The turbine wheel is connected to the shaft of the dynamo.

The diameter of the turbine wheel in feet is

$$D = \frac{4{,}812}{3.1416 \times 900} = 1.6.$$

330. The weight of water discharged *per second from a turbine wheel is equal to the volume of water discharged in cubic feet multiplied by 62.5, the weight in lbs. per cubic foot.* Or, $W = Q \times 62.5$. Let W = the weight in pounds. Let Q = the volume in cubic feet per second.

331. The volume of water discharged *in cubic feet per second equals the area of the jet in square feet multiplied by*

the velocity in feet per second. Or, $Q = A \times V$. Let A = the area in square feet. Let V = the velocity in feet per second. Area $= D^2 \times .7854$. Or, area $= R^2 \times 3.1416$. $W = A \times V \times 62.5$.

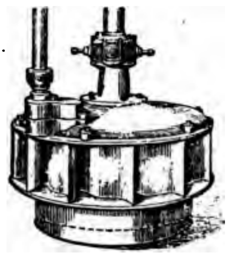

FIGURE 59

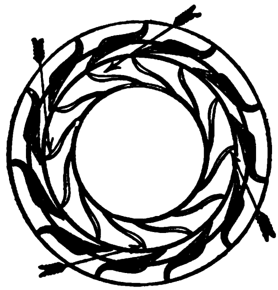

FIGURE 60

PROBLEMS

1. What is the horsepower of a turbine wheel which is discharging 1,350 lbs. of water per minute under a head of 625 ft.?

2. What should be the diameter of a turbine wheel connected to the shaft of a dynamo, if the head of water is 900 ft. and the dynamo makes 800 R. P. M.?

3. A turbine wheel has a jet diameter of $1\frac{1}{2}$ in., and a velocity of 130.34 ft. per second. What is the weight of water discharged per second?

4. What is the horsepower of a turbine which is discharging 100 cu. ft. of water per minute under a head of 50 ft.?

5. A turbine has a jet diameter of $1\frac{3}{4}$ in., and a jet velocity of 150 ft. under a head of 484 ft. What is the horsepower?

6. What is the horsepower of a turbine which discharges 675 lbs. of water under a head of 400 ft.?

7. What should be the diameter of a turbine connected to a dynamo shaft which is making 850 R. P. M., if the head of water is 625 ft.?

8. What is the velocity of a falling body at the end of a fall of 250 ft.? What should be the velocity of the rim of a turbine wheel with a head of 250 ft.?

9. The turbine pits at Niagara Falls are 136 ft. deep, and each turbine discharges 25,000 cu. ft. of water per minute. What is the horsepower of each turbine?

10. A turbine discharges 180 cu. ft. of water per minute. The head is 390 ft. What is the horsepower?

11. What should be the velocity of the rim of a turbine wheel, if the total fall of water is 400?

12. What is the horsepower of a turbine wheel which is discharging 4,000 cu. ft. of water per minute under a head of 900 ft.?

13. What weight of water is discharged per second by a turbine wheel, if the cross-section area of the jets is 9 sq. in. and the velocity of the jets is 80 ft. per second?

14. The power supplied the turbine at Niagara Falls is 31,250,000 foot-pounds per second. What is the horsepower of the turbine?

STOCK AND FORGING

332. To find what sized piece of stock to use to make a given forging, we must find the weight of the forging and also an equivalent weight of a bar of stock of the size to be used. In some cases, as in welding, a little more must be added—an allowance in length equal to the thickness of the stock.

In measuring a forging, measure along the central line or axis of the piece and not along the outside or inside curve. There is a difference in bent circular pieces, whether we measure along the outside of a curve, inside of a curve or along the central axis. A certain curve measures $8\frac{1}{2}$ in. outside measurement, 7 in. inside measurement, and $7\frac{3}{4}$ in.

along the central line or central axis. For round stock, from the outside diameter subtract, or to the inside diameter add, one thickness of the stock, and multiply by $\frac{22}{7}$ (π).

PROBLEMS

1. What is the length of stock for a flat ring $\frac{1}{4}$ in. thick, $1\frac{1}{4}$ in. wide, and $4\frac{1}{4}$ in. inside diameter?

2. A disk $4\frac{1}{4}$ in. in diameter and 1 in. thick is to be forged. How long a piece of stock from a round bar $3\frac{1}{2}$ in. in diameter must be cut off for the disk?

3. A ring 8 in. outside diameter, $\frac{3}{4}$ in. thick, and 4 in. wide is to be made. The nearest size of flat bar in stock is $2\frac{3}{4}$ in. by $1\frac{1}{2}$ in. How long a piece of the bar must be used?

4. A flat ring whose inside diameter is 8 in., outside diameter $10\frac{1}{2}$ in., and thickness $\frac{1}{2}$ in. is to be made. How much stock 4 in. wide is required?

5. How long a piece of bar must be used from flat bar stock 3 in. by $1\frac{3}{4}$ in., to make a ring 6 in. outside diameter, $\frac{3}{4}$ thick, and 2 in. wide?

BLAST FURNACE

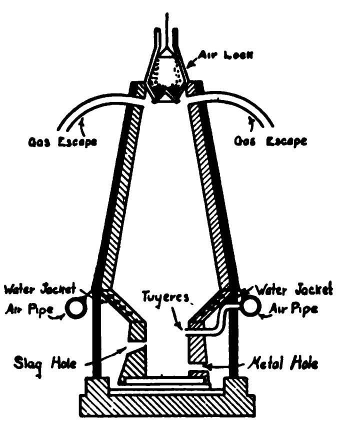

FIGURE 61

333. Fig. 61 represents a blast furnace into which hot, dry, blasts of air are forced over the burning mass by blowers to assist in the melting and reduction of iron ore. This furnace is a cylindrical steel shell, 75 to 90 ft. high and lined with fire brick. A charge of ore, coke and flux-limestone is dropped in at the top of the furnace from time to time. Cold water is made to circulate through hollow castings where

the heat is most intense. This is just above the tuyeres, or blow pipes, where the powerful hot, dry air blasts enter. The impure molten iron settles to the bottom of the furnace, from where it is drawn off from time to time through the tap hole in the bottom.

The limestone flux and other impurities are called slag and are drawn off about every two hours, and the molten iron about every six hours. The gases produced are led away at the top of the furnace and utilized for heating.

The slag is generally regarded as useless and thrown away. The iron is allowed to flow out into trenches made in beds of sand. The iron, as soon as cool enough, is broken into pieces about three feet long, and is called pig iron.

PROBLEMS

1. A cupola melts 45,000 lbs. of iron with 5,500 lbs. of coal. What is the percentage of fuel to iron that is used?

2. A cupola measures 36 in. inside diameter and 12 in. from the sand bottom to the tuyeres, or top of the ore. How many pounds of iron does it hold? (One cubic foot of cast-iron weighs 450 lbs.)

3. A cupola measures 32 in. inside diameter and 16 in. from sand bottom to bottom of tuyeres. How many pounds of melted iron will the cupola hold?

4. How many pounds of fuel will it take to melt 3,000 lbs. of iron, if the ratio of fuel to iron is 1 to 7?

5. A cupola melts 7,000 lbs. of iron with 800 lbs. of coke. What is the ratio of fuel to iron?

6. A cupola charge has 25,000 lbs. of iron worth $18 per ton and 4,000 lbs. of coke worth $4 per ton. What will the iron cost per pound?

7. A blast furnace charge is 4,000 lbs. of ore containing 60% of iron, 800 lbs. of limestone, and 2,000 lbs. of coke.

What is the number of pounds of each per ton of pig iron produced?

8. A furnace requires 2,400 lbs. of coal to produce one ton of iron, when the ore is 60% iron. What proportion of coal to iron ore must the charge contain?

9. A charge is made up of different kinds of iron as follows: 25% iron, 20% iron and 60% iron. How many pounds of each in a total charge of 4,000 lbs.?

10. A charge is made up of 8,000 lbs. of one kind of iron, 8,500 lbs. of another kind, and 9,000 lbs. of a third kind. What is the percentage of each?

HEAT AND SPECIFIC HEAT

334. The specific heat of water is 1; of ice is .5; of silver is .057; of copper is .095; of zinc is .096; of lead is .031; of wrought iron is .11; of cast iron is .13; of brass is .094.

The specific heat of a substance is the number of B. T. U. (British thermal units) of heat to raise the temperature of 1 lb. of the substance through 1° F. It is the ratio between the amount of heat required to heat the body through 1° and the amount of heat required to heat an equal weight of water through 1°. To heat any weight of copper through 1° requires only .095 as much heat as to heat an equal weight of water through 1°. Hence the specific heat of copper is .095.

The latent heat of fusion of a substance is the amount of heat given up by a liquid in freezing or absorbed by a solid in melting.

The amount of heat to change 1 lb. of ice at 32° F. to water at the same temperature is 147 B. T. U.

The latent heat of steam or vaporization is 967 B. T. U. That is the amount of heat required to change 1 lb. of boiling water at 212° F. to steam at the same temperature.

One B. T. U. of heat = 778 ft.-lbs. of work.

How many B. T. U. of heat are required to melt 20 lbs. of ice?

The amount of heat to melt 1 lb. of ice is 147 B. T. U. 20×147 B. T. U. = 2,940 B. T. U., which is the amount of heat required to melt 20 lbs. of ice.

How many B. T. U. of heat are required to raise the temperature of 20 lbs. of ice after melting to 60° F.? The specific heat of ice is .5 B. T. U. Therefore, $20 \times .5$ (the specific heat of ice) B. T. U. = 10 B. T. U.

How many B. T. U. would be required to melt 20 lbs. of ice and then raise its temperature to 60° F.? 2,940 B. T. U. + 10 B. T. U. = 2,950 B. T. U.

PROBLEMS

1. How many B. T. U. are given out by a steam radiator when 50 lbs. of steam have been condensed into water in it? 50×967 B. T. U. per lb. = 48,350 B. T. U.

2. How many B. T. U. are liberated by the boiling away of 4 lbs. of water?

3. If the temperature of fusion for lead is 626° F. and the latent heat of fusion is 9.67, how much heat will be required to melt 20 lbs. of lead from a temperature of 30° F.? $20 \times (626 - 30) \times .0314$ (sp. heat) = 374,288 B.T.U. 20×9.67 latent heat of fusion = 193.40 B.T.U. 374.288 B.T.U. + 193.40 B.T.U. = 567.688 B. T. U.

4. How many pounds of coal will it take to melt 20 lbs. of zinc from 40° F. if the latent heat of fusion of zinc is 50.63, the temperature of fusion is 680° F., and there is no heat lost?

5. How many pounds of tin can be melted from 46° F. by 100 lbs. of coal, if the latent heat of fusion of tin is 25.65, the temperature of fusion is 446° F. and the specific heat of tin is .0562? (Let x = the number of lbs. of tin.)

6. How many B. T. U. are carried into a kitchen by steam when 1 qt. of water has been allowed to boil away. (1 qt. weighs about 2 lbs.)

2×967 B. T. U. per lb. = 1,934 B. T. U.

7. How many units of heat are required to raise the temperature of 10 lbs. of zinc from 60° F. to 680° F.? $10 \times (680 - 60) \times .0956 = 592.72$.

8. If it requires 47.55 B. T. U. of heat to warm 25 lbs. of copper 20° F., what is the specific heat of copper?

9. How much heat is needed to heat 100 lbs. of steel 2,520° F.? Specific heat of steel is .1175.

10. How many pounds of good coal will be required to melt 50 lbs. of zinc from 56° F.? Latent heat of fusion is 50.63. Temperature of fusion is 680° F.

11. If 50 lbs. of good coal is used under a boiler to run an engine, how many foot-pounds of work will be done, if the boiler and engine works at 15% efficiency?

12. How many tons of coal will be required to melt one ton of ice at 32° F. and change it to steam at 212° F., if no heat is lost? (The latent for ice to water is 147° F. and for water to steam is 967° F.)

LINEAR EXPANSION

335. The coefficient of linear expansion is the ratio of increase in length per degree of rise in temperature to the total length.

336. The following table gives the coefficient of expansion for some substances per degree Fahrenheit.

Brass	.00001036	Cast-iron	.00000618
Wrought iron	.00000686	Steel tempered	.00000702
Steel	.00000698	Copper	.00000955
Untempered steel	.00000599	Zinc	.00001644

Two times these numbers will give surface expansion. Three times these numbers will give cubic expansion.

PROBLEMS

1. A wrought iron bar 30 ft. long is heated from 80° to 300°. How much will it lengthen? 300° − 80° = 220°. $220 \times 30 \times .00000686 = .045276$, the number of feet.

2. A cast-iron bar 20 ft. long is heated from 70° to 350°. How much will it lengthen?

3. A copper bar 1 ft. long is heated from 100° F. to 250° F. How much will it lengthen?

4. A cast-iron steam pipe 400 ft. long has expansion collars. Each of these collar gives $1\frac{1}{2}$ in. free play. How many must be put in to allow for a range of temperature from 32° F. to 232° F.?

5. How many expansion collars $1\frac{1}{2}$ in. free play must be put into a 700 ft. steam pipe to allow for a range of temperature of 200° F.?

6. A ring of cast-iron has an inside diameter of 5 ft. when at a temperature of 932° F. What is the diameter at 32° F.?

7. A wrought iron connecting rod is 20 ft. long at 18° F. What is the increase of length at 144° F.?

8. The volume of a mass of cast iron is 5 cu. ft. at 18° F. What is its volume at 140° F.?

9. A cast-iron steam pipe is 100 ft. long at 32° F. What is its length when steam at 215°F. passes through it?

10. A carriage wheel is 7 ft., $6\frac{3}{8}$ in. in circumference. A cast-iron tire is 7 ft., 6 in. on its inner circumference at 27° F. To what temperature must the tire be heated to just slip on the wheel?

11. A wrought iron bar 28 ft. long is heated from 40° F. to 400° F. How much will it lengthen?

12. A cast iron plate 6′ × 4′ is heated from 38° F. to 300° F. What will be the surface expansion? What will be its dimensions after heating?

13. A block of copper whose dimensions are $4' \times 3' \times 2'$ is heated from 34° F. to 400° F. How much does it expand?

14. A steel cable is one mile long. By how many feet does its length vary between a winter day when the temperature is $-4°$ F. and a summer day when the temperature is 86° F. (Iron expands .0000066 of its length for each degree increase in temperature, F.

15. If iron rails are 30 ft. long, and if the variation of temperature through the year is 90° F., what space must be left between their ends?

16. If an iron steam pipe is 60 ft. long at 32° F., what is its length when steam passes through it at 212° F.?

17. In long steam iron pipes expansion joints are inserted every 200 ft. If the range of temperature is from 22° F. to 257° F., what play must be allowed for at the expansion joint?

18. A line shaft is 200 ft. long at the temperature of 50° F. How long will it be at 90° F.?

19. A shaft is 2.45 in. in diameter. To what size must a collar be bored to give a solid fit? (For a solid fit, or driving fit, .001 per inch of diameter is allowed.)

HOT-WATER HEATING

337. To find approximately the number of heat units *to heat a building of any number of cubic feet capacity to any temperature.* The specific heat of air is .238. That is, .238 heat units will raise the temperature of 1 lb. of air 1°F. And at zero 1 cu. ft. of air weighs .081 lb. Then 1 divided by .081 = 12.34, which represents the number of cubic feet of air occupied by 1 lb. of air. 12.34 divided by .238 = 51+, cu. ft., raised 1° by one B. T. U.; but in practice it is called 50 cu. ft. Therefore, 1 British thermal unit, or B.T.U., will raise the temperature of 50 cubic feet of air through 1°F.

To raise the temperature of 1,000 cu. ft. of air through 1°F. will require as many B. T. U. as 50 is contained times in 1,000, or 20. To raise the temperature of 500 cu. ft. of air through 1°F. will require (500 divided by 50) 10 B. T. U. If we wish to heat 1,000 cu. ft. of air through 30 degrees, it will take 30 times 20 B. T. U., or 600 B. T. U. If we wish to heat 500 cu. ft. of air through 40 degrees, it will take 40 times 10 B. T. U., or 400 B. T. U.

A room is 20′ × 30′ and 10′ high. How many heat units will be required to raise the temperature of the room from 32° to 68°F.?

20 × 30 × 10 = 6,000. 1 B. T. U. will raise the temperature of 50 cu. ft. of air 1°F. To raise through 1°F. the temperature of 6,000 cu. ft. of air, which is 120 times 50 cu. ft., will require 120 times 1 B. T. U., or 120 B. T. U. To raise the temperature from 32° to 68° will require 36 × 120 B. T. U. = 4,320 B. T. U.

Heat Losses in B. T. U. per Square Foot of Surface per Hour Southern Exposure

MATERIAL	Difference between inside and outside temperature									
	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°
8 in. Brick Wall	5	9	13	18	22	27	31	36	40	45
12 in. Brick Wall	4	7	10	13	16	20	23	26	30	35
16 in. Brick Wall	3	5	8	10	13	16	19	22	24	27
20 in. Brick Wall	2.8	4.5	7	9	11	14	16	18	20	23
24 in. Brick Wall	2.5	4	6	8	10	12	14	16	18	20
28 in. Brick Wall	2	3.5	5	7	9	11	13	14	16	18
32 in. Brick Wall	1.5	3	4.5	6	8	10	11	13	15	16
Single Window	12	24	36	49	60	73	85	93	110	122
Double Window	8	16	24	32	40	48	56	62	70	78
Single Skylight	11	21	31	42	52	63	73	84	94	104
Double Skylight	7	14	20	28	35	42	48	56	62	70
1 in. Wooden Door	4	8	12	16	20	24	28	32	36	40
2 in. Wooden Door	3	5	8	11	14	17	20	23	25	28
2 in. Solid Plaster Partition	6	12	18	24	30	36	42	48	54	60
3 in. Solid Plaster Partition	5	10	15	20	25	30	35	40	45	50
Concrete Floor on Brick Arch	2	4	6.5	9	11	13	15	18	20	22
Wood Floor on Brick Arch	1.5	3	4.5	6	7	9	10	12	13	15
Double Wood Floor	1	2	3	4	5	6	7	8	9	10
Walls of Ordinary Wood Dwellings	3	5	8	10	13	16	19	22	24	27

The above table applies only to a southern exposure; for other exposures multiply the heat loss given in table above by the factors given in the table below:

Factors for Calculating Heat Losses for Other than Southern Exposure

EXPOSURE	FACTOR
North	1.32
East	1.12
West	1.20
Northeast	1.22
Northwest	1.26
Southeast	1.06
Southwest	1.10
N. S. E. and West or total exposure	1.16

A room is 10 × 15 ft., with 10-ft ceiling, and the walls are 20″ brick. There are two windows 3 × 5 ft. each, and one window 4 × 5 ft.

The exposure is northwest. How many square feet of hot-water radiation will be required to keep the temperature at 70°F. when it is 10°F. below outside? Total exposed surface = 250 sq. ft. Glass surface = 50 sq. ft. Net wall surface = 250 − 50 = 200 sq. ft. Find 18 for multiplier from Table of Heat Losses, for 20 in. brick walls and 80° difference in temperature. Find 93 for single window glass surface and 80° difference in temperature. Get the sum of these products. Find from table the factor 1.26 for northwest exposure. Multiply the total 8,250 sq. ft. for walls and glass surface by 1.26 = 10,395, the number of B. T. U. required to heat the surfaces. For the air in the room, divide 1,500 cu. ft. capacity by 50 and multiply by 80°, difference in temperature. This gives 2,400, the number of B. T. U. required to heat the air in the room. The total B. T. U. is 10,395 + 2,400 = 12,795 B. T. U. For hot-water radiation divide 12,795 B. T. U. by 150 = 85.3, which represents the number of square feet of radiation.

338. Rule: *The cubic feet of air space in the room is divided by 50, and this quotient is multiplied by the number of*

degrees of temperature through which the temperature of the room is to be raised. This result gives the B. T. U. of heat.

The sum of these three gives the total heat required in B. T. U.

Divide this total heat required in B. T. U. by 150 B. T. U. for hot-water radiation. This gives the number of square feet of hot-water radiation required.

339. For solid stone walls, multiply the figures for brick of the same thickness by 1.7.

Where rooms have a cold attic above, or a cold cellar below, multiply the heat losses through walls and windows by 1.1.

The figures given in the tables apply only to the most thoroughly constructed buildings.

340. For the average well built house the figures should be increased by about 10%; for fairly well constructed, by 20%; for poorly constructed, by 30%.

For example: A room is $10' \times 15' \times 10'$ ceiling. The walls are 20 in. brick. The construction is the very best. The room has two windows $3' \times 5'$ and one window $4' \times 5'$. The exposure is north and west to be heated with hot-water apparatus. The average temperature of the water is to be 180 degrees. The temperature of the room is to be kept at 70 degrees when it is 10 degrees below zero outside.

341 In computing heat losses through walls, only those exposed to the outside air are to be considered.

Total exposed surface:

$$15 + 10 = 25.$$
$$25 \times 10 = 250\text{—}250 \text{ sq. ft.}$$

Glass surface:

$$\begin{aligned} &2 \text{ windows } 3 \times 5 \times 2 = 30 \\ &1 \text{ window } 4 \times 5 \quad\;\; = 20 \\ &\overline{50 \text{ sq. ft. glass}} \end{aligned}$$

250 sq. ft. − 50 sq. ft. = 200 sq. ft. of net wall surface.

From the table of heat losses, 20-in. brick will lose 18 B. T. U. per square foot per hour when the temperature difference is 80 degrees, which is the difference when the temperature of the room is to be kept at 70 degrees in 10 degrees below zero weather. 200 × 18 = 3,600, the units of heat loss through the wall. 50 × 93 = 4.650 , the units of of heat loss through the windows, at 80 degrees difference in temperature. The total units of heat loss through windows and walls is 3,600 + 4,650 = 8,250. The factor for northwest exposure is 1.26. 8,250 × 1.26 = 10,395, the number of B. T. U. of heat loss for northwest exposure.

342. The air in a room should change once per hour. One B. T. U. will heat 50 cu. ft. of air one degree.

The cubical contents of the room is 15′ × 10′ × 10′ = 1,500 cu. ft. 1,500 × 80, which is the number of degrees the air is to be heated = 120,000 (the air outside 10 degrees below and the air inside 70 degrees).

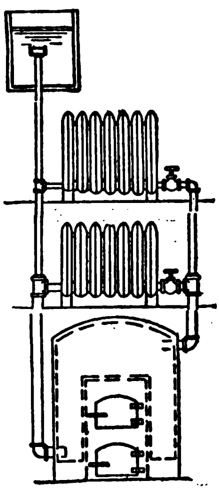

FIGURE 62
HOT-WATER HEATING

120,000 divided by 50 = 2,400. We then have the heat loss through the walls and windows, 10,395 B. T. U. and the heat for heating the changing air in the room 2,400 B. T. U. 10,395 B. T. U. + 2,400 B. T. U. = 12,795 B. T. U.

By experiment it is found that one square foot of hot-water radiation standing in air of 70 degrees will yield 187 B. T. U. per hour, and one square foot of steam radiation will yield 240 B. T. U. In practice, one square foot of hot-water radiation, is assumed to yield 150 B. T. U. per hour. 12,795 divided by 150 = 82.6 which is the number of square feet of hot-water

radiation which should be placed in the room. The number of square feet of heating surface in tubes equals the number of tubes multiplied by the diameter of the tube in inches, and by its length in feet, and by .2618.

One pound of water evaporated at and from 212° F. = 965.7 B. T. U.

343. Each person breathes on an average about 28 cu. ft. of air in an hour.

A room in a grammar school is 28′ × 32′ × 12 and is to accommodate 50 pupils. The walls are of brick, 16 in. thick, and there are 6 single windows in the room, each 3′ × 6′. There are warm rooms above and below. Exposure is S. E. How many B. T. U. will be required per hour for warming the room and how many for ventilation in zero weather, assuming the building to be of average construction? Air requirements for ventilation in grammar schools are 1,800 cu. ft. per hour per pupil.

Exposed walls are 28′ + 32′ = 60′. Height 12′ × 60′ = 720 sq. ft. For the windows, 3′ × 6′ = 18 sq. ft. and 6′ × 18′ = 108 sq. ft. Square feet of net wall surface = 720 − 108 = 612 sq. ft. From the table of Heat Losses we get the factor 19 which multiplied by 612 = 612 × 19 = 11,628 B. T. U. From the table we get 85 the factor for single windows, which multiplied by 108 = 108 × 85 = 9,180, the number of B. T. U. required. Then 11,628 B. T. U. + 9,180 B. T. U. = 20,808 B.T.U. And 20,808 B. T. U. × 1.06 × 1.1 = 24,262 B. T. U., heat requirements for warming the room.

1,800 cu. ft. × 50 = 90,000 cu. ft. And 90,000 cu. ft. × 70 ÷ 50 = 126,000, the number of B. T. U. for ventilation. 24,262 + 126,000 = 150,262 B. T. U., total heat requirements. If the room is to be heated by steam at 21 lbs. gauge pressure, the temperature of steam at this pressure is 220 degrees. 1 sq. ft. of radiation yields 240 B. T. U. per hour.

Divide 150,262 B.T. U. by 240 = 626.09 B. T. U., the total amount of radiation required.

STEAM HEAT

344. Find how many square feet of steam radiation will be required to heat a room on the first floor of a house, where rooms above, hall and room on each side, and cellar, are warm. The room is 10′ × 10′ × 10′ with two windows 3′ × 5′ each, and the exposure is northwest.

The number of square feet of exposed surface is 10 + 10 = 20 × 10 = 200 sq. ft. Window surface is 3 × 5 = 15 sq. ft. each. For two windows it is 30 sq. ft.

200 sq. ft. of exposed surface less 30 sq. ft. of window surface = 170 sq. ft. of exposed wall surface. The difference of temperature, 70° inside and 10° below outside = 80°.

With a wind velocity of about 12½ miles per hour, the rate of heat loss per square foot of single thick common glass is about 1.20 B. T. U. per degree of difference in temperature between outside and inside air. For double thick glass the loss per square foot is about .80 B. T. U. per degree of difference in temperature.

For ordinary dwellings of wood the rate of heat losses per square foot through the walls is about .27 B. T. U. per degree of temperature.

For the loss of heat by the glass surface, single glass, we have 1.20 × 80 difference in temperature × 30 = 2,880, which is the number of B. T. U. required.

For the loss of heat by the wall surface, we have .27 × 80 difference in temperature × 170 = 3,672, which is the number of B. T. U. required.

345. To find the number of heat units ***necessary to raise the temperature of the air to the required temperature,*** we have 1,000 cu. ft. divided by 50 cu. ft. = 20, which is the number

of B. T. U. required to heat the air 1°; and 20 × 80 = 1,600, the number of B. T. U. required.

346. Rule: *Find the heat losses through the walls. Find the heat losses through the windows. Find the cubic feet of room space, and how much heat to heat this room space to the required temperature. This result gives the total required amount of heat for the building in B. T. U.*

The area of the wall is multiplied by the heat loss factor for the kind of wall and exposure. Only walls exposed to outside air are considered.

The area of windows is multiplied by the heat loss factor for windows.

The cubic feet of air space in the room is multiplied by the amount of heat required for one cubic foot of air. The sum of these three gives the total heat required in B. T. U. The cubic feet of air space is divided by 50 and this quotient multiplied by the number of degrees of temperature through which the temperature of the room is to be raised.

One square foot of steam radiation yields about 240 B. T. U. of heat. All radiator companies furnish the number of square feet of radiation for their radiators.

The total number of heat units lost through walls, glass, and to heat the air in the rooms, is 3,672 for walls + 2,880 for glass × 1,600 for air in the room = 8,152 B. T. U.

It is known that one square foot of steam radiation standing in air of 70°F. and the temperature of the steam 212°F. will yield about 240 B. T. U.

So, we have 8,152 divided by 240 = 33.96, the number of square feet of radiation + 26%, or multiplied by 1.26 for northwest exposure = 42.79, the number of square feet of radiation required for the room.

Or, we may say that 80 × .27 = 21.60, the multiplier for the square feet of exposed walls. And 80 × 1.20 = 96, the multiplier for the square feet of glass surface. And

1,000 (cu. ft. of cubical contents of the room) divided by 50 = 20, the number of B. T. U. required to heat the air 1°F. And 20 × 80 = 1,600, the number of B. T. U. Then for wall, 170 × 21.6 = 3,672. For the glass, 30 × 96 = 2,880. For the air volume, 1,600 B. T. U. And 3,672 + 2,880 + 1,600 = 8,152. The total B. T. U. loss, 8,152, divided by 240 = 33.96, the number of square feet of radiation. This + 26% of itself or multiplied by 1.26 = 42.79, the number of square feet of radiation required.

PROBLEMS

1. A room is 20′ × 40′ × 10′ high. How many heat units will be required to raise the temperature of the room from 32° to 70° F.?

2. A room is 20′ × 20′ × 10′ high. How many heat units will be required to raise the temperature of the room from 40° to 70° F.?

3. How many heat units are required to warm a room from 34° to 75° F., if the room is 16′ × 20′ × 8′ high?

4. A four-room house has the following sizes of rooms: 10′ × 12′; 12′ × 14′; 16′ × 18′; and 16′ × 20′, all 9 feet high. How many heat units are required to warm these rooms from 36° to 75° F.?

5. A schoolroom in a frame building is 30′ × 30′ × 15′ ceiling with a northeast exposure, hall and adjoining room and above and cellar are warm. There are 4 windows, each 4′ × 7′. How many square feet of steam radiation are required to keep the room at a temperature of 70°, when the temperature outside is 10° below zero?

6. The 4 rooms on the lower floor of a house are each 12′ × 18′ × 10′ ceiling. The rooms above are warm and the cellar below is warm. There are 4 windows, each 4′ × 7′, one in each room. How many square feet of steam radiation are required to keep the rooms at a temperature of 70°, when the outside temperature is 10° below zero?

7. A schoolroom in a frame building is 24′× 36′ × 15′ ceiling with a northwest exposure; hall, adjoining and upper rooms and cellar are warm. There are 6 windows, each 4′ × 7′. How many square feet of steam radiation are required to keep the room at a temperature of 70°, when the temperature outside is 15° below zero?

8. The 4 rooms on the upper floor of a house are each 12′ × 16′ × 9′ ceiling. The attic above is cold and the rooms below are warm. There are 4 windows, each 4′ × 7′, one in each room. How many square feet of steam radiation are required to keep the rooms at a temperature of 70°, when the outside temperature is 12° below zero?

ELECTRICITY

347. In the electric circuit, the unit of *resistance* is the *ohm,* which is equal to the resistance of about 152 ft. of number 18 copper wire.

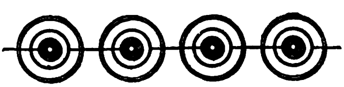

FIGURE 63—CELLS IN SERIES

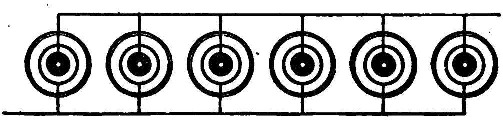

FIGURE 64—CELLS IN PARALLEL

The unit of current strength is the *ampere,* which is the *quantity* of electricity that will deposit .001118 gram of silver per second from a silver salt solution.

The unit of electromotive force is the *volt,* which is the electric *pressure* that will force a current of one ampere through a resistance of one ohm. The formula is, current in amperes:

$$(C) = \frac{\text{Electromotive force } (E)}{\text{Resistance in ohms } (R)}; \text{ or, } C = \frac{E}{R}; \text{ or, amperes} = \frac{\text{volts}}{\text{ohms}}; \text{ or, } C = \frac{E.\,M.\,F.}{R}; \text{ or, } C = \frac{E}{R + r}; \text{ or, } C = \frac{n\,E}{R + \frac{nr}{m}}.$$

R = external resistance; r = internal resistance; n = number cells in series; m = number cells in parallel.

348. Cells in series give an E. M. F. of a single cell multiplied by the number of cells. Cells in parallel give an E. M. F. of a single cell. Cells in series give an internal resistance of a single cell multiplied by the number of cells. Cells in parallel give an internal resistance of a single cell divided by the number of cells.

$$C = \frac{nE}{R + nr} \text{ (in series)}. \quad C = \frac{E}{R + \frac{r}{m}} \text{ (in parallel.)}$$

Connect in series when R is large in comparison with r. Connect in parallel when r is large in comparison with R.

349. With cells in series Figure 63, the pressure and the resistance are caused by the current passing through all the cells, and each is equal, respectively, to that of one cell multiplied by the number of cells.

From the six cells in parallel, Figure 64, there are six times as many conductors, or openings, for electric passage as there are from one cell. Hence, the resistance is one sixth as much as from one cell. The current is from each cell, separately, to the conductors. Hence the pressure from any number of cells is the same as that from a singlecell.

350. Rule: *Any number of cells in series have an E. M. F. of a single cell, and an internal resistance of a single cell, multiplied by the number of cells.*

Any number of cells in parallel have an E. M. F. of a single cell. They have an internal resistance of a single cell divided by the number of cells.

PROBLEMS

1. The resistance of a circuit is 5 ohms, the E. M. F. 210 volts. Find the current in amperes. $C = \frac{E}{R}$.

2. The resistance of a circuit is 10,000 ohms, the E. M. F. is 250 volts. What is the current in amperes?

3. A battery has a resistance of 4 ohms, and sends a current of .05 ampere through a conductor whose resistance is 50 ohms. What is the E. M. F. of the battery? (Total resistance is 4 ohms + 50 ohms = 54 ohms.)

4. A battery of 50 cells in series with an internal resistance of 100 ohms, gives an E. M. F. of 75 volts, and sends a current through an external resistance of 122 ohms. What is the strength of the current?

5. The resistance of a circuit is 5 ohms, the current is 50 amperes. What is the E. M. F.?

6. The E. M. F. in a circuit is 550 volts, the current is 10 amperes. What is the resistance?

7. The E. M. F. at a lamp is 110 volts, the resistance is 250 ohms. What is the current in amperes?

8. A battery of 2 ohms resistance sends a current of .05 ampere through a wire whose resistance is 50 ohms. What is the E. M. F. of the battery?

9. A battery of 50 volts E. M. F. and 20 ohms resistance has opposed to it in the same circuit a battery of 30 volts E. M. F. and 25 ohms resistance. A current of $\frac{1}{4}$ ampere is maintained in the circuit. What is the resistance thus connected?

50 volts − 30 volts = 20 volts.

$$C = \frac{E}{R};\ R = \frac{E}{C};\ R = \frac{20}{\frac{1}{4}} = 80, \text{ the number of ohms.}$$

The internal resistance is (20 + 25) 45 ohms. The resistance connected is 80 ohms − 45 ohms = 35 ohms.

10. A battery of 40 volts E. M. F. and 15 ohms resistance has opposed to it in the same circuit a battery of 30 volts E. M. F. and 20 ohms resistance. A current of $\frac{1}{4}$ ampere is maintained in the circuit. What is the resistance?

11. Four conductors in parallel have resistances of 25 ohms, 30 ohms, 40 ohms and 50 ohms respectively. What part of the whole current passes through each conductor?

The proportion of the current passing through each is $\frac{1}{25}$; $\frac{1}{30}$; $\frac{1}{40}$; $\frac{1}{50}$. Reducing to a common denominator, $\frac{24}{600} + \frac{20}{600} + \frac{15}{600} + \frac{12}{600} = \frac{71}{600}$. Numerators = 71. $\frac{24}{71}$; $\frac{20}{71}$; $\frac{15}{71}$; $\frac{12}{71}$ = parts of current that passes through each conductor.

12. How many amperes will a battery of 6 cells furnish, arranged 3 in series and 2 in parallel, the E. M. F. of a cell being 1.2 volts and the internal resistance being .5 of an ohm?

$$C = \frac{nE}{R + \frac{nr}{m}}. \quad C = \frac{3 \times 1.2}{0 + \frac{3 \times .5}{2}} = \frac{3.6}{\frac{1.5}{2}} = \frac{3.6 \times 2}{1.5} = 4.8$$
amperes.

13. How many amperes will a battery of 12 cells furnish, arranged 4 in series and 3 in parallel, the E. M. F. of a cell being 1.2 volts and the internal resistance being .5 of an ohm?

14. A battery of 48 cells is arranged 16 in series and 3 in parallel. The external resistance is 12 ohms, the E. M. F. is 1 volt and internal resistance 1.5 ohms per cell. Find the current.

15. What is the strength of 20 cells arranged in parallel? The E. M. F. is 1 volt, resistance 40 ohms for each cell and external resistance 1.5 ohms.

16. What number of cells must be used to pass a current of .025 amperes through an external resistance of 1,200 ohms, the cells being connected in series, and each having an E. M. F. of .8 volt and an internal resistance of 1.5 ohms?

$$\text{Use } C = \frac{nE}{R + nr}.$$

17. Eight cells are connected in series, each with an E. M. F. of 1.05 volts and 3.5 ohms. Three wires of 21 ohms each are connected in parallel to the poles of the battery. What is the current? (The battery E. M. F. is 8.40 volts and the internal resistance is 28 ohms. Three wires of 21 ohms each in parallel give an external resistance of 21 ohms. ÷ 3 = 7 ohms.)

18. A battery of 32 cells, 1 volt and 3 ohms each, is arranged 8 in parallel and 4 in series. What is the resistance and the E. M. F. of the battery?

19. Twelve cells are connected in series, each with an E. M. F. of 1.2 volts and 3.5 ohms. Three wires of 12 ohms each are connected in parallel to the poles of the battery. What is the current?

351. The resistance of a conductor is proportional to its length. The resistance of one mile of a certain conductor is 5 ohms. What is the resistance of 24 miles of the same wire?

The required resistance is to the given resistance as the length for the required resistance is to the length for the given resistance.

Let x = required resistance. $x : 5 :: 24 : 1$. $x = 120$ ohms.

The resistance of a certain conductor is 5 ohms, and the resistance of a mile of the same conductor is 20 ohms. What is the length of the conductor? The length for the required resistance: the length for the given resistance :: the required resistance: the given resistance.

$$x : 1 :: 5 : 20. \quad 20x = 5. \quad x = \tfrac{1}{4} \text{ mile}.$$

The resistance of 200 ft. of a conductor is 2 ohms. What length of wire has a resistance of 20 ohms?

$$2 \text{ ohms} : 20 \text{ ohms} :: 200 \text{ ft.} : x \text{ ft.}$$

The resistance of a conductor is inversely proportional to the area of its cross-section, or inversely proportional to the square of the diameter. The required resistance : the given resistance :: the area for the given resistance: is the area for the required resistance. Or, as the square of the diameter for the given resistance : the square of the diameter for the required resistance. If the resistance of a wire having a cross-section area of .008 sq. in. is 1.8 ohms, what would be its resistance if the area of its cross-section were .09 sq. in.?

x ohms : 1.8 ohms :: .008 : 09. x = .16 ohm.

The resistance of a wire having a diameter of .1 in. is 14 ohms. What would be its resistance if the diameter were .3 in.?

$$x : 14 :: .1^2 : .3^2. \quad x = 1.55 \text{ ohms.}$$

PROBLEMS

1. The resistance of a mile of copper wire 70 mils in diameter is 10.82 ohms. What is the diameter of a mile of copper wire which has a resistance of 43.28 ohms?

(A mil = .001 of an inch.)

2. What is the diameter of a wire, if 1,000 ft. of it has a resistance of 14 ohms, when the same length of wire with a diameter of 95 mils has a resistance of 1.15 ohms?

3. What is the resistance of 880 yds. of copper wire 160 mils in diameter, if the resistance of 1 mile of copper wire 230 mils in diameter is 1 ohm? The resistance of 880 yds. 230 mils in diameter = 880 ÷ 1,760 = .5 ohms. (1,760 yds. = 1 mile.)

4. If the resistance of a wire 3 miles long and 40 mils in diameter is 40 ohms, what is the resistance of a wire 9 miles long and 50 mils in diameter?

5. The resistance of 390 ft. of wire $\frac{1}{16}$ in. in diameter is 1 ohm. What is the resistance of the same length of wire $\frac{1}{8}$ in. in diameter? The resistance of 1 mile of copper wire 80 mils in diameter is 8.29 ohms. What is the resistance of a mile of the same wire 50 mils in diameter?

6. If the resistance of 500 ft. of a certain wire 95 mils in diameter is .57 ohm, what is the diameter of 1,000 ft. of the same wire, if the resistance is 10.09 ohms?

7. A wire 1,800 ft. long and .01 in. in diameter has a resistance of 150 ohms. What will be the resistance of 400 ft. of wire .022 in. in diameter?

MAGNETISM

352. Magnetic flux is the number of magnetic lines of force flowing through a magnet or field, and is similar to amperes in current electricity.

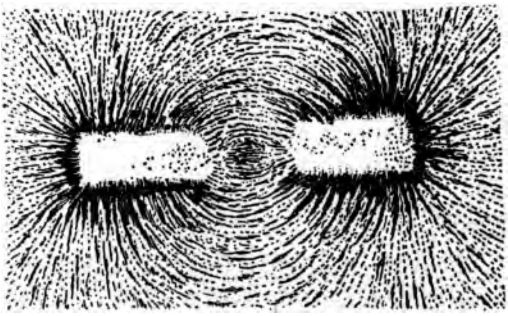

FIGURE 65—MAGNETIC FIELD AROUND A BAR MAGNET.

353. Magneto motive force is the difference of magnetic potential, or it is magnetic pressure, and is similar to electromotive force or volts in current electricity.

354. Reluctance is similar to resistance.

Let N = magnetic flux; $M.\ M.\ F.$ = magnetomotive force; R = reluctance.

$$M.\ M.\ F. = N. \times R. \quad N = \frac{M.\ M.\ F.}{R.}$$

A magnetic circuit composed of a ring of iron has 1,000 units of magnetic pressure, and a reluctance of .001 unit. How many lines of magnetic flux are passing through it?

$N = \dfrac{M.\ M.\ F.}{R.} = \dfrac{1{,}000}{.001} = 1{,}000{,}000$, the number of lines of magnetic force passing through it.

The number of lines of force set up per square inch through an air path one inch long is 3.2 for each ampere-turn. (Ampere-turns are the number of amperes multiplied by the number of turns.)

Let B = the number of lines of force set up per square inch of area through an air path; T = the number of turns per inch of length; C = the number of amperes. $B = 3.2 \times C \times T$.

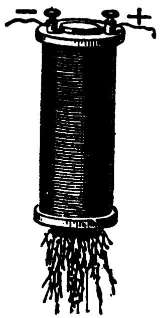

FIGURE 66
A SOLENOID

A solenoid 36 in. long, Figure 66, has 288 turns, and a current of 3 amperes flowing through it. How many lines of force are there per square inch flowing inside the coil? $288 \div 36 = 8$ turns per inch of length of coil. 8 turns $\times$ 3 amperes = 24 ampere-turns. 3.2 for each ampere-turns is $24 \times 3.2 = 76.8$ lines of force per square inch of coil.

To obtain 1,000 lines per square inch with 10 amperes, we have a certain number of turns of wire to the inch.

Since each ampere-turn produces 3.2 lines of force, 1,000 lines of force divided by 3.2 = 312.4 ampere-turns. Since we have 10 amperes, 312.4 ampere-turns divided by 10 = 31.24 turns per inch.

How many amperes will be required to produce 3,200 lines per square inch in a helix 13 inches long consisting of 2,600 turns?

Since there are 2,600 turns in 13 inches in length, or 200 turns to the inch in length, it will take 1,000 ampere-turns divided by 200 = 500 , the number of amperes.

355. The permeability of a body is the relative ease with which magnetic lines of force may be produced in the body. The permeability of air is unity. If through a bar of iron placed in a helix 50,000 lines of force are passed, and if the number of lines passing through the air space before the iron was placed there was 100, then the ratio of permeability of the iron is $\frac{50{,}000}{100} = 500$.

The permeability (M) of a substance is the magnetic density (B) divided by the intensity (H) of the magnetomotive force.

$$\text{Or, } M = \frac{B}{H}.$$

Magnetic materials, such as iron and the like are good conductors of magnetic lines of force; that is, they possess magnetic permeability.

When a magnetic force acts upon an air space, caused by an electric current circulating in a surrounding coil, there results a certain number of magnetic lines of force in that space. We say that the coil produces H magnetic lines per square inch in air, if H equals the intensity of the magnetic force. If the space were filled with iron instead of air, there would be a greater number of magnetic lines per square inch in the iron than in the air space.

This larger number of lines of force in iron is the degree of magnetization in iron and is symbolized by B. The ratio of B to H is the permeability of the metal and is represented by M.

For example: A certain magnetic force produces 300 lines of force per square inch in an air space, and when the space was filled with iron there were 96,300 lines per square inch. Then 96,300 lines per square inch in iron divided by 300 lines per square inch in air gives the relative number of lines of force with which iron is permeated.

Permeability diminishes as magnetization is pushed beyond a certain limit. The limit for magnetization (B) in good wrought iron is about 125,000 lines per square inch; and in cast-iron is about 70,000 lines per square inch. If a piece of iron is of low permeability, a larger piece must be used, or there must be more copper wire wound upon it.

PROBLEMS

1. If the number of magnetic lines of force per square inch in air (H) is 6.45 and the number of lines in wrought iron (B) is 30,000, what is the permeability of the iron?

$$M = \frac{B}{H}.$$

2. What is the magnetizing force, or what is the number of lines per square inch in air, required to produce in wrought iron a magnetization of 110,000 lines per square inch, if the permeability of the iron is 166?

3. A magnetic circuit composed of a ring of iron has 1,500 units of magnetic pressure, and a reluctance of .002 units. How many lines of magnetic flux are passing through it?

4. A solenoid 36 in. long has 360 turns, and a current of 2 amperes passing through it. How many lines of force are there per square inch flowing inside the coil?

5. How many amperes will be required to produce 3,600 lines per square inch in a helix 12 in. long, consisting of 2,400 turns?

6. How many amperes will be required to produce 2,560 lines per square inch in a helix 14 in. long, consisting of 2,800 turns?

7. A solenoid 24 in. long consists of 240 turns of wire, and has 4 amperes passing through it. How many lines of force are there per square inch inside the coil?

TRACTIVE FORCE OF MAGNETS

356. A bar of iron is magnetized to 60,000 lines per square inch, and has a cross-section area of 16 square inches. How many pounds weight can it sustain?

Let P = pull in pounds; B = density or number of lines per square inch; A = cross-section area. Then,

$$P = \frac{B^2 \times A}{72{,}134{,}000}.$$

$$P \text{ (Pounds)} = \frac{3{,}600{,}000{,}000}{72{,}134{,}000} \times 16 = 50 \text{ about.}$$

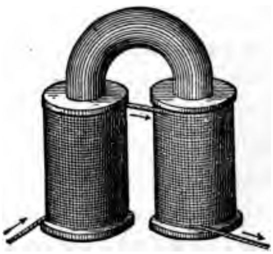

FIGURE 67
HORSESHOE ELECTROMETER

357. To find the tractive force of magnets *in pounds, multiply the number of square inches of contact by the square of the number of lines of force per square inch, and divide by 72,134,000.*

358. To find the number of lines of force *per square inch, when the pull in pounds is known, multiply the pull in pounds by 72,134,000, divide by the area of contact and extract the square root.*

PROBLEMS

1. A bar of iron is magnetized to 15,000 lines per square inch; its cross-section area is 3 sq. in. What weight can it sustain?

2. A magnet with 4 sq. in. cross-section area sustains a weight of 32 lbs. What is the number of magnetic lines of induction per square inch?

3. A magnet with 12 sq. in. cross-section area sustains a weight of 600 lbs. What is the magnetization in lines per square inch?

4. A bar of iron magnetized to 30,000 lines per square inch, has a cross-section area of 12 sq. in. What weight will it sustain?

5. An electromagnet is magnetized to 45,000 lines per square inch, and the poles of the magnet have a total area of 1 sq. in. How many pounds pull will they sustain?

ALTERNATING CURRENTS

359. Alternating currents reverse their direction at very short intervals. Starting at zero they increase to a positive maximum, then decrease to zero, then increase again to a negative maximum, decreasing to zero, and start through the cycle of operations again.

The time from one positive maximum to the next positive maximum is a period or cycle of the current.

There are two alternations in each period or cycle. One alternation is one half a period, or cycle. From positive to negative, or from negative to positive, is an alternation.

360. A **frequency** is the number of periods in a second.

A frequency is equal to the number of revolutions per second multiplied by one half the number of poles.

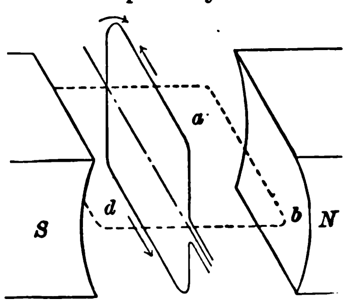

FIGURE 68

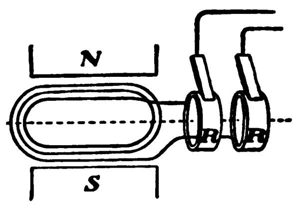

FIGURE 69

If we start with the armature coil in a vertical position, as in Figure 68, the current for the first half revolution flows around the coil in one direction, and for the second half, in the opposite direction. The current is, therefore, always alternating.

In the alternating current dynamo the current is conducted from the armature to the brushes by means of separate rings (Figure 69), one attached to each end of the wire of the armature. The dynamo then sends to the line an alternating current.

The frequency of an alternating current is the speed of the armature in seconds (*R. P. S.*) multiplied by one half the number of poles. Let Fr = frequency in cycles; P = the number of poles.

$$Fr = \frac{R.\,P.\,S. \times P}{2}.$$

$R.\,P.\,S.$ = revolutions per second.

What is the frequency of an 8-pole machine revolving 12 R. P. S.? $Fr = 12 \times \frac{8}{2} = 48$, the number of cycles.

ELECTRICAL POWER

361. The electrical unit of power is the *watt*. One watt = .00134 H. P. One H. P. = 746 watts. 1 watt = $C \times E$. (C = current in amperes. E = volts.) 1 watt = $C^2 \times R$. 1 watt = $\frac{E^2}{R}$. R = resistance in ohms. (E is the same as *E. M. F.*, or electromotive force.)

Let E = the E. M. F. of the line current; e = the E. M. F. per phase, or per armature coil; C = the line current in amperes; c = the current per phase or per armature coil in amperes; W = the total watts output; Fr = the frequency; P = the number of poles. F = flux. $E = \frac{e}{\sqrt{3}} = 1.732.$

PROBLEMS

1. What is the frequency in cycles of a current from an alternator, having 10 poles and 1,500 R. P. M.?

$$Fr = Cy = 2\,Al. \quad Fr = \frac{R.\,P.\,M.}{60} \times \frac{P}{2} = 125.$$

What is the number of alternations?

$$Al\text{ (alternations)} = \frac{R.\,P.\,M.}{60} \times P. \quad Al = \frac{10 \times 1{,}500}{60} = 250.$$

2. An alternator has 24 poles, and its armature runs at a speed of 300 R. P. M. What is the frequency? What is the number of alternations?

3. What is the frequency of a 40-pole machine which makes 125 R. P. M.? What is the number of alternations?

4. A 10-pole machine makes 600 R. P. M. What is the frequency? What is the number of alternations?

5. An 8-pole machine makes 800 R. P. M. What is the frequency? What is the number of alternations?

6. What number of poles must be used to secure a frequency of 60 cycles and have a speed of 600 R. P. M.?

$$P = \frac{120\ Fr}{R.\ P.\ M.}$$

7. What is the frequency of an 18-pole machine, making 400 R. P. M.?

8. How many cycles are made by a 60-pole dynamo running at a speed of 180 R. P. M.?

9. What is the speed, or the R. P. M., of a dynamo which has 8 poles, and a frequency of 60 cycles?

10. A generator delivers a current of 800 amperes at an E. M. F. of 500 volts. Find the number of watts delivered. $W = C \times E = 800 \times 500 = 400{,}000$ watts.

11. How many watts are required for 100 incandescent lamps in parallel, each lamp taking $\frac{1}{2}$ ampere and an E. M. F. of 110 volts?

12. What power in watts will be required to send a current of 10 amperes through a conductor of 8 ohms resistance?

13. What power in watts will be required to maintain a current of 110 volts E. M. F. through a resistance of $\frac{1}{4}$ ohm?

14. A current of 1,000 amperes is flowing through a conductor whose resistance is .08 of an ohm. What power is required to maintain this current at this resistance?

15. What power is expended in lighting 500 incandescent lamps in parallel each rated at 110 volts and 220 ohms?

16. An E. M. F. of 1,500 volts is maintained through a circuit whose resistance is 200 ohms. What is the horsepower required?

17. A current of 20 amperes is maintained through a resistance of 50 ohms. What is the horsepower? (Or electrical horsepower.)

18. An electrical motor takes a current of 200 amperes from a 500 volt circuit. What is the electrical horsepower?

362. The dynamo formula for a direct current 2-pole armature is this:

$$E = \frac{N \times c \times n}{10^8 \times 60}.$$

Let E = the voltage, or the E. M. F; N = the R. P. M. of the armature; n = the number of lines of force from one pole through the armature; c = the number of conductors on the surface of the armature of a bipolar machine; $10^8 = 100,000,000$. One watt = 100,000,000 ergs per second, and equals .00134 H. P. The *erg* is a unit of work. We use 60 in the denominator, because we used R. P. M., revolutions per minute, and not revolutions per second.

PROBLEMS

1. What is the E. M. F. of a dynamo which has a R. P. M. of 1,800; a flux, or number of lines of force, of 1,000,000; and conductors 500?

$$E = \frac{N \times c \times n}{10^8 \times 60} = \frac{1,800 \times 1,000,000 \times 500}{100,000,000 \times 60} = 150 \text{ volts.}$$

2. What is the E. M. F. of a dynamo which has R. P. M. 1,500; conductors 400; and lines of force 3,500,000?

3. What is the flux required to produce an E. M. F. 120 volts with 1,380 R. P. M. and 480 conductors?

4. What should be the number of conductors on the surface of the armature of a bipolar dynamo, if the flux is 1,000,000, the R. P. M. 1,200, and the voltage 100?

5. What should be the R. P. M. of a dynamo, if the number of conductors is 3,200, the E. M. F. is 1,000, and the flux is 947,000?

6. What is the E. M. F. of a bipolar dynamo, if the R. P. M. is 1,200, the conductors 500, and the flux 1,000,000?

7. What is the E. M. F. of a bipolar machine, if the R. P. M. is 2,400, the flux 2,000,000, and the conductors 400?

8. What should the number of conductors be on the armature of a bipolar machine, if the E. M. F. is 100, the flux 1,000,000, and the R. P. M. 600?

363. Multipolar dynamos. The E. M. F. of a multipolar dynamo with a series winding is $E = \frac{frequency \times c \times n}{100,000,000}$. The frequency is revolutions per second, the R. P. S., multiplied by $\frac{1}{2}$ the number of poles. The frequency of a 20-pole machine, making 600 R. P. S. is $\frac{1}{2}$ of 20 = 10, and 10 times 600 = 6,000. The frequency of a 24-pole machine making 600 R. P. M. is 600 ÷ 60 = 10 R. P. S. 24 ÷ 2 = 12, and 10 times 12 = 120 frequency.

The E. M. F. of a multipolar dynamo with multiple winding is $E = \frac{R.\,P.\,S. \times c \times n}{100,000,000}$

PROBLEMS

1. A 10-pole multipolar dynamo has a speed of 10 R. P. S., a flux of 8,200,000, and 580 conductors in series. What is the E. M. F.?

2. An 8-pole multipolar dynamo has a speed of 10 R. P. S., a flux of 600,000 and 450 conductors in series. What is the E. M. F.?

3. An 8-pole multipolar dynamo has a speed of 180 R. P. M., a flux of 7,900,000 lines of force, and 320 con-

ductors. This machine is multiple-wound. What is the E. M. F.?

4. A 6-pole multipolar dynamo with series winding, has 250 conductors, and gives an E. M. F. of 100 volts, at 240 R. P. M. What is the flux?

5. A 4-pole multipolar dynamo with 500 conductors, multiple-wound, is to give 500 volts at 600 R. P. M. What is the flux?

6. A 4-pole multipolar dynamo, multiple-wound, has a flux of 16,000,000 lines and gives 500 volts at 600 R. P. M. What is the number of conductors?

364. To find how much E. M. F. will be required *to drive an electric current through a certain number of arc lights, multiply the voltage required for each lamp by the number of lamps, and add the E. M. F. required to overcome the resistance of the line.*

What is the E. M. F. of a dynamo to light 100 arc lights, if each of the lamps requires 50 volts and 9 amperes, and the resistance of the line is 25 ohms? $100 \times 50 = 5{,}000$, the number of volts required for the 100 lamps. $E = C \times R$; or, $9 \times 25 = 225$, the number of volts for the line. And 5,000 volts + 225 volts = 5,225 volts.

How many watts per lamp are consumed? Watts $= C \times E = 9 \times 50 = 450$ watts consumed by each lamp. The total number of watts for the line and lamps = 5,225 volts $\times$ 9 amperes = 47,025 watts.

What is the E. M. F. of a dynamo to light 50 arc lamps, each requiring 110 volts and 4.9 amperes, the resistance of the line being 50 ohms? How many watts are consumed by each lamp?

$110 \times 50 = 5{,}500$, the number of volts required.

$4.9 \times 50 = 245$, the number of volts required for the line. $(E = C \times R.)$

5,500 volts + 245 volts = 5,745 volts required at the terminals of the dynamo, or the E. M. F. of the dynamo.

110 × 4.9 = 539, the number of watts. (Watts = $C \times E$)

365. In incandescent lighting and lamps in parallel, *the current through the dynamo is the sum of the currents through all the lamps, and the E. M. F. through any lamp is less than the E. M. F. through the dynamo by the amount of the drop in potential on the line.*

366. The drop in the line, or the loss of E. M. F., *between any two points on the line in indicated by the* $E = C\,R$; *or, is the current in that part of the line multiplied by the resistance of the line between the points.*

Each of a group of 12 lamps 100 feet from a dynamo and a group of 10 lamps 200 feet beyond, takes one half ampere of current at 120 volts and the resistance of the line is .0004 ohm per foot. What is the loss of E. M. F. at the lamps? What is the voltage of each group? How many watts are consumed by each group?

The current at the lamps of the first is 12 lamps × ½ ampere = 6 amperes. The resistance to the first group is 100 × .0004 ohm = .04 ohm.

The drop in potential, or the loss in voltage, at the first group is (6 × .04) .24 volt. The voltage at the first group is 120 volts − .24 volt = 119.76 volts.

The current at the second group is (10 × ½) 5 amperes. The resistance from the first group to the second group is 200 × .0004 ohm = .08 ohm.

The loss in voltage between the first and second groups is 5 (amperes) × .08 (ohm) = 4 volt.

The voltage at the second group is 120 volts − (.24 volt + .4 volt) = 119.36 volts. The total drop from dynamo to second group is .64 volt.

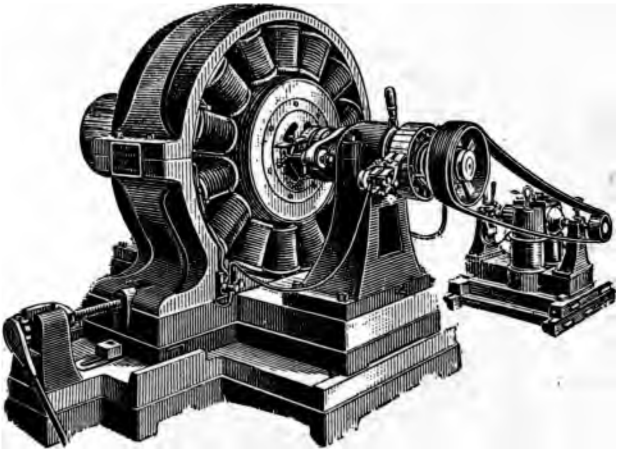

FIGURE 70—ALTERNATING CURRENT DYNAMO AND EXCITUR

367. The watts consumed by each group of lamps *are equal to the volts at the group terminals, multiplied by the current passing through the group.*

$W = C \times E$, or the watts consumed by a lamp equal the volts at the lamp terminals multiplied by the current passing through the lamp.

The current passing through the first group = 6 amperes. The volts at the first group terminals = 119.76 volts.

6 × 119.76 = 718.56, the number of watts consumed by first group.

The current passing through the second group = 5 amperes. The volts at the second group terminals = 119.36 volts.

5 × 119.36 = 596.80, the number of watts consumed by the second group. The power absorbed = watts divided by 746. $H.P. = \frac{W}{746}$

PROBLEMS

1. There are on a circuit forty 16-candle power incandescent lamps, each taking .5 ampere at 110 volts; and 4 arc lights, each taking 6.8 amperes at 50 volts. How many watts and how many horsepower are required to operate these lights?

2. In an electric light circuit there are 40 arc lights, each taking 50 volts, and 10 miles of wire having a resistance of 2 ohms per mile. If the current is 9.6 amperes, how many watts are required to run the lights?

3. An arc light has a circuit of 80 lamps, each requiring 9.6 amperes and 50 volts at the lamp terminals, the resistance of the line being 25 ohms. What is the E. M. F. required for all the lamps? What is the E. M. F. for the line? What is the E. M. F. required at the machine? If 10% of the energy should be lost in the line, what energy should be supplied by the dynamo?

4. An arc light circuit has 100 lamps, requires 4.9 amperes and 110 volts at the lamp terminals, and there is a loss of 147 watts by choke coil. How many volts should be supplied by the dynamo?

5. Dynamo to run 75 (16-*c. p.*) incadescent lamps, each lamp taking 110 volts, will give how many watts effective output? If the maximum voltage is 125 volts, and losses by resistances and by heat are 10%, what will be the efficiency of the machine? What will be the maximum power of the machine in watts? How many amperes will the machine produce? If the engine to drive the dynamo is 80% efficiency, what is its horsepower?

6. Two hundred incandescent lamps are connected in parallel to a dynamo circuit. The resistance of the line is .8 of an ohm, and the resistance of the lamps is 220 ohms each. The P. D. at the dynamo terminals is 112 volts. What current flows through the circuit?

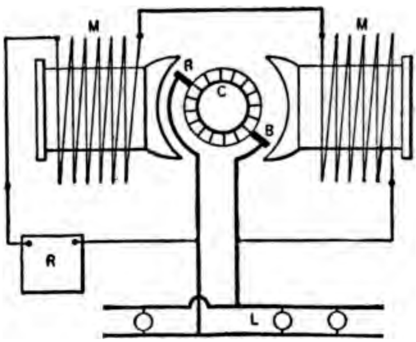

FIGURE 71 DYNAMO AND ELECTRIC LIGHT DIAGRAM

7. Fifty lamps, each requiring $\frac{1}{2}$ ampere and 100 volts, are connected in series. The P.D. at the dyname terminals is 500 volts, Figure 71. How much current flows from the power station?

8. In an electric light station the voltameter shows 115 volts, the ammeter shows 330 amperes. This is a two-wire direct current system and uses 55-watt, 110-volt incandescent lamps. How many lamps are burning?

ELECTRIC RAILWAY

368. A tractive force of 25 lbs. per ton is usually taken for a car on a level road, running at 10 miles per hour. An eleven-ton car will require 11 × 25 lbs. = 275 lbs. tractive force on a level.

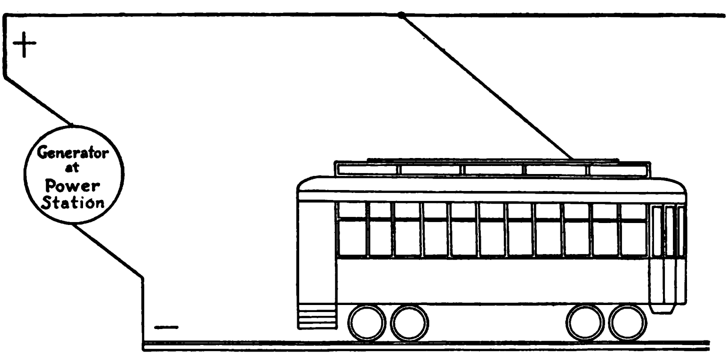

FIGURE 72.—DIAGRAM OF ELECTRIC RAILWAY SYSTEM

369. To find the H. P. required *to propel a car at a certain speed on a level road, multiply the tractive force by the speed in feet per minute and divide by 33,000. To find the watts required, or the watts output, by the motor to propel the car on a level road, multiply the H. P. by 746; and to get the watts required by the motor divide the watts output of the motor by the efficiency of the motor.*

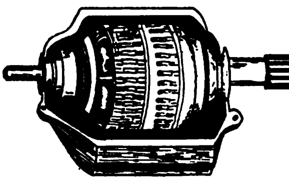
FIGURE 73—ARMATURE OF MOTOR

How many watts are supplied to the motors of a street car weighing 8 tons, tractive force per ton 25 lbs., running 10 miles per hour on a level road? Efficiency is 75%.

Tractive force is $8 \times 25 = 200$.

Feet per minute is $\frac{10 \times 5,280}{60} = 880$.

Foot-pounds is $880 \times 200 = 176,000$.

Horsepower $= \frac{176,000}{33,000} = 5.3+$.

Watts $= 5.3 \times 746 = 3,953.8$ watts.

Watts supplied to motors $= 3,953.8 \div .75 = 5,271.73$ watts.

370. To find the energy of a car *to enable it to overcome a grade, find the number of feet it is raised vertically per minute by multiplying the speed in feet per minute by the percentage of the grade. Then multiply by the weight of the car in pounds and divide by 33,000 for the horsepower.*

How many watts must be supplied to the motors of the car in the last example to enable it to climb a 10% grade at 10 miles per hour?

The vertical rise in feet per minute $= 880 \times .10 = 88$.

Weight of car in pounds $= 16,000$ lbs.

Foot-pounds $= 16,000 \times 88 = 1,408,000$.

Horsepower $= \frac{1,408,000}{33,000} = 42.66$.

Watts = 42.66 × 746 = 31,824.36.

F = tractive force = 25 lbs. per ton.

Eff = efficiency of motor in per cent.

Wt = weight of car and passengers in tons.

S = Speed of car in feet per minute.

W = watts required by motor on level road.

$$W = \frac{Wt. \times F \times S \times 746}{33{,}000} \div Eff.$$

PROBLEMS

1. A street car weighs 8 tons and the passengers weigh 2 tons; the tractive force is 25 lbs. per ton; efficiency of motor is 75%. How many watts are supplied to the motor when the car is running at the rate of 10 miles per hour?

2. A street car weighs 12 tons and the passengers weigh 3 tons; the tractive force is 25 lbs. per ton; the efficiency of the motor is 75%. How many watts are supplied to the motor when the car is running at the rate of 10 miles per hour? How many watts must be supplied to the motor to enable the car to climb a grade of 10% at 10 miles per hour?

3. How many horsepower must be supplied to a street car weighing 10 tons, tractive force 25 lbs. per ton, running on a level road at 10 miles per hour?

4. How many watts are required to propel a street car weighing 14 tons, tractive force 30 lbs. per ton, running at 20 miles per hour on a level road, if the efficiency is 70%?

5. How many watts are required to propel a street car weighing 15 tons, tractive force 25 lbs. per ton, up a 10% grade at 15 miles per hour?

6. How many horsepower are necessary to run a 15-ton street car at 20 miles per hour, level road, and 25-lb, tractive force?

TRANSFORMERS

371. A **transformer** is composed of a primary coil, a secondary coil, and a magnetic core.

The induced electromotive force set up in the secondary coil is to that in the primary coil, nearly, as the number of coils in the secondary is to the number of coils in the primary. If the primary has 200 turns of wire and the secondary has 2,000 turns, then the induced E. M. F. in the secondary will be about 10 times as great as that in the primary.

If the secondary contains 100 turns of wire and the primary 200 turns, then the secondary will have $\frac{1}{2}$ the E. M. F. of the primary.

The E. M. F. in either coil of a transformer equals 4.44 times the number of turns in the coil, times the maximum flux, times the frequency divided by 100,000,000. (10^8.)

Let T = the number of primary turns; t = the number of secondary turns; E = the primary E. M. F.; e = the secondary E. M. F.; n = the frequency or the number of cycles; Fm = the maximum flux.

$$\text{Then } E.\ M.\ F. = \frac{4.44 \times T \times Fm \times n}{100{,}000{,}000}.$$

A transformer has 400 turns in the primary coil and 50 turns in the secondary coil, a frequency of 80, and a maximum flux of 30,000 lines. What is the E. M. F. in each coil?

$$E = \frac{4.44 \times 400 \times 80 \times 30{,}000}{100{,}000{,}000} = 42.6, \text{ the number of volts.}$$

$$e = \frac{4.44 \times 50 \times 80 \times 30{,}000}{100{,}000{,}000} = 5.3, \text{ the number of volts.}$$

The ratio of the number of volts in the primary to the number of volts in the secondary is as 42.6 is to 5.3. Or,

the ratio (*Ra.*) of the transformer is, $Ra = \frac{E}{e}$. Also,

$$Ra = \frac{C''}{C'}. \quad \begin{array}{l} C'' = \text{secondary current.} \\ C' = \text{primary current.} \end{array}$$

The maximum flux through the core of a transformer is

$$Fm = \frac{E \times 100{,}000{,}000}{4.44 \times n \times T}.$$

PROBLEMS

1. A transformer has 720 turns in the primary coil and 36 turns in the secondary coil. The frequency is 136 and the maximum flux is 160,000 lines. What is the E. M. F. in each coil?

2. A transformer has 600 turns in the primary coil and 60 turns in the secondary coil. The frequency is 90 and the maximum flux is 60,000 lines. What is the E. M. F.?

3. The E. M. F. of a primary coil is 100 and the ratio of transformation is 40. What is the E. M. F. of the secondary coil?

4. The ratio of transformation is 20, the secondary E. M. F. is 200. What is the E. M. F. of the primary?

5. What is the secondary E. M. F. if the ratio of transformation is 20 and the primary E. M. F is 800?

6. If the ratio of transformation is 20, the secondary E. M. F. is 400, and the secondary turns 40, what is the primary E. M. F.?

7. If a secondary E. M. F. is 60, the primary turns 600, the frequency 60, and the ratio of transformation 30, what is the maximum flux?

8. The ratio of transformation is 30, the secondary E. M. F. is 300, and the secondary turns 60. What is the primary E. M. F.?

9. A transformer has 840 turns in the primary and 48 in the secondary. The frequency is 140 and the maximum flux is 180,000 lines. What is the E. M. F. in each coil? What is the ratio of transformation?

10. A transformer has 400 turns in the primary coil, a maximum flux of 30,000 lines and a voltage of 42. What is the frequency?

Transformers are used generally to change from very high and dangerous voltage and low current to lower and more safe voltage and larger current.

LOCOMOTIVES

372. Locomotives are classified by the number of wheels from front to back, as the front truck wheels, the drivers,

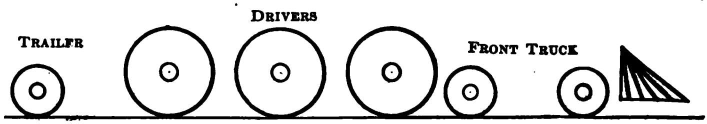

FIGURE 74.

the trailers. In Figure 74, 2 front truck wheels, 3 drivers, and 1 trailer are seen on one side. So there are 4 front truck wheels, 6 drivers, and 2 trailers on this locomotive. The type number is 4-6-2. This type is read "four-six-two type." Read from front.

A locomotive with 2 front trucks, 6 drivers, and no trailers, is a 2-6-0 type, or a "two-six-0" type. The cypher shows no trailers. The weight is given in the nearest even thousandths of pounds. If this locomotive weighs 16,000 lbs., it is given, "2-6-0-16." If it weighs 204,572 lbs., it is given "2-6-0-204."

If the tender is on the engine, T is placed where the dash is, as "2-6-0T204."

If there is a compound cylinder, then C is in place of T, and if a compound cylinder and tank are on the engine. then use both letters, as "2-6-0TC204."

If a 4-drive wheel locomotive has each drive wheel fast so it cannot turn, but must slide, and there is a pull of 40,000 pounds on the drawbar, pulling the locomotive ahead, what is the sliding force on the rail where each wheel rests? 40,000 ÷ 4 = 10,000 lbs. where each wheel rests on the rail.

If the drive wheel is 60 in. in diameter, and the piston stroke is 20 in., how many pounds force would be needed at each crank pin when the crank pins are at right angles to the axle of the wheel, to resist the 10,000 lbs. where each wheel rests on the rail? The diameter of the wheel is three times the length of the piston stroke, or the ratio of the diameter of the wheel to the piston stroke is 3 to 1. Therefore, it would require (3 × 10,000) 30,000 lbs. at each crank pin to counteract the 10,000 lbs. resistance where each wheel rests on the rail. If the right and left crank-pins are placed at right angles to each other, those on the other side of the locomotive would be on the same horizontal line as the axle and would not help to overcome the resistance on the rail. Therefore, it would require 2 × 30,000 lbs., or 60,000 lbs., at each crank pin which is on the

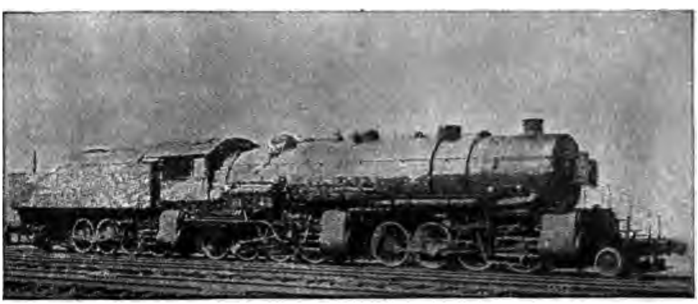

FIGURE 75—"MATT H. SHAY," POWERFUL LOCOMOTIVE CAPABLE OF HAULING 640 FREIGHT CARS IN A TRAIN 4¾ MILES LONG. LENGTH 105 FEET; WEIGHT, 853,050 POUNDS.
Courtesy Erie Railroad.

side at right angles to the axle of the wheel to overcome the resistance of all the wheels.

A 4-6-0 type of locomotive has 78-in. wheels and 26-in. stroke, and is required to exert a pull of 30,000 lbs. What force is required at each crank pin on one side when the pin on the other side is at the top of the stroke, or at right angles to the pin on this one side?

A pull of 30,000 ÷ 6 (the number of drivers) = 5,000 lbs., the sliding force at each wheel. As the wheel diameter is 78 in. and the piston stroke is 26 in., the diameter of the wheel is 3 times the length of the piston stroke. 3 × 5,000 lbs. = 15,000 lbs. As the crank pins on the opposite sides are at right angles to each other, it will require 2 × 15,000 lbs. = 30,000 lbs. to apply to each of the crank pins to keep the wheels on both sides moving.

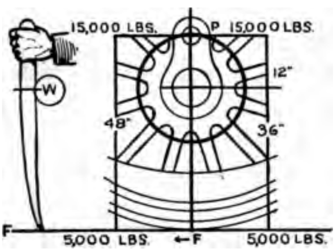

FIGURE 76

In Figure 76 is shown a circle marking the path of the crank pin (to which the power moving the wheel is applied) around the central point of the wheel.

The diameter of this circle is the "stroke of the engine." The diameter of the circle in Figure 76 is 24 in. Therefore, the length of the stroke of the engine is 24 in. The diameter of the driving wheel is 72 in. Or, 2 × 36″ = 72 in. In this case, also, the diameter of the wheel is 3 times the piston stroke.

If a power of 15,000 lbs. is applied at the crank pin, with what force will the axle push forward, if the length of the piston stroke is 24 in. and the diameter of the wheel is 72 in.? The wheel is like a lever, having its fulcrum at F, its power at P, and its weight at W. With levers, the

power multiplied by its distance from the fulcrum equals the weight multiplied by its distance from the fulcrum. The power, 15,000 lbs. × its distance, 48″ = the weight *W* × its distance from the fulcrum, 36 in. 48 × 15,000 lbs. = 720,000 lbs. And 720,000 lbs. ÷ 36 = 20,000 lbs. = *W*. Therefore, when 15,000 lbs. is applied at the crank pin, the axle will push forward with a force of 20,000 lbs., if the diameter of the wheel is 3 times the length of the piston stroke.

One revolution of the driving wheels is one "step of the locomotive" (the iron horse). The distance which a locomotive moves forward at each revolution of the drive wheels equals the circumference of the drive wheels. The circumference equals the diameter multiplied by 3.1416.

PROBLEMS

1. How many feet will a locomotive travel in one revolution of its driving wheels, if the wheels are 78 in. in diameter? 78″ = $6\frac{1}{2}$ft. 3.1416 × $6\frac{1}{2}$ ft. = 20.42 ft.

2. How many feet will a locomotive travel in one revolution of its driving wheels, if they are 72 in. in diameter?

3. How many feet will a locomotive travel in one revolution of its driving wheels, if they are 66 in. in diameter?

4. How many revolutions will the driving wheels of a locomotive make in going 204.204 ft., if they are 78 in. in diameter?

5. How many feet will a locomotive travel in 20 revolutions of its driving wheels, if they are 72 in. in diameter?

6. How many revolutions will be made per mile by wheels 78 in. in diameter?

7. How many revolutions will be made per mile by wheels 60 in. in diameter?

8. The driving wheels of a locomotive are 78 in. in diameter and make 324 revolutions per minute. How many miles will the locomotive travel per minute?

9. The driving wheels of a locomotive are 60 in. in diameter and make 420 revolutions per minute. How many miles will the locomotive travel per minute?

10. At what speed in miles per hour will a locomotive run, whose driving wheels are 70 in. in diameter, and make 300 revolutions per minute?

11. The drive wheels of a 6-drive wheel locomotive are fast and must slide, and there is a pull of 60,000 lbs. on the drawbar. What is the sliding force on the rail where each wheel rests?

12. The drive wheel is 72 in. in diameter, and the piston stroke is 24 in. How many pounds force must there be at each crank pin, when the crank pins are at right angles to the axles of the wheel, to resist 20,000 lbs. where each wheel rests on the rail?

13. A 4-6-0 type engine has 60-in. drive wheels and 20-in. stroke, and exerts a pull of 60,000 lbs. What force is required at each crank pin on one side, when the pin on the other side is at the top of the stroke?

373. The Pulling Power of Engines. The relation of the position of crank pin, axle and point of contact at the rail is always changing as the wheel revolves. The crank-pin may be at the top or the side, or at the bottom of the stroke. When figuring the pull of an engine the average power of the wheel is considered for the entire revolution. This is about .637 of the maximum tractive force per side. There are several resistances offered to moving trains, as, friction, the resistance offered to being moved, etc.

A horizontal force of about $5\frac{1}{2}$ lbs. is needed to keep one ton (2,000 lbs.) moving at 10 miles per hour, overcoming these resistances. A loaded train of 2,000 tons would require a power of $(2{,}000 \times 5\frac{1}{2})$ 11,000 lbs. to keep it moving.

374. Grade Resistance. A 1% grade is one that has a rise equal to 1% of its length. Or, a rise of 52.8 ft. in a length of 5,280 ft., or one mile.

If a car is raised 1% of one mile (5,280 ft.) it will take 1% of the weight of the car to pull it up the incline. To pull one ton up a grade of $1\frac{1}{2}$% would take a power of $1\frac{1}{2}$% of 2,000 lbs. = 30 lbs. To pull one ton up a grade of $2\frac{1}{2}$% would take $2\frac{1}{2}$% of 2,000 lbs. = 50 lbs.

What force will be needed to haul 1,600 tons up a grade of 1%?

To haul 1 ton up a grade of 1% requires 20 lbs. and to haul 1,600 tons up this grade of 1% will take $1{,}600 \times 20$ lbs. = 32,000 lbs. To these figures must be added $5\frac{1}{2}$ lbs. for friction, per ton, if the train is moved 10 miles per hour up the incline. 20 lbs. per ton + $5\frac{1}{2}$ lbs. per ton = $25\frac{1}{2}$ lbs. per ton. $1{,}600 \times 25\frac{1}{2}$ lbs. per ton = 40,800 lbs. There must be provided sufficient tractive force to overcome this resistance.

375. Tractive Force and Adhesive Weight. Friction is necessary to prevent slipping, and weight is necessary to produce friction. The necessary friction is obtained by giving to the wheels a weight of $4\frac{1}{2}$ times the slipping force. In Figure 76 the pressure at F is $15{,}000 \times 12''$ (the distance from P to the fulcrum F) $\div$ 36″ (the distance from W to the fulcrum). Or, the pressure at F is 5,000 lbs. $(15{,}000 \times 12 = 180{,}000$. And $180{,}000 \div 36 = 5{,}000$ lbs.)

This 5,000 lbs. is a backward pressure, or slipping force.

It is, therefore, necessary to provide sufficient friction at F, between the wheel and the rail to withstand this

backward pressure of 5,000 lbs. This is $4\frac{1}{2} \times 5{,}000$ lbs. = 22,500 lbs. This is called "adhesion" and the $4\frac{1}{2}$ is the factor of adhesion.

If the power at the crank pin of a 2-6-0-type engine is 10,000 lbs., the drive wheels are 70 in. in diameter, and the stroke is 28 in., what is the tractive force, and what weight of engine would be required to produce a factor of adhesion of $4\frac{1}{2}$ lbs.? $(10{,}000 \times 14) \div 35 = 4{,}000$. By section 373, we use .637 as a multiplier. $.637 \times 4{,}000$ lbs. = 2,548 lbs. per wheel. $6 \times 2{,}548$ lbs. = 15,288 lbs. tractive force. $4\frac{1}{2} \times 15{,}288$ lbs. = 68,796 lbs. adhesive weight.

Tractive force, when diameter of cylinder and boiler pressure are given.

$$\text{Tractive Force } (T.\ F.) = \frac{.85 \times P \times A \times S \times .637 \times 2}{D}.$$

P = boiler pressure; A = area of cylinder; S = length of stroke; .637 = factor from section 373; D = diameter of drive wheel; .85 is factor to give more correct result, or average pressure.

A locomotive type 2-6-0 has a 20-in. cylinder, a 28-in. stroke, 50-in. diameter driving wheels, and a boiler pressure of 180 lbs. What is the tractive force?

$.85\ (P) \times 180 = 153$ lbs. average pressure. $20'' \times 20'' \times .7854 = 314.16$ sq. in. (A). $314.16 \times 153 = 48{,}066.48$, the average number of pounds of piston pressure.

$28 \times 48{,}066.48$ lbs. = 1,345,861.44 lbs. average piston stroke pound pressure.

1,345,861.44 lbs. $\div$ 50 = 26,917.23 lbs. tractive force per each side drive wheels.

$.637 \times 26{,}917.23$ lbs. = 17,140.2755 lbs. average tractive force per side.

$2 \times 17{,}140.2755$ lbs. = 34,280.55 lbs. total tractive force for both sides.

$$T.\ F. = \frac{.85 \times 20^2 \times .7854 \times 28 \times 180 \times .637 \times 2}{50} = ?$$

A locomotive engine has a cylinder of 19-in. diameter, a 26-in. stroke, a driving wheel 63-in. in diameter, and steam pressure of 190 lbs. per square inch at beginning of the stroke. What is the tractive force of the engine at 10 miles per hour? As in the above problem, let A = the piston area in square inches; P = the steam pressure in pounds per square inches; S = the length of stroke in inches; D = the diameter of the driving wheel in inches.

The formula for tractive force is:

$$T.F. = \frac{1.08 \times P \times A \times S}{D}.$$

(.85 × .687 × 2 = 1.08.)

$$T.\ F. = \frac{1.08 \times 190 \times 283.53 \times 26}{63} = 24{,}010.94.$$

PROBLEMS

1. With 30,000 lbs. power at the crank pin, the diameter of the wheel 72 in. and length of stroke 24 in., what is the slipping force, and what weight must be given to the wheels to counteract this?

2. With 15,000 lbs. at the crank pin, the diameter of the wheel being 72 in., and the length of stroke being 24 in., what is the slipping force and what weight must be given the wheels to counteract this?

3. With 10,000 lbs. power at the crank pin, the diameter of the wheel 60 in., and the length of stroke 24 in., what is the slipping force and what weight must be given to the wheel to counteract this?

4. At 40 miles per hour, how many revolutions will a drive wheel 60 in. in diameter make per minute?

5. An engine is running at 40 miles per hour. The driving axle is 8½ in. in diameter and the driver is 60 in. in diameter. What velocity will a point on the journal of the axle make in feet per minute?

$\frac{8\frac{1}{2} \times 3.1416}{12} = 2.23$ feet = the circumference = one revolution of the wheel or axle. 60 in. = 5 ft. 3.1416 × 5 ft. = 15.71 ft. = circumference of the driver.

5,280 ÷ 15.71 = 336.09 = times the circumference of the driver is contained in one mile. $\frac{336.09 \times 40}{60} = 224.06$ revolutions per minute of driver. And 2.23 ft. per revolution times the number of revolutions per minute equals 2.23 × 224.06 = 499.65, the number of feet per minute.

6. An engine is running 60 miles per hour. How many revolutions will a truck wheel, which is 33 in. in diameter make per minute?

7. A train is running at 44 ft. per second. How many miles will it travel in one hour?

8. A train is traveling at the rate of 3.87 miles in 6 minutes. How many miles per hour is it traveling?

9. A train takes 120 seconds to go one mile. What is its speed in miles per hour?

10. At the rate of 80 seconds per mile, how fast is a train moving in miles per hour?

11. At the rate of 55 miles an hour, how many seconds will there be between mileposts?

12. A watch shows 55 seconds between mileposts. What is the speed in miles per hour?

13. How many feet will a locomotive travel in one revolution of the driving wheel which is 78 in. in diameter?

14. How many revolutions will a driving wheel make which is 60 in. in diameter in going 10 miles?

15. A locomotive engine, type 2-6-0, has a 20-in. cylinder, a 28-in. stroke, a driving wheel 50 in diameter and a boiler pressure of 180 lbs. What is the tractive force?

$$T.\ F.\ \text{(Tractive Force)} = \frac{.85 \times P \times A \times S \times .637 \times 2}{D}$$

376. Curve resistance. The sharpest curve has a radius of 573 ft, measured on the middle of the track between the rails. A railroad curve having a radius of 5,730 ft. is called a one degree curve without regard to length.

The degree of curvature is 5,730 ÷ the radius of the curvature in feet. 5,730 ÷ 573 = 10. A curve having a radius of 2,865 ft., or 5,730 ÷ 2865, is a 2-degree curve.

The train resistance on curves is about .8 lb. per ton of train for each degree of curvature. With a two-degree curve we must provide the additional tractive force to overcome 5½ lbs. per ton of train resistance. This is 2 × .8 lb. = 1.6 lbs. per ton.

What force will be needed to haul a train of 1,600 tons up a grade of 1% and on a curve of 10%?

1,600 × 20 lbs. = 32,000 lbs. If the train moves at 10 miles per hour, we add 5½ lbs. per ton for friction. 1,600 × 5½ lb. = 8,800 lbs. A 10% curve requires 10 × .8 lb. = 8 lbs. per ton. 8 × 1,600 lbs. = 12,800 lbs.

32,000 lbs. + 8,800 lbs. + 12,800 lbs. = 53,600 lbs. Or, adding 20 lbs., 5½ lbs., and 8 lbs., we have 33½ lbs. And 1,600 × 33½ lbs. = 53,600 lbs.

Train resistance for a train moving at 10 miles per hour on a level is about 5½ lbs. per ton.

Train resistance for a train moving up a grade is 20 lbs. per ton for each 1 % of grade.

Train resistance for a train moving on a curve is .8 lb. per ton for each degree of curvature.

377. The number of driving wheels. Wheel loads on rails are based on the weight of the rail per yard of length. The safe load per wheel is 250 times the weight of the rail per yard. From section 375, a wheel must have a weight of 4½ times the slipping force to withstand the necessary backward pressure.

lengths in card I and measure and add those in card II, and see that they are the same or nearly so. Divide either of these amounts by the number of spaces, and multiply by the number of the spring. This gives the M. E. P. in pounds per square inch.

In the figure given, Figure 54, the sum of No. 1 card is $\frac{116}{16}$ in. and the same for No. II card. We use either one. Divide $\frac{116}{16}$, or $7\frac{1}{4}$, by 13 (spaces) $= \frac{29}{52}$. $\frac{29}{52}$ divided by 40, scale number, $= 22.3$, the M. E. P.

The cylinder area is $16^2 \times .7854 = 201$ sq. in.

The length of stroke is $\frac{42 \times 2}{12} = 7$, length of stroke in feet (double).

Revolutions $= N = 68$.

$$H.P. = \frac{P\,L\,A\,N}{33{,}000} = \frac{22.3 \times 7 \times 201 \times 68}{33{,}000} = 64.6.$$

To make the problem simple we have not considered the piston rod. If we were to consider the piston rod we would have the area of the piston A and the area of the rod a, and then $\frac{2A - a}{2}$ = area of piston.

308. These indicator diagrams were taken from a Rice & Sargent Reciprocating Engine, rated at 1,500 H. P. and 118 R. P. M. The cylinder was of the tandem compound type. Diameter of high pressure cylinder was $24\frac{1}{4}$ in., diameter of low pressure cylinder 44 in., and stroke 42 in. High pressure piston rod diameter $4\frac{1}{2}$ in., low pressure piston rod diameter 6 in.

A Crosby Steam Indicator containing an 80-lb. spring was connected to the high pressure cylinder of the engine and one containing a 24-lb. spring was connected to the low pres-

sure cylinder. Indicator cards were taken at frequent intervals during the test. One card shows diagram from the high pressure engine and the other from the low pressure engine.

309. The indicated horsepower is obtained from the following formula:

$$I.\,H.\,P. = \frac{P\,L\,A\,N}{33{,}000}$$

P = mean effective pressure, which is the average effective pressure behind the piston in pounds.

L = the length of the engine stroke in feet.

A = the effective area of the piston head on which the steam acts in square inches.

N = the number of revolutions per minute of the engine.

310. The Indicated Diagrams were integrated with a planimeter and the area determined. The scale of the planimeter was such as to read directly the mean effective pressure indicated by the diagram for a 40-lb. spring.

In this case, since an 80-lb. spring was used on the high pressure cylinder, the planimeter readings were multiplied by 2, and since a 24-lb. spring was used on the low pressure cylinder, the planimeter readings were multiplied by .6, to give the mean effective pressure.

Steam Engine High Pressure Indicator Card

311. No. 1 Engine; No. H. P. Cylinder; Card No. 8.

Diameter of cylinder, $\frac{24.3''}{44''}$; diameter of rod, $\frac{4.5''}{6''}$; stroke, 42 in.; clearance, 00 in.; date, Oct., 1912; time, 11:10; end of cylinder, both; scale of spring, 80; boiler gauge, 155 lbs.; Vacuum gauge 00 lbs.; revolutions per minute, 118.

Steam Low Pressure Indicator Card

312. No. 1 engine; No. L. P cylinder; card No. 8.

Diameter of cylinder, $\frac{24.3''}{44''}$; diameter of rod, $\frac{4.5''}{6''}$; stroke

42 in.; clearance, 00 in.; date, Oct. 1912; time, 11:10; end of cylinder, both; scale of spring, 24; boiler gauge, 155 lbs; vacuum gauge, 00 lbs.; revolutions per minute, 118.

313. Area of piston head head, high pressure cylinder, head end. Since the steam pressure is exerted over the entire piston head area, we take the diameter of the high pressure cylinder, which is 24.3 in. This gives a cross-section area of 462 sq. in. (Area $= \frac{1}{4}\pi D^2$).

314. Area of piston head, high pressure cylinder, crank end. Steam pressure is exerted on total piston head area minus area of high pressure piston rod, which is $4\frac{1}{2}$ in. in diameter. Therefore, area $= 462 - \frac{1}{4}\pi(4\frac{1}{2})^2 = 446.1$.

315. Area of piston head, cylinder and head end. Here pressure is exerted on the piston head minus the area of the piston rod. The diameter of the piston head is 44 in., and the diameter of the piston rod is $4\frac{1}{2}$ in.; hence the area of the head end is: area $= \frac{1}{4}\pi(44)^2 - \frac{1}{4}\pi(4\frac{1}{2})^2 = 1{,}506.1$, the number of square inches.

316. Area of piston head, cylinder and crank end. Here pressure is exerted on entire piston head area. Area $= \frac{1}{4}\pi(44)^2 = 1{,}583.36$, number of square inches.

The spring used should be of such a strength or number that the diagram will not be over $1\frac{3}{4}$ in. high. The proper spring may be found by dividing the boiler pressure in pounds by the height of the diagram in inches. If the boiler pressure is 70 lbs., then $\frac{70}{1\frac{3}{4}} = 40$, the number of the spring to be used. It is generally more convenient to make the diagram $2\frac{1}{2}$ in. to $3\frac{1}{2}$ in. long, and the height $1\frac{1}{2}$ in. to $1\frac{3}{4}$ in. above the atmospheric line.

The planimeter by being run over the diagram shows the number of square inches in it. If we divide this number of square inches by the length of the diagram, we have its average height. The average height multiplied by the num-

ber of the spring used gives the mean effective pressure in pounds per square inch on the piston of the engine.

317. **Proof of Actual Indicator Card**

Taken........Sept. 22, 1915 **at**......Menomonie, Wis.
Which Cylinder............. **Which End**..........Both
Boiler Pres........90 lbs. **Vac.**.......**ins.** **Revs.**.......68
Scale....40 **Stroke**....42 in. **Diam. of cylinder**....16
Diam. of Rod......3

$16^2 \times .7854 = 201$ piston area.

$$\frac{42 \times 2}{12} = 7, \text{ length of stroke in feet.}$$

$$H.P. = \frac{PLAN}{33{,}000} = \frac{22.3 \times 7 \times 201 \times 68}{33{,}000} = 64.6.$$

Revolutions for double stroke.

L. P. = Low Pressure. *H. P.* = High Pressure.

Result of Calculations

	Planimeter read	*P*	*L*	*A*	*N*	*IHP*
H. P. Head End	29.2	58.4	3.5	462	118	338
Crank End	31.8	63.6	3.5	446.1	118	354
L. P. Head End	28.3	16.9	3.5	1,506.1	118	318
Crank End	28.5	15.1	3.5	1,495	118	283
						1,293

318. The planimeter readings in the first column H. P. are multiplied by 2, because the planimeter was to read directly, for a 40-lb. spring, and an 80-lb. spring was used. The planimeter readings in the first column, L. P., is multiplied by .6 because the planimeter was to read directly for a 40-lb. and a 24-lb. spring was used.

319. To find the H. P. of an engine, it is necessary to ascertain separately three factors, and then find the product

of the three. The first is the area of the two ends of the piston, the head end and the crank end; the second is the total travel of the piston in feet per minute; and the third is the mean effective pressure, M. E. P. of the steam urging the piston forward. The total travel of the piston in feet per minute is twice the length of the stroke in feet multiplied by the number of revolutions of the crank shaft per minute. The piston area at the back end is the same as the area of a cross-section of the cylinder; at the head end it is the same less the area of the cross-section of the piston rod.

The M. E. P computed from an indicator card taken from an air cylinder compressor is 30.6 lbs. per square inch; diameter of cylinder is 28 in.; stroke is 48 in.; number of strokes per minute is 108. What is the H. P.?

$$v = \frac{28^2 \times .7854 \times 48 \times 108}{1{,}728} \text{ cu. ft. per minute.}$$

$H.P. = \dfrac{144\,v\,p}{33{,}000}$. v = volume of the fluid used or discharged in cubic feet per minute; p = average pressure in pounds per square inch.

$$\frac{144\,v\,p}{33{,}000} = \frac{144 \times 28^2 \times .7854 \times 48 \times 108 \times 30.6}{1.728 \times 33{,}000} = 246.66 \text{ H. P.}$$

A ventilating fan delivers 5,000 cu. ft. of air per minute at a pressure of .56 lb. above the atmospheric pressure; what is the theoretical H. P. to drive the fan?

$$H.P = \frac{144\,v\,p}{33{,}000} = \frac{144 \times 5{,}000 \times .56}{33{,}000} = 12.218.$$

PROBLEMS

1. A double acting engine has a cylinder 26 in. in diameter, and a 30-in. stroke. It makes 80 R. P. M. The indicator card being divided up gives a length of mean ordinate 2.5 in. A No. 40 spring is used and gives 2.5 $\times$ 40 = 100 lbs. per square inch as the M. E. P. on the piston. What is the indicated horsepower of the engine?

2. A non-condensing engine has a head end and crank end of cylinder the same 16 in. in diameter. The diameter of piston rod is 3 in. and there is a 30-in. stroke. It runs at a speed of 150 R. P. M. The card is taken with a 60-lb. spring. The card measured with a planimeter shows a M. E. P. of 48 lbs. What is the I. H. P.? The average area = (.7854 $\times$ 16^2 – 7 sq. in., area of piston rod) = 198 sq. in.

$$I.H.P. = \frac{P\,L\,A\,N}{33{,}000} = \frac{48 \times 2.5 \times 198 \times 300}{33{,}000}$$

N = number of strokes per minute = number of revolutions $\times$ 2 = 150 $\times$ 2 = 300.

SAFETY VALVE

320. In the equation for the safety valve, three weights are given: the weight of the ball, the lever, and the valve. Three lengths are given: the length of the lever, the distance from the center of gravity to the fulcrum, and the distance from the valve to the fulcrum. Find the center of gravity of the lever, by balancing the valve and lever over some sharp edge and measure the distance from this point to the fulcrum.

The equation is the following: $S \times b = V \times b + L \times h + W \times y$.

S = the pressure indicated by the steam gauge $\times$ the area of the valve.

b = the distance from the valve to the fulcrum in inches.

V = the weight of the valve in pounds.

L = the weight of the lever in pounds.

h = the distance from the center of gravity to the fulcrum in inches.

W = the weight of the ball in pounds.

y = the distance from the ball to the fulcrum in inches.

$S \times b$ = power $\times$ power arm.
$W \times y$ = weight $\times$ weight arm.
$V \times b$ = weight of valve $\times$ power arm.
$L \times h$ = weight of lever $\times$ weight arm.

What weight ball must be put on a 3-in. safety valve that it may blow off at 100 lbs.? The weight of the valve is 4

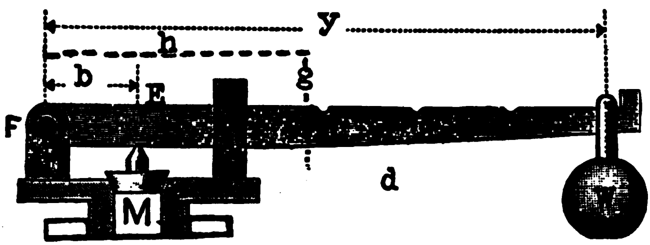

FIGURE 58

lbs.; the weight of the lever is 20 lbs.; the distance from the ball to the fulcrum is 34 in.; the distance from the center of gravity to the fulcrum is 12 in.; the distance from the valve to the fulcrum is 4 in. $3^2 \times .7854 \times 100$ lbs. = 707 lbs. = S. $S \times b = 4 \times 707$ lbs. = 2,828 lbs. $V \times b = 4 \times 4$ lbs. = 16 lbs. $L \times h = 12 \times 20$ lbs. = 240 lbs. $W \times y = W \times 34$. 16 + 240 + 34 $\times$ W. $W \times 34 + 16 + 240 = 2{,}828$. $34 \times W$ = 2,572 lbs. W = 75.64 lbs.

PROBLEMS

1. If the area of a safety valve is 7 sq. in., the weight of lever is 7 lbs., the weight of valve is 4 lbs., the weight of ball is 50 lbs., the center of gravity is 7 in. from fulcrum, the valve is 2 in. from fulcrum, the pressure on the valves is 80 lbs. per square in., where must W be placed to blow off at 80 lbs.?

2. With a 2-in. safety valve where must the 35-lb. weight be placed to blow off at 85 lbs., if the weight of lever is 8 lbs., the weight of valve is 4 lbs., the distance from the

center of gravity to the fulcrum is 12 in., the distance from the valve to the fulcrum is 3 in., the distance from the ball to the fulcrum is 30 in.?

3. The diameter of a safety valve is 4 in.; the weight on the lever is 90 lbs.; the weight of the lever is 12 lbs.; the weight of the valve is 4 lbs.; the distance from the valve to the fulcrum is 2.5 in.; the length of the lever is 30 in.: the distance from the center of gravity to the fulcrum is $11\frac{1}{2}$ in. How far from the fulcrum must the weight be placed so as to blow off at 80 lbs.?

4. A safety valve is $2\frac{1}{2}$ in. in diameter, and weighs 3 lbs. The distance from the valve to the fulcrum is 5 in.; the distance from the center of gravity to the fulcrum is 10 in.; the distance from the ball to the fulcrum is 24 in.; the weight of the lever is 15 lbs. What weight must be used to blow off at 90 lbs.?

5. What weight valve, whose diameter is 2 in., must be used to blow off at 90 lbs., if the ball weighs 60 lbs., the lever weighs 8 lbs.; the distance from the ball to the fulcrum is 12.7 in., the distance from the center of gravity to the fulcrum is 10 in. and the distance from the valve to the fulcrum is 3 in.?

6. How far must a valve, whose diameter is 4 in., be placed from the fulcrum, if the valve weighs 8 lbs., the ball weighs 64 lbs. and blows off at 90 lbs., the lever weighs 24 lbs., the distance from the ball to the fulcrum is 38 in. and the distance from the center of gravity to the fulcrum is 16 in.?

PUMPS

321. Let P = the pressure in pounds per square inch on the piston to lift the piston. Let H = the height in feet through which the water is lifted. Let L = the length of the stroke of the piston in feet. Let D = the diameter of the piston in inches. Let Q = the number of cubic feet of water dis-

charged per minute. Let N = the number of strokes per minute. Then Q = the area of the piston times the length of the stroke in feet times the number of strokes per minute ÷ by 144.

$$\text{Or, } Q = \frac{D^2 \times .7854 \times L \times N}{144}.$$

Work in feet per minute = $Q \times 62.5 \times H$. One cubic foot of water weighs 62.5 lbs. H = the height in feet to which the water is lifted.

H. P. = foot-pounds per minute ÷ 33,000.

$$\text{Or, } H.P. = \frac{Q \times 62.5 \times H}{33{,}000}.$$

Work done per stroke in foot-pounds = $D^2 \times .7854 \times L \times P$.

The work done per stroke by a pump whose piston area is 40 sq. in., stroke 2 ft., and piston pressure is 40 lbs. per square inch is $D^2 \times .7854$ = 40 sq. in., piston area. L = 2 ft. P = 40 lbs. per square inch. $40 \times 2 \times 40$ = 3,200, the number of foot-pounds.

PROBLEMS

1. What is the H. P. of a pump whose plunger is 10 in. in diameter; stroke of piston 2 ft., pressure 60 lbs. per sq. in.; and number of strokes per minute 40?

2. How many gallons will be delivered per minute by a pump whose plunger diameter is 4 in., length of stroke 12 in., and number of strokes per minute 24? (1 cu. ft. = 7.48 gal.)

3. How many cubic feet of water will be delivered per minute by a pump whose plunger diameter is 6 in., length of stroke 20 in., and number of strokes per minute 36?

4. How many cubic feet of water will be delivered per minute by a pump whose plunger diameter is 5 in., length of stroke 16 in., and number of strokes 40?

PRESSURE OF WATER AT DEPTH

322. The pressure of water at any depth with a free upper surface, is the weight of the overlying water, and is 1 lb. per square inch for every 2.3 ft. of depth. A cubic foot of water weighs 62.5 lbs. A cubic inch of water weighs .03617 lb. A column of water 12 in. long and 1 sq. in. in cross-section weighs (.03617 × 12) .434 lb. Let $P =$ the pressure in pounds per square inch; $H =$ the head of water in feet; W = the weight of a column of water 1 ft. long and 1 sq. in. in cross section. Then $P = WH$, or, $P = .434\ H$.

If the depth of water in a standpipe is 100 ft., the pressure of water in pounds per inch on the bottom of the pipe is $P = .434 \times H = .434 \times 100 = 43.4$ lbs. per square inch. If it is required to find the head of water for a given pressure per square inch., $H = \frac{P}{W}$. Or, $H = \frac{P}{.434}$. To find the head of water in a standpipe to give a pressure of 100 lbs. per square inch, $H = \frac{100}{.434} = 230.4$, the head in feet.

PROBLEMS

1. If the difference in pressure in pounds per square inch, as found by a pressure gauge, between two faucets in a building is 23 lbs., what is the difference in level between the two points in feet? $23 \div 2.3 = 10$, the difference in feet.

2. A submarine boat goes 50 ft. below the surface. What is the pressure in pounds per square inch on the surface? $P = .434 \times H = .434 \times 50 = 21.70$, the number of pounds per square inch. What is the pressure on a part of the surface of the boat 2 ft. by 2 ft.?

3. What is the pressure per square foot on the surface of a diver who is working 40 ft. below the surface of a lake?

4. What is the pressure per square inch corresponding to a head of 350 ft.?

5. What is the head of water which will give a pressure of 90 lbs. per square inch?

6. What is the pressure of water per square inch striking the blades of a turbine wheel when the head of water is 96 ft.?

TURBINES

323. High Pressure Turbines: They have elevated heads and revolve at high speed.

324. Low Pressure Turbines: They have large volumes of water at low head and revolve at low speed.

325. Reaction Turbines: All parts are filled with moving water and only a part of the energy of the water is converted into velocity. A greater part is in the form of pressure due to the reaction of the moving water as it issues from the vanes of the wheel.

326. Impulse Turbines: These are only partly filled with water, which strikes the turbine blades in the form of jets. The water energy is converted into velocity before it acts upon the moving parts of the turbine. The acting force is the pressure due to the impulse of the jets of water issuing from the portals.

327. The weight of water used *per second in an impulse wheel is equal to the cross-section area of the jet in square feet multiplied by the velocity of the jet in feet per second multiplied by 62.5.*

$$W = \frac{D^2 \times .7854 \times V \times 62.5}{144}.$$

W = the weight in pounds. V = the velocity in feet per second. 62.5 = the weight in pounds of 1 cu. ft. of water.

$$H.\ P. = \frac{W \times H}{550}.$$

H = the height of the head of water above the jet. The velocity of a body falling from a height, at end of fall, is, $V = 8.02 \sqrt{H}$ ft. per second.

An impulse turbine wheel is working to the best advantage when the velocity of the rim of the wheel is one half the velocity of the jet.

328. To find the diameter of a turbine wheel which works to the best advantage with a dynamo. Let D = the diameter and V = the velocity of a turbine wheel.

$$V = D \times 3.1416 \times R.\,P.\,M. \quad D = \frac{V}{3.1416 \times R.\,P.\,M.}$$

329. Rule: *The diameter of a turbine wheel equals the velocity of the rim of the turbine wheel in feet per minute divided by 3.1416 × R. P. M of the dynamo.*

What should be the diameter (in feet) of a turbine wheel connected to the shaft of a dynamo, if the head of water is 400 ft. and the dynamo makes 900 R. P. M.? $V = 8.02 \sqrt{H}$, or $V = 8.02 \sqrt{400} = 160.40$ ft. per second. The velocity of the rim of the turbine should be one half of this. Or, $\frac{1}{2}$ of $160.4 = 80.2$, the number of feet per second. And $60 \times 80.2 = 4{,}812$, the number of feet per per minute = the velocity of the rim of the turbine wheel in feet per minute.

The turbine wheel is connected to the shaft of the dynamo.

The diameter of the turbine wheel in feet is

$$D = \frac{4{,}812}{3.1416 \times 900} = 1.6.$$

330. The weight of water discharged *per second from a turbine wheel is equal to the volume of water discharged in cubic feet multiplied by 62.5, the weight in lbs. per cubic foot.* Or, $W = Q \times 62.5$. Let W = the weight in pounds. Let Q = the volume in cubic feet per second.

331. The volume of water discharged *in cubic feet per second equals the area of the jet in square feet multiplied by*

the velocity in feet per second. Or, $Q = A \times V$. Let A = the area in square feet. Let V = the velocity in feet per second. Area $= D^2 \times .7854$. Or, area $= R^2 \times 3.1416$. $W = A \times V \times 62.5$.

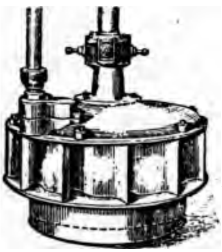

FIGURE 59

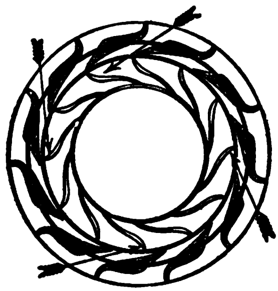

FIGURE 60

PROBLEMS

1. What is the horsepower of a turbine wheel which is discharging 1,350 lbs. of water per minute under a head of 625 ft.?

2. What should be the diameter of a turbine wheel connected to the shaft of a dynamo, if the head of water is 900 ft. and the dynamo makes 800 R. P. M.?

3. A turbine wheel has a jet diameter of $1\frac{1}{2}$ in., and a velocity of 130.34 ft. per second. What is the weight of water discharged per second?

4. What is the horsepower of a turbine which is discharging 100 cu. ft. of water per minute under a head of 50 ft.?

5. A turbine has a jet diameter of $1\frac{3}{4}$ in., and a jet velocity of 150 ft. under a head of 484 ft. What is the horsepower?

6. What is the horsepower of a turbine which discharges 675 lbs. of water under a head of 400 ft.?

7. What should be the diameter of a turbine connected to a dynamo shaft which is making 850 R. P. M., if the head of water is 625 ft.?

8. What is the velocity of a falling body at the end of a fall of 250 ft.? What should be the velocity of the rim of a turbine wheel with a head of 250 ft.?

9. The turbine pits at Niagara Falls are 136 ft. deep, and each turbine discharges 25,000 cu. ft. of water per minute. What is the horsepower of each turbine?

10. A turbine discharges 180 cu. ft. of water per minute. The head is 390 ft. What is the horsepower?

11. What should be the velocity of the rim of a turbine wheel, if the total fall of water is 400?

12. What is the horsepower of a turbine wheel which is discharging 4,000 cu. ft. of water per minute under a head of 900 ft.?

13. What weight of water is discharged per second by a turbine wheel, if the cross-section area of the jets is 9 sq. in. and the velocity of the jets is 80 ft. per second?

14. The power supplied the turbine at Niagara Falls is 31,250,000 foot-pounds per second. What is the horsepower of the turbine?

STOCK AND FORGING

332. To find what sized piece of stock to use to make a given forging, we must find the weight of the forging and also an equivalent weight of a bar of stock of the size to be used. In some cases, as in welding, a little more must be added—an allowance in length equal to the thickness of the stock.

In measuring a forging, measure along the central line or axis of the piece and not along the outside or inside curve. There is a difference in bent circular pieces, whether we measure along the outside of a curve, inside of a curve or along the central axis. A certain curve measures $8\frac{1}{2}$ in. outside measurement, 7 in. inside measurement, and $7\frac{3}{4}$ in.

along the central line or central axis. For round stock, from the outside diameter subtract, or to the inside diameter add, one thickness of the stock, and multiply by $\frac{22}{7}$ (π).

PROBLEMS

1. What is the length of stock for a flat ring $\frac{1}{4}$ in. thick, $1\frac{1}{4}$ in. wide, and $4\frac{1}{4}$ in. inside diameter?

2. A disk $4\frac{1}{4}$ in. in diameter and 1 in. thick is to be forged. How long a piece of stock from a round bar $3\frac{1}{2}$ in. in diameter must be cut off for the disk?

3. A ring 8 in. outside diameter, $\frac{3}{4}$ in. thick, and 4 in. wide is to be made. The nearest size of flat bar in stock is $2\frac{3}{4}$ in. by $1\frac{1}{2}$ in. How long a piece of the bar must be used?

4. A flat ring whose inside diameter is 8 in., outside diameter $10\frac{1}{2}$ in., and thickness $\frac{1}{2}$ in. is to be made. How much stock 4 in. wide is required?

5. How long a piece of bar must be used from flat bar stock 3 in. by $1\frac{3}{4}$ in., to make a ring 6 in. outside diameter, $\frac{3}{4}$ thick, and 2 in. wide?

BLAST FURNACE

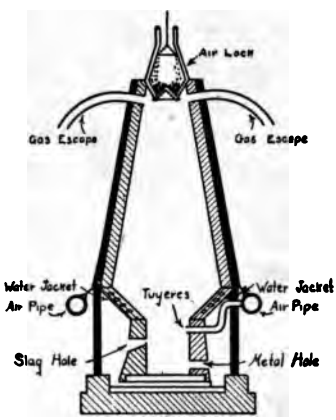

FIGURE 61

333. Fig. 61 represents a blast furnace into which hot, dry, blasts of air are forced over the burning mass by blowers to assist in the melting and reduction of iron ore. This furnace is a cylindrical steel shell, 75 to 90 ft. high and lined with fire brick. A charge of ore, coke and flux-limestone is dropped in at the top of the furnace from time to time. Cold water is made to circulate through hollow castings where

the heat is most intense. This is just above the tuyeres, or blow pipes, where the powerful hot, dry air blasts enter. The impure molten iron settles to the bottom of the furnace, from where it is drawn off from time to time through the tap hole in the bottom.

The limestone flux and other impurities are called slag and are drawn off about every two hours, and the molten iron about every six hours. The gases produced are led away at the top of the furnace and utilized for heating.

The slag is generally regarded as useless and thrown away. The iron is allowed to flow out into trenches made in beds of sand. The iron, as soon as cool enough, is broken into pieces about three feet long, and is called pig iron.

PROBLEMS

1. A cupola melts 45,000 lbs. of iron with 5,500 lbs. of coal. What is the percentage of fuel to iron that is used?

2. A cupola measures 36 in. inside diameter and 12 in. from the sand bottom to the tuyeres, or top of the ore. How many pounds of iron does it hold? (One cubic foot of cast-iron weighs 450 lbs.)

3. A cupola measures 32 in. inside diameter and 16 in. from sand bottom to bottom of tuyeres. How many pounds of melted iron will the cupola hold?

4. How many pounds of fuel will it take to melt 3,000 lbs. of iron, if the ratio of fuel to iron is 1 to 7?

5. A cupola melts 7,000 lbs. of iron with 800 lbs. of coke. What is the ratio of fuel to iron?

6. A cupola charge has 25,000 lbs. of iron worth $18 per ton and 4,000 lbs. of coke worth $4 per ton. What will the iron cost per pound?

7. A blast furnace charge is 4,000 lbs. of ore containing 60% of iron, 800 lbs. of limestone, and 2,000 lbs. of coke.

What is the number of pounds of each per ton of pig iron produced?

8. A furnace requires 2,400 lbs. of coal to produce one ton of iron, when the ore is 60% iron. What proportion of coal to iron ore must the charge contain?

9. A charge is made up of different kinds of iron as follows: 25% iron, 20% iron and 60% iron. How many pounds of each in a total charge of 4,000 lbs.?

10. A charge is made up of 8,000 lbs. of one kind of iron, 8,500 lbs. of another kind, and 9,000 lbs. of a third kind. What is the percentage of each?

HEAT AND SPECIFIC HEAT

334. The specific heat of water is 1; of ice is .5; of silver is .057; of copper is .095; of zinc is .096; of lead is .031; of wrought iron is .11; of cast iron is .13; of brass is .094.

The specific heat of a substance is the number of B. T. U. (British thermal units) of heat to raise the temperature of 1 lb. of the substance through 1° F. It is the ratio between the amount of heat required to heat the body through 1° and the amount of heat required to heat an equal weight of water through 1°. To heat any weight of copper through 1° requires only .095 as much heat as to heat an equal weight of water through 1°. Hence the specific heat of copper is .095.

The latent heat of fusion of a substance is the amount of heat given up by a liquid in freezing or absorbed by a solid in melting.

The amount of heat to change 1 lb. of ice at 32° F. to water at the same temperature is 147 B. T. U.

The latent heat of steam or vaporization is 967 B. T. U. That is the amount of heat required to change 1 lb. of boiling water at 212° F. to steam at the same temperature.

One B. T. U. of heat = 778 ft.-lbs. of work.

How many B. T. U. of heat are required to melt 20 lbs. of ice?

The amount of heat to melt 1 lb. of ice is 147 B. T. U. 20 × 147 B. T. U. = 2,940 B. T. U., which is the amount of heat required to melt 20 lbs. of ice.

How many B. T. U. of heat are required to raise the temperature of 20 lbs. of ice after melting to 60° F.? The specific heat of ice is .5 B. T. U. Therefore, 20 × .5 (the specific heat of ice) B. T. U. = 10 B. T. U.

How many B. T. U. would be required to melt 20 lbs. of ice and then raise its temperature to 60° F.? 2,940 B. T. U. + 10 B. T. U. = 2,950 B. T. U.

PROBLEMS

1. How many B. T. U. are given out by a steam radiator when 50 lbs. of steam have been condensed into water in it? 50 × 967 B. T. U. per lb. = 48,350 B. T. U.

2. How many B. T. U. are liberated by the boiling away of 4 lbs. of water?

3. If the temperature of fusion for lead is 626° F. and the latent heat of fusion is 9.67, how much heat will be required to melt 20 lbs. of lead from a temperature of 30° F.? 20 × (626 − 30) × .0314 (sp. heat) = 374,288 B.T.U. 20 × 9.67 latent heat of fusion = 193.40 B.T.U. 374.288 B.T. U. + 193.40 B.T.U. = 567.688 B. T. U.

4. How many pounds of coal will it take to melt 20 lbs. of zinc from 40° F. if the latent heat of fusion of zinc is 50.63, the temperature of fusion is 680° F., and there is no heat lost?

5. How many pounds of tin can be melted from 46° F. by 100 lbs. of coal, if the latent heat of fusion of tin is 25.65, the temperature of fusion is 446° F. and the specific heat of tin is .0562? (Let x = the number of lbs. of tin.)

6. How many B. T. U. are carried into a kitchen by steam when 1 qt. of water has been allowed to boil away. (1 qt. weighs about 2 lbs.)

2×967 B. T. U. per lb. $= 1{,}934$ B. T. U.

7. How many units of heat are required to raise the temperature of 10 lbs. of zinc from 60° F. to 680° F.? $10 \times (680 - 60) \times .0956 = 592.72$.

8. If it requires 47.55 B. T. U. of heat to warm 25 lbs. of copper 20° F., what is the specific heat of copper?

9. How much heat is needed to heat 100 lbs. of steel 2,520° F.? Specific heat of steel is .1175.

10. How many pounds of good coal will be required to melt 50 lbs. of zinc from 56° F.? Latent heat of fusion is 50.63. Temperature of fusion is 680° F.

11. If 50 lbs. of good coal is used under a boiler to run an engine, how many foot-pounds of work will be done, if the boiler and engine works at 15% efficiency?

12. How many tons of coal will be required to melt one ton of ice at 32° F. and change it to steam at 212° F., if no heat is lost? (The latent for ice to water is 147° F. and for water to steam is 967° F.)

LINEAR EXPANSION

335. The coefficient of linear expansion is the ratio of increase in length per degree of rise in temperature to the total length.

336. The following table gives the coefficient of expansion for some substances per degree Fahrenheit.

Brass	.00001036	Cast-iron	.00000618
Wrought iron	.00000686	Steel tempered	.00000702
Steel	.00000698	Copper	.00000955
Untempered steel	.00000599	Zinc	.00001644

Two times these numbers will give surface expansion. Three times these numbers will give cubic expansion.

PROBLEMS

1. A wrought iron bar 30 ft. long is heated from 80° to 300°. How much will it lengthen? 300° − 80° = 220°. 220 × 30 × .00000686 = .045276, the number of feet.

2. A cast-iron bar 20 ft. long is heated from 70° to 350°. How much will it lengthen?

3. A copper bar 1 ft. long is heated from 100° F. to 250° F. How much will it lengthen?

4. A cast-iron steam pipe 400 ft. long has expansion collars. Each of these collar gives $1\frac{1}{2}$ in. free play. How many must be put in to allow for a range of temperature from 32° F. to 232° F.?

5. How many expansion collars $1\frac{1}{2}$ in. free play must be put into a 700 ft. steam pipe to allow for a range of temperature of 200° F.?

6. A ring of cast-iron has an inside diameter of 5 ft. when at a temperature of 932° F. What is the diameter at 32° F.?

7. A wrought iron connecting rod is 20 ft. long at 18° F. What is the increase of length at 144° F.?

8. The volume of a mass of cast iron is 5 cu. ft. at 18° F. What is its volume at 140° F.?

9. A cast-iron steam pipe is 100 ft. long at 32° F. What is its length when steam at 215°F. passes through it?

10. A carriage wheel is 7 ft., $6\frac{3}{8}$ in. in circumference. A cast-iron tire is 7 ft., 6 in. on its inner circumference at 27° F. To what temperature must the tire be heated to just slip on the wheel?

11. A wrought iron bar 28 ft. long is heated from 40° F. to 400° F. How much will it lengthen?

12. A cast iron plate 6′ × 4′ is heated from 38° F. to 300° F. What will be the surface expansion? What will be its dimensions after heating?

13. A block of copper whose dimensions are $4' \times 3' \times 2'$ is heated from 34° F. to 400° F. How much does it expand?

14. A steel cable is one mile long. By how many feet does its length vary between a winter day when the temperature is $-4°$ F. and a summer day when the temperature is 86° F. (Iron expands .0000066 of its length for each degree increase in temperature, F.

15. If iron rails are 30 ft. long, and if the variation of temperature through the year is 90° F., what space must be left between their ends?

16. If an iron steam pipe is 60 ft. long at 32° F., what is its length when steam passes through it at 212° F.?

17. In long steam iron pipes expansion joints are inserted every 200 ft. If the range of temperature is from 22° F. to 257° F., what play must be allowed for at the expansion joint?

18. A line shaft is 200 ft. long at the temperature of 50° F. How long will it be at 90° F.?

19. A shaft is 2.45 in. in diameter. To what size must a collar be bored to give a solid fit? (For a solid fit, or driving fit, .001 per inch of diameter is allowed.)

HOT-WATER HEATING

337. To find approximately the number of heat units *to heat a building of any number of cubic feet capacity to any temperature.* The specific heat of air is .238. That is, .238 heat units will raise the temperature of 1 lb. of air 1°F. And at zero 1 cu. ft. of air weighs .081 lb. Then 1 divided by .081 = 12.34, which represents the number of cubic feet of air occupied by 1 lb. of air. 12.34 divided by .238 = 51+, cu. ft., raised 1° by one B. T. U.; but in practice it is called 50 cu. ft. Therefore, 1 British thermal unit, or B. T. U., will raise the temperature of 50 cubic feet of air through 1°F.

To raise the temperature of 1,000 cu. ft. of air through 1°F. will require as many B. T. U. as 50 is contained times in 1,000, or 20. To raise the temperature of 500 cu. ft. of air through 1°F. will require (500 divided by 50) 10 B. T. U. If we wish to heat 1,000 cu. ft. of air through 30 degrees, it will take 30 times 20 B. T. U., or 600 B. T. U. If we wish to heat 500 cu. ft. of air through 40 degrees, it will take 40 times 10 B. T. U., or 400 B. T. U.

A room is 20′ × 30′ and 10′ high. How many heat units will be required to raise the temperature of the room from 32° to 68°F.?

20 × 30 × 10 = 6,000. 1 B. T. U. will raise the temperature of 50 cu. ft. of air 1°F. To raise through 1°F. the temperature of 6,000 cu. ft. of air, which is 120 times 50 cu. ft., will require 120 times 1 B. T. U., or 120 B. T. U. To raise the temperature from 32° to 68° will require 36 × 120 B. T. U. = 4,320 B. T. U.

Heat Losses in B. T. U. per Square Foot of Surface per Hour Southern Exposure

MATERIAL	Difference between inside and outside temperature									
	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°
8 in. Brick Wall	5	9	13	18	22	27	31	36	40	45
12 in. Brick Wall	4	7	10	13	16	20	23	26	30	35
16 in. Brick Wall	3	5	8	10	13	16	19	22	24	27
20 in. Brick Wall	2.8	4.5	7	9	11	14	16	18	20	23
24 in. Brick Wall	2.5	4	6	8	10	12	14	16	18	20
28 in. Brick Wall	2	3.5	5	7	9	11	13	14	16	18
32 in. Brick Wall	1.5	3	4.5	6	8	10	11	13	15	16
Single Window	12	24	36	49	60	73	85	93	110	122
Double Window	8	16	24	32	40	48	56	62	70	78
Single Skylight	11	21	31	42	52	63	73	84	94	104
Double Skylight	7	14	20	28	35	42	48	56	62	70
1 in. Wooden Door	4	8	12	16	20	24	28	32	36	40
2 in. Wooden Door	3	5	8	11	14	17	20	23	25	28
2 in. Solid Plaster Partition	6	12	18	24	30	36	42	48	54	60
3 in. Solid Plaster Partition	5	10	15	20	25	30	35	40	45	50
Concrete Floor on Brick Arch	2	4	6.5	9	11	13	15	18	20	22
Wood Floor on Brick Arch	1.5	3	4.5	6	7	9	10	12	13	15
Double Wood Floor	1	2	3	4	5	6	7	8	9	10
Walls of Ordinary Wood Dwellings	3	5	8	10	13	16	19	22	24	27

The above table applies only to a southern exposure; for other exposures multiply the heat loss given in table above by the factors given in the table below:

Factors for Calculating Heat Losses for Other than Southern Exposure

EXPOSURE	FACTOR
North	1.32
East	1.12
West	1.20
Northeast	1.22
Northwest	1.26
Southeast	1.06
Southwest	1.10
N. S. E. and West or total exposure	1.16

A room is 10 × 15 ft., with 10-ft ceiling, and the walls are 20″ brick. There are two windows 3 × 5 ft. each, and one window 4 × 5 ft.

The exposure is northwest. How many square feet of hot-water radiation will be required to keep the temperature at 70°F. when it is 10°F. below outside? Total exposed surface = 250 sq. ft. Glass surface = 50 sq. ft. Net wall surface = 250 − 50 = 200 sq. ft. Find 18 for multiplier from Table of Heat Losses, for 20 in. brick walls and 80° difference in temperature. Find 93 for single window glass surface and 80° difference in temperature. Get the sum of these products. Find from table the factor 1.26 for northwest exposure. Multiply the total 8,250 sq. ft. for walls and glass surface by 1.26 = 10,395, the number of B. T. U. required to heat the surfaces. For the air in the room, divide 1,500 cu. ft. capacity by 50 and multiply by 80°, difference in temperature. This gives 2,400, the number of B. T. U. required to heat the air in the room. The total B. T. U. is 10,395 + 2,400 = 12,795 B. T. U. For hot-water radiation divide 12,795 B. T. U. by 150 = 85.3, which represents the number of square feet of radiation.

338. Rule: *The cubic feet of air space in the room is divided by 50, and this quotient is multiplied by the number of*

degrees of temperature through which the temperature of the room is to be raised. This result gives the B. T. U. of heat.

The sum of these three gives the total heat required in B. T. U.

Divide this total heat required in B. T. U. by 150 B. T. U. for hot-water radiation. This gives the number of square feet of hot-water radiation required.

339. For solid stone walls, multiply the figures for brick of the same thickness by 1.7.

Where rooms have a cold attic above, or a cold cellar below, multiply the heat losses through walls and windows by 1.1.

The figures given in the tables apply only to the most thoroughly constructed buildings.

340. For the average well built house the figures should be increased by about 10%; for fairly well constructed, by 20%; for poorly constructed, by 30%.

For example: A room is 10′ × 15′ × 10′ ceiling. The walls are 20 in. brick. The construction is the very best. The room has two windows 3′ × 5′ and one window 4′ × 5′. The exposure is north and west to be heated with hot-water apparatus. The average temperature of the water is to be 180 degrees. The temperature of the room is to be kept at 70 degrees when it is 10 degrees below zero outside.

341 In computing heat losses through walls, only those exposed to the outside air are to be considered.

Total exposed surface:

$$15 + 10 = 25.$$
$$25 \times 10 = 250\text{—}250 \text{ sq. ft.}$$

Glass surface:

2 windows	3 × 5 × 2 =	30
1 window	4 × 5 =	20
		50 sq. ft. glass

250 sq. ft. − 50 sq. ft. = 200 sq. ft. of net wall surface.

From the table of heat losses, 20-in. brick will lose 18 B. T. U. per square foot per hour when the temperature difference is 80 degrees, which is the difference when the temperature of the room is to be kept at 70 degrees in 10 degrees below zero weather. 200 × 18 = 3,600, the units of heat loss through the wall. 50 × 93 = 4.650 , the units of of heat loss through the windows, at 80 degrees difference in temperature. The total units of heat loss through windows and walls is 3,600 + 4,650 = 8,250. The factor for northwest exposure is 1.26. 8,250 × 1.26 = 10,395, the number of B. T. U. of heat loss for northwest exposure.

342. The air in a room should change once per hour. One B. T. U. will heat 50 cu. ft. of air one degree.

The cubical contents of the room is 15′ × 10′ × 10′ = 1,500 cu. ft. 1,500 × 80, which is the number of degrees the air is to be heated = 120,000 (the air outside 10 degrees below and the air inside 70 degrees).

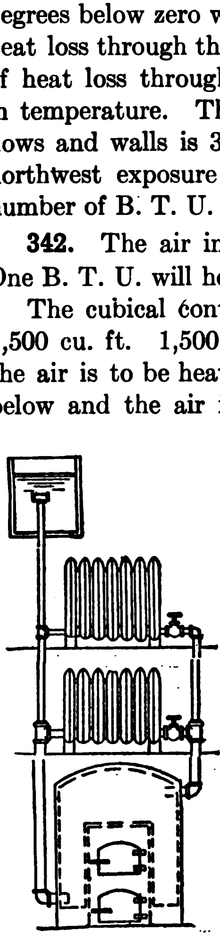

FIGURE 62
HOT-WATER HEATING

120,000 divided by 50 = 2,400. We then have the heat loss through the walls and windows, 10,395 B. T. U. and the heat for heating the changing air in the room 2,400 B. T. U. 10,395 B. T. U. + 2,400 B. T. U. = 12,795 B. T. U.

By experiment it is found that one square foot of hot-water radiation standing in air of 70 degrees will yield 187 B. T. U. per hour, and one square foot of steam radiation will yield 240 B. T. U. In practice, one square foot of hot-water radiation, is assumed to yield 150 B. T. U. per hour. 12,795 divided by 150 = 82.6 which is the number of square feet of hot-water

radiation which should be placed in the room. The number of square feet of heating surface in tubes equals the number of tubes multiplied by the diameter of the tube in inches, and by its length in feet, and by .2618.

One pound of water evaporated at and from 212° F. = 965.7 B. T. U.

343. Each person breathes on an average about 28 cu. ft. of air in an hour.

A room in a grammar school is 28′ × 32′ × 12 and is to accommodate 50 pupils. The walls are of brick, 16 in. thick, and there are 6 single windows in the room, each 3′ × 6′. There are warm rooms above and below. Exposure is S. E. How many B. T. U. will be required per hour for warming the room and how many for ventilation in zero weather, assuming the building to be of average construction? Air requirements for ventilation in grammar schools are 1,800 cu. ft. per hour per pupil.

Exposed walls are 28′ + 32′ = 60′. Height 12′ × 60′ = 720 sq. ft. For the windows, 3′ × 6′ = 18 sq. ft. and 6′ × 18′ = 108 sq. ft. Square feet of net wall surface = 720 − 108 = 612 sq. ft. From the table of Heat Losses we get the factor 19 which multiplied by 612 = 612 × 19 = 11,628 B. T. U. From the table we get 85 the factor for single windows, which multiplied by 108 = 108 × 85 = 9,180, the number of B. T. U. required. Then 11,628 B. T. U. + 9,180 B. T. U. = 20,808 B.T.U. And 20,808 B. T. U. × 1.06 × 1.1 = 24,262 B. T. U., heat requirements for warming the room.

1,800 cu. ft. × 50 = 90,000 cu. ft. And 90,000 cu. ft. × 70 ÷ 50 = 126,000, the number of B. T. U. for ventilation. 24,262 + 126,000 = 150,262 B. T. U., total heat requirements. If the room is to be heated by steam at 21 lbs. gauge pressure, the temperature of steam at this pressure is 220 degrees. 1 sq. ft. of radiation yields 240 B. T. U. per hour.

Divide 150,262 B.T. U. by 240 = 626.09 B. T. U., the total amount of radiation required.

STEAM HEAT

344. Find how many square feet of steam radiation will be required to heat a room on the first floor of a house, where rooms above, hall and room on each side, and cellar, are warm. The room is $10' \times 10' \times 10'$ with two windows $3' \times 5'$ each, and the exposure is northwest.

The number of square feet of exposed surface is $10 + 10 = 20 \times 10 = 200$ sq. ft. Window surface is $3 \times 5 = 15$ sq. ft. each. For two windows it is 30 sq. ft.

200 sq. ft. of exposed surface less 30 sq. ft. of window surface = 170 sq. ft. of exposed wall surface. The difference of temperature, 70° inside and 10° below outside = 80°.

With a wind velocity of about $12\frac{1}{2}$ miles per hour, the rate of heat loss per square foot of single thick common glass is about 1.20 B. T. U. per degree of difference in temperature between outside and inside air. For double thick glass the loss per square foot is about .80 B. T. U. per degree of difference in temperature.

For ordinary dwellings of wood the rate of heat losses per square foot through the walls is about .27 B. T. U. per degree of temperature.

For the loss of heat by the glass surface, single glass, we have 1.20×80 difference in temperature $\times 30 = 2{,}880$, which is the number of B. T. U. required.

For the loss of heat by the wall surface, we have $.27 \times 80$ difference in temperature $\times 170 = 3{,}672$, which is the number of B. T. U. required.

345. To find the number of heat units ***necessary to raise the temperature of the air to the required temperature,*** we have 1,000 cu. ft. divided by 50 cu. ft. = 20, which is the number

of B. T. U. required to heat the air 1°; and 20 × 80 = 1,600, the number of B. T. U. required.

346. Rule: *Find the heat losses through the walls. Find the heat losses through the windows. Find the cubic feet of room space, and how much heat to heat this room space to the required temperature. This result gives the total required amount of heat for the building in B. T. U.*

The area of the wall is multiplied by the heat loss factor for the kind of wall and exposure. Only walls exposed to outside air are considered.

The area of windows is multiplied by the heat loss factor for windows.

The cubic feet of air space in the room is multiplied by the amount of heat required for one cubic foot of air. The sum of these three gives the total heat required in B. T. U. The cubic feet of air space is divided by 50 and this quotient multiplied by the number of degrees of temperature through which the temperature of the room is to be raised.

One square foot of steam radiation yields about 240 B. T. U. of heat. All radiator companies furnish the number of square feet of radiation for their radiators.

The total number of heat units lost through walls, glass, and to heat the air in the rooms, is 3,672 for walls + 2,880 for glass × 1,600 for air in the room = 8,152 B. T. U.

It is known that one square foot of steam radiation standing in air of 70°F. and the temperature of the steam 212°F. will yield about 240 B. T. U.

So, we have 8,152 divided by 240 = 33.96, the number of square feet of radiation + 26%, or multiplied by 1.26 for northwest exposure = 42.79, the number of square feet of radiation required for the room.

Or, we may say that 80 × .27 = 21.60, the multiplier for the square feet of exposed walls. And 80 × 1.20 = 96, the multiplier for the square feet of glass surface. And

1,000 (cu. ft. of cubical contents of the room) divided by 50 = 20, the number of B. T. U. required to heat the air 1°F. And $20 \times 80 = 1{,}600$, the number of B. T. U. Then for wall, $170 \times 21.6 = 3{,}672$. For the glass, $30 \times 96 = 2{,}880$. For the air volume, 1,600 B. T. U. And $3{,}672 + 2{,}880 + 1{,}600 = 8{,}152$. The total B. T. U. loss, 8,152, divided by $240 = 33.96$, the number of square feet of radiation. This + 26% of itself or multiplied by $1.26 = 42.79$, the number of square feet of radiation required.

PROBLEMS

1. A room is $20' \times 40' \times 10'$ high. How many heat units will be required to raise the temperature of the room from 32° to 70° F.?

2. A room is $20' \times 20' \times 10'$ high. How many heat units will be required to raise the temperature of the room from 40° to 70° F.?

3. How many heat units are required to warm a room from 34° to 75° F., if the room is $16' \times 20' \times 8'$ high?

4. A four-room house has the following sizes of rooms: $10' \times 12'$; $12' \times 14'$; $16' \times 18'$; and $16' \times 20'$, all 9 feet high. How many heat units are required to warm these rooms from 36° to 75° F.?

5. A schoolroom in a frame building is $30' \times 30' \times 15'$ ceiling with a northeast exposure, hall and adjoining room and above and cellar are warm. There are 4 windows, each $4' \times 7'$. How many square feet of steam radiation are required to keep the room at a temperature of 70°, when the temperature outside is 10° below zero?

6. The 4 rooms on the lower floor of a house are each $12' \times 18' \times 10'$ ceiling. The rooms above are warm and the cellar below is warm. There are 4 windows, each $4' \times 7'$, one in each room. How many square feet of steam radiation are required to keep the rooms at a temperature of 70°, when the outside temperature is 10° below zero?

7. A schoolroom in a frame building is 24′ × 36′ × 15′ ceiling with a northwest exposure; hall, adjoining and upper rooms and cellar are warm. There are 6 windows, each 4′ × 7′. How many square feet of steam radiation are required to keep the room at a temperature of 70°, when the temperature outside is 15° below zero?

8. The 4 rooms on the upper floor of a house are each 12′ × 16′ × 9′ ceiling. The attic above is cold and the rooms below are warm. There are 4 windows, each 4′ × 7′, one in each room. How many square feet of steam radiation are required to keep the rooms at a temperature of 70°, when the outside temperature is 12° below zero?

ELECTRICITY

347. In the electric circuit, the unit of *resistance* is the *ohm,* which is equal to the resistance of about 152 ft. of number 18 copper wire.

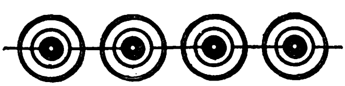

FIGURE 63—CELLS IN SERIES

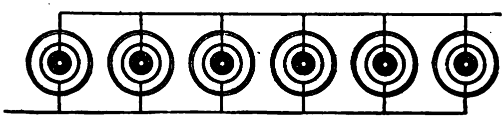

FIGURE 64—CELLS IN PARALLEL

The unit of current strength is the *ampere,* which is the *quantity* of electricity that will deposit .001118 gram of silver per second from a silver salt solution.

The unit of electromotive force is the *volt,* which is the electric *pressure* that will force a current of one ampere through a resistance of one ohm. The formula is, current in amperes:

$$(C) = \frac{\text{Electromotive force } (E)}{\text{Resistance in ohms } (R)}; \text{ or, } C = \frac{E}{R}; \text{ or, amperes } = \frac{\text{volts}}{\text{ohms}}; \text{ or, } C = \frac{E.\,M.\,F.}{R}; \text{ or, } C = \frac{E}{R + r}; \text{ or, } C = \frac{n\,E}{R + \frac{nr}{m}}.$$

R = external resistance; r = internal resistance; n = number cells in series; m = number cells in parallel.

348. Cells in series give an E. M. F. of a single cell multiplied by the number of cells. Cells in parallel give an E. M. F. of a single cell. Cells in series give an internal resistance of a single cell multiplied by the number of cells. Cells in parallel give an internal resistance of a single cell divided by the number of cells.

$$C = \frac{nE}{R + nr} \text{ (in series)}. \quad C = \frac{E}{R + \frac{r}{m}} \text{ (in parallel.)}$$

Connect in series when R is large in comparison with r. Connect in parallel when r is large in comparison with R.

349. With cells in series Figure 63, the pressure and the resistance are caused by the current passing through all the cells, and each is equal, respectively, to that of one cell multiplied by the number of cells.

From the six cells in parallel, Figure 64, there are six times as many conductors, or openings, for electric passage as there are from one cell. Hence, the resistance is one sixth as much as from one cell. The current is from each cell, separately, to the conductors. Hence the pressure from any number of cells is the same as that from a singlecell.

350. Rule: *Any number of cells in series have an E. M. F. of a single cell, and an internal resistance of a single cell, multiplied by the number of cells.*

Any number of cells in parallel have an E. M. F. of a single cell. They have an internal resistance of a single cell divided by the number of cells.

PROBLEMS

1. The resistance of a circuit is 5 ohms, the E. M. F. 210 volts. Find the current in amperes. $C = \frac{E}{R}$.

2. The resistance of a circuit is 10,000 ohms, the E. M. F. is 250 volts. What is the current in amperes?

3. A battery has a resistance of 4 ohms, and sends a current of .05 ampere through a conductor whose resistance is 50 ohms. What is the E. M. F. of the battery? (Total resistance is 4 ohms + 50 ohms = 54 ohms.)

4. A battery of 50 cells in series with an internal resistance of 100 ohms, gives an E. M. F. of 75 volts, and sends a current through an external resistance of 122 ohms. What is the strength of the current?

5. The resistance of a circuit is 5 ohms, the current is 50 amperes. What is the E. M. F.?

6. The E. M. F. in a circuit is 550 volts, the current is 10 amperes. What is the resistance?

7. The E. M. F. at a lamp is 110 volts, the resistance is 250 ohms. What is the current in amperes?

8. A battery of 2 ohms resistance sends a current of .05 ampere through a wire whose resistance is 50 ohms. What is the E. M. F. of the battery?

9. A battery of 50 volts E. M. F. and 20 ohms resistance has opposed to it in the same circuit a battery of 30 volts E. M. F. and 25 ohms resistance. A current of $\frac{1}{4}$ ampere is maintained in the circuit. What is the resistance thus connected?

50 volts − 30 volts = 20 volts.

$$C = \frac{E}{R};\ R = \frac{E}{C};\ R = \frac{20}{\frac{1}{4}} = 80, \text{ the number of ohms.}$$

The internal resistance is (20 + 25) 45 ohms. The resistance connected is 80 ohms − 45 ohms = 35 ohms.

10. A battery of 40 volts E. M. F. and 15 ohms resistance has opposed to it in the same circuit a battery of 30 volts E. M. F. and 20 ohms resistance. A current of $\frac{1}{4}$ ampere is maintained in the circuit. What is the resistance?

11. Four conductors in parallel have resistances of 25 ohms, 30 ohms, 40 ohms and 50 ohms respectively. What part of the whole current passes through each conductor?

The proportion of the current passing through each is $\frac{1}{25}$; $\frac{1}{30}$; $\frac{1}{40}$; $\frac{1}{50}$. Reducing to a common denominator, $\frac{24}{600} + \frac{20}{600} + \frac{15}{600} + \frac{12}{600} = \frac{71}{600}$. Numerators = 71. $\frac{24}{71}$; $\frac{20}{71}$; $\frac{15}{71}$; $\frac{12}{71}$ = parts of current that passes through each conductor.

12. How many amperes will a battery of 6 cells furnish, arranged 3 in series and 2 in parallel, the E. M. F. of a cell being 1.2 volts and the internal resistance being .5 of an ohm?

$$C = \frac{nE}{R + \frac{nr}{m}}. \quad C = \frac{3 \times 1.2}{0 + \frac{3 \times .5}{2}} = \frac{3.6}{\frac{1.5}{2}} = \frac{3.6 \times 2}{1.5} = 4.8$$
amperes.

13. How many amperes will a battery of 12 cells furnish, arranged 4 in series and 3 in parallel, the E. M. F. of a cell being 1.2 volts and the internal resistance being .5 of an ohm?

14. A battery of 48 cells is arranged 16 in series and 3 in parallel. The external resistance is 12 ohms, the E. M. F. is 1 volt and internal resistance 1.5 ohms per cell. Find the current.

15. What is the strength of 20 cells arranged in parallel? The E. M. F. is 1 volt, resistance 40 ohms for each cell and external resistance 1.5 ohms.

16. What number of cells must be used to pass a current of .025 amperes through an external resistance of 1,200 ohms, the cells being connected in series, and each having an E. M. F. of .8 volt and an internal resistance of 1.5 ohms?

$$\text{Use } C = \frac{nE}{R + nr}.$$

17. Eight cells are connected in series, each with an E. M. F. of 1.05 volts and 3.5 ohms. Three wires of 21 ohms each are connected in parallel to the poles of the battery. What is the current? (The battery E. M. F. is 8.40 volts and the internal resistance is 28 ohms. Three wires of 21 ohms each in parallel give an external resistance of 21 ohms. ÷ 3 = 7 ohms.)

18. A battery of 32 cells, 1 volt and 3 ohms each, is arranged 8 in parallel and 4 in series. What is the resistance and the E. M. F. of the battery?

19. Twelve cells are connected in series, each with an E. M. F. of 1.2 volts and 3.5 ohms. Three wires of 12 ohms each are connected in parallel to the poles of the battery. What is the current?

351. The resistance of a conductor is proportional to its length. The resistance of one mile of a certain conductor is 5 ohms. What is the resistance of 24 miles of the same wire?

The required resistance is to the given resistance as the length for the required resistance is to the length for the given resistance.

Let x = required resistance. $x : 5 :: 24 : 1$. $x = 120$ ohms.

The resistance of a certain conductor is 5 ohms, and the resistance of a mile of the same conductor is 20 ohms. What is the length of the conductor? The length for the required resistance: the length for the given resistance :: the required resistance: the given resistance.

$$x : 1 :: 5 : 20. \quad 20x = 5. \quad x = \tfrac{1}{4} \text{ mile.}$$

The resistance of 200 ft. of a conductor is 2 ohms. What length of wire has a resistance of 20 ohms?

$$2 \text{ ohms} : 20 \text{ ohms} :: 200 \text{ ft.} : x \text{ ft.}$$

The resistance of a conductor is inversely proportional to the area of its cross-section, or inversely proportional to the square of the diameter. The required resistance : the given resistance :: the area for the given resistance: is the area for the required resistance. Or, as the square of the diameter for the given resistance : the square of the diameter for the required resistance. If the resistance of a wire having a cross-section area of .008 sq. in. is 1.8 ohms, what would be its resistance if the area of its cross-section were .09 sq. in.?

$$x \text{ ohms} : 1.8 \text{ ohms} :: .008 : 09. \quad x = .16 \text{ ohm.}$$

The resistance of a wire having a diameter of .1 in. is 14 ohms. What would be its resistance if the diameter were .3 in.?

$$x : 14 :: .1^2 : .3^2. \quad x = 1.55 \text{ ohms.}$$

PROBLEMS

1. The resistance of a mile of copper wire 70 mils in diameter is 10.82 ohms. What is the diameter of a mile of copper wire which has a resistance of 43.28 ohms?

(A mil = .001 of an inch.)

2. What is the diameter of a wire, if 1,000 ft. of it has a resistance of 14 ohms, when the same length of wire with a diameter of 95 mils has a resistance of 1.15 ohms?

3. What is the resistance of 880 yds. of copper wire 160 mils in diameter, if the resistance of 1 mile of copper wire 230 mils in diameter is 1 ohm? The resistance of 880 yds. 230 mils in diameter = 880 ÷ 1,760 = .5 ohms. (1,760 yds. = 1 mile.)

4. If the resistance of a wire 3 miles long and 40 mils in diameter is 40 ohms, what is the resistance of a wire 9 miles long and 50 mils in diameter?

5. The resistance of 390 ft. of wire $\frac{1}{16}$ in. in diameter is 1 ohm. What is the resistance of the same length of wire $\frac{1}{8}$ in. in diameter? The resistance of 1 mile of copper wire 80 mils in diameter is 8.29 ohms. What is the resistance of a mile of the same wire 50 mils in diameter?

6. If the resistance of 500 ft. of a certain wire 95 mils in diameter is .57 ohm, what is the diameter of 1,000 ft. of the same wire, if the resistance is 10.09 ohms?

7. A wire 1,800 ft. long and .01 in. in diameter has a resistance of 150 ohms. What will be the resistance of 400 ft. of wire .022 in. in diameter?

MAGNETISM

352. Magnetic flux is the number of magnetic lines of force flowing through a magnet or field, and is similar to amperes in current electricity.

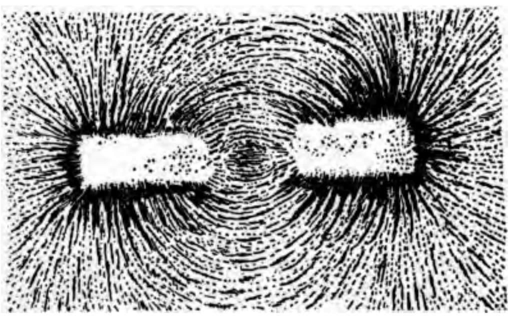

FIGURE 65—MAGNETIC FIELD AROUND A BAR MAGNET.

353. Magneto motive force is the difference of magnetic potential, or it is magnetic pressure, and is similar to electromotive force or volts in current electricity.

354. Reluctance is similar to resistance.

Let N = magnetic flux; $M.\ M.\ F.$ = magnetomotive force; R = reluctance.

$$M.\ M.\ F. = N. \times R. \quad N = \frac{M.\ M.\ F.}{R.}$$

A magnetic circuit composed of a ring of iron has 1,000 units of magnetic pressure, and a reluctance of .001 unit. How many lines of magnetic flux are passing through it?

$N = \dfrac{M.\ M.\ F.}{R.} = \dfrac{1{,}000}{.001} = 1{,}000{,}000$, the number of lines of magnetic force passing through it.

The number of lines of force set up per square inch through an air path one inch long is 3.2 for each ampere-turn. (Ampere-turns are the number of amperes multiplied by the number of turns.)

Let B = the number of lines of force set up per square inch of area through an air path; T = the number of turns per inch of length; C = the number of amperes. $B = 3.2 \times C \times T$.

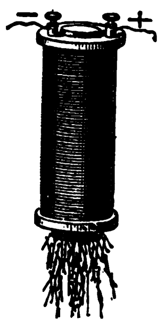

FIGURE 66
A SOLENOID

A solenoid 36 in. long, Figure 66, has 288 turns, and a current of 3 amperes flowing through it. How many lines of force are there per square inch flowing inside the coil? $288 \div 36 = 8$ turns per inch of length of coil. 8 turns $\times$ 3 amperes = 24 ampere-turns. 3.2 for each ampere-turns is $24 \times 3.2 = 76.8$ lines of force per square inch of coil.

To obtain 1,000 lines per square inch with 10 amperes, we have a certain number of turns of wire to the inch.

Since each ampere-turn produces 3.2 lines of force, 1,000 lines of force divided by 3.2 = 312.4 ampere-turns. Since we have 10 amperes, 312.4 ampere-turns divided by 10 = 31.24 turns per inch.

How many amperes will be required to produce 3,200 lines per square inch in a helix 13 inches long consisting of 2,600 turns?

Since there are 2,600 turns in 13 inches in length, or 200 turns to the inch in length, it will take 1,000 ampere-turns divided by 200 = 500 , the number of amperes.

355. The permeability of a body is the relative ease with which magnetic lines of force may be produced in the body. The permeability of air is unity. If through a bar of iron placed in a helix 50,000 lines of force are passed, and if the number of lines passing through the air space before the iron was placed there was 100, then the ratio of permeability of the iron is $\frac{50,000}{100} = 500$.

The permeability (M) of a substance is the magnetic density (B) divided by the intensity (H) of the magnetomotive force.

$$\text{Or, } M = \frac{B}{H}.$$

Magnetic materials, such as iron and the like are good conductors of magnetic lines of force; that is, they possess magnetic permeability.

When a magnetic force acts upon an air space, caused by an electric current circulating in a surrounding coil, there results a certain number of magnetic lines of force in that space. We say that the coil produces H magnetic lines per square inch in air, if H equals the intensity of the magnetic force. If the space were filled with iron instead of air, there would be a greater number of magnetic lines per square inch in the iron than in the air space.

This larger number of lines of force in iron is the degree of magnetization in iron and is symbolized by B. The ratio of B to H is the permeability of the metal and is represented by M.

For example: A certain magnetic force produces 300 lines of force per square inch in an air space, and when the space was filled with iron there were 96,300 lines per square inch. Then 96,300 lines per square inch in iron divided by 300 lines per square inch in air gives the relative number of lines of force with which iron is permeated.

Permeability diminishes as magnetization is pushed beyond a certain limit. The limit for magnetization (B) in good wrought iron is about 125,000 lines per square inch; and in cast-iron is about 70,000 lines per square inch. If a piece of iron is of low permeability, a larger piece must be used, or there must be more copper wire wound upon it.

PROBLEMS

1. If the number of magnetic lines of force per square inch in air (H) is 6.45 and the number of lines in wrought iron (B) is 30,000, what is the permeability of the iron?

$$M = \frac{B}{H}.$$

2. What is the magnetizing force, or what is the number of lines per square inch in air, required to produce in wrought iron a magnetization of 110,000 lines per square inch, if the permeability of the iron is 166?

3. A magnetic circuit composed of a ring of iron has 1,500 units of magnetic pressure, and a reluctance of .002 units. How many lines of magnetic flux are passing through it?

4. A solenoid 36 in. long has 360 turns, and a current of 2 amperes passing through it. How many lines of force are there per square inch flowing inside the coil?

5. How many amperes will be required to produce 3,600 lines per square inch in a helix 12 in. long, consisting of 2,400 turns?

6. How many amperes will be required to produce 2,560 lines per square inch in a helix 14 in. long, consisting of 2,800 turns?

7. A solenoid 24 in. long consists of 240 turns of wire, and has 4 amperes passing through it. How many lines of force are there per square inch inside the coil?

TRACTIVE FORCE OF MAGNETS

356. A bar of iron is magnetized to 60,000 lines per square inch, and has a cross-section area of 16 square inches. How many pounds weight can it sustain?

Let P = pull in pounds; B = density or number of lines per square inch; A = cross-section area. Then,

$$P = \frac{B^2 \times A}{72{,}134{,}000}.$$

$$P \text{ (Pounds)} = \frac{3{,}600{,}000{,}000}{72{,}134{,}000} \times 16 = 50 \text{ about.}$$

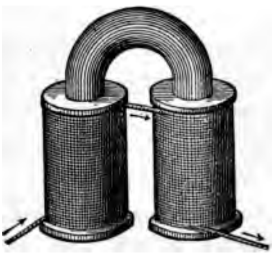

FIGURE 67
HORSESHOE ELECTROMETER

357. To find the tractive force of magnets *in pounds, multiply the number of square inches of contact by the square of the number of lines of force per square inch, and divide by 72,134,000.*

358. To find the number of lines of force *per square inch, when the pull in pounds is known, multiply the pull in pounds by 72,134,000, divide by the area of contact and extract the square root.*

PROBLEMS

1. A bar of iron is magnetized to 15,000 lines per square inch; its cross-section area is 3 sq. in. What weight can it sustain?

2. A magnet with 4 sq. in. cross-section area sustains a weight of 32 lbs. What is the number of magnetic lines of induction per square inch?

3. A magnet with 12 sq. in. cross-section area sustains a weight of 600 lbs. What is the magnetization in lines per square inch?

4. A bar of iron magnetized to 30,000 lines per square inch, has a cross-section area of 12 sq. in. What weight will it sustain?

5. An electromagnet is magnetized to 45,000 lines per square inch, and the poles of the magnet have a total area of 1 sq. in. How many pounds pull will they sustain?

ALTERNATING CURRENTS

359. Alternating currents reverse their direction at very short intervals. Starting at zero they increase to a positive maximum, then decrease to zero, then increase again to a negative maximum, decreasing to zero, and start through the cycle of operations again.

The time from one positive maximum to the next positive maximum is a period or cycle of the current.

There are two alternations in each period or cycle. One alternation is one half a period, or cycle. From positive to negative, or from negative to positive, is an alternation.

360. A **frequency** is the number of periods in a second.

A frequency is equal to the number of revolutions per second multiplied by one half the number of poles.

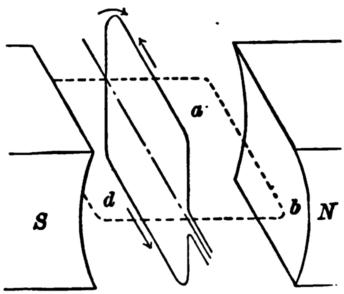

FIGURE 68

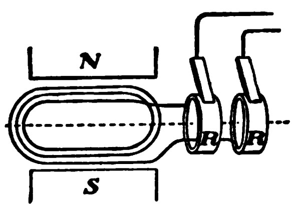

FIGURE 69

If we start with the armature coil in a vertical position, as in Figure 68, the current for the first half revolution flows around the coil in one direction, and for the second half, in the opposite direction. The current is, therefore, always alternating.

In the alternating current dynamo the current is conducted from the armature to the brushes by means of separate rings (Figure 69), one attached to each end of the wire of the armature. The dynamo then sends to the line an alternating current.

The frequency of an alternating current is the speed of the armature in seconds (*R. P. S.*) multiplied by one half the number of poles. Let Fr = frequency in cycles; P = the number of poles.

$$Fr = \frac{R.\,P.\,S. \times P}{2}.$$

R. P. S. = revolutions per second.

What is the frequency of an 8-pole machine revolving 12 R. P. S.? $Fr = 12 \times \frac{8}{2} = 48$, the number of cycles.

ELECTRICAL POWER

361. The electrical unit of power is the *watt*. One watt = .00134 H. P. One H. P. = 746 watts. 1 watt = $C \times E$. (C = current in amperes. E = volts.) 1 watt = $C^2 \times R$. 1 watt = $\frac{E^2}{R}$. R = resistance in ohms. (E is the same as *E. M. F.*, or electromotive force.)

Let E = the E. M. F. of the line current; e = the E. M. F. per phase, or per armature coil; C = the line current in amperes; c = the current per phase or per armature coil in amperes; W = the total watts output; Fr = the frequency; P = the number of poles. F = flux. $E = \frac{e}{\sqrt{3}} = 1.732.$

PROBLEMS

1. What is the frequency in cycles of a current from an alternator, having 10 poles and 1,500 R. P. M.?

$$Fr = Cy = 2\,Al. \quad Fr = \frac{R.\,P.\,M.}{60} \times \frac{P}{2} = 125.$$

What is the number of alternations?

$$Al \text{ (alternations)} = \frac{R.\,P.\,M.}{60} \times P. \quad Al = \frac{10 \times 1{,}500}{60} = 250.$$

2. An alternator has 24 poles, and its armature runs at a speed of 300 R. P. M. What is the frequency? What is the number of alternations?

3. What is the frequency of a 40-pole machine which makes 125 R. P. M.? What is the number of alternations?

4. A 10-pole machine makes 600 R. P. M. What is the frequency? What is the number of alternations?

5. An 8-pole machine makes 800 R. P. M. What is the frequency? What is the number of alternations?

6. What number of poles must be used to secure a frequency of 60 cycles and have a speed of 600 R. P. M.?

$$P = \frac{120\ Fr}{R.\ P.\ M.}$$

7. What is the frequency of an 18-pole machine, making 400 R. P. M.?

8. How many cycles are made by a 60-pole dynamo running at a speed of 180 R. P. M.?

9. What is the speed, or the R. P. M., of a dynamo which has 8 poles, and a frequency of 60 cycles?

10. A generator delivers a current of 800 amperes at an E. M. F. of 500 volts. Find the number of watts delivered. $W = C \times E = 800 \times 500 = 400{,}000$ watts.

11. How many watts are required for 100 incandescent lamps in parallel, each lamp taking $\frac{1}{2}$ ampere and an E. M. F. of 110 volts?

12. What power in watts will be required to send a current of 10 amperes through a conductor of 8 ohms resistance?

13. What power in watts will be required to maintain a current of 110 volts E. M. F. through a resistance of $\frac{1}{4}$ ohm?

14. A current of 1,000 amperes is flowing through a conductor whose resistance is .08 of an ohm. What power is required to maintain this current at this resistance?

15. What power is expended in lighting 500 incandescent lamps in parallel each rated at 110 volts and 220 ohms?

16. An E. M. F. of 1,500 volts is maintained through a circuit whose resistance is 200 ohms. What is the horsepower required?

17. A current of 20 amperes is maintained through a resistance of 50 ohms. What is the horsepower? (Or electrical horsepower.)

18. An electrical motor takes a current of 200 amperes from a 500 volt circuit. What is the electrical horsepower?

362. The dynamo formula for a direct current 2-pole armature is this:

$$E = \frac{N \times c \times n}{10^8 \times 60}.$$

Let E = the voltage, or the E. M. F; N = the R. P. M. of the armature; n = the number of lines of force from one pole through the armature; c = the number of conductors on the surface of the armature of a bipolar machine; $10^8 = 100,000,000$. One watt = 100,000,000 ergs per second, and equals .00134 H. P. The *erg* is a unit of work. We use 60 in the denominator, because we used R. P. M., revolutions per minute, and not revolutions per second.

PROBLEMS

1. What is the E. M. F. of a dynamo which has a R. P. M. of 1,800; a flux, or number of lines of force, of 1,000,000; and conductors 500?

$$E = \frac{N \times c \times n}{10^8 \times 60} = \frac{1,800 \times 1,000,000 \times 500}{100,000,000 \times 60} = 150 \text{ volts}.$$

2. What is the E. M. F. of a dynamo which has R. P. M. 1,500; conductors 400; and lines of force 3,500,000?

3. What is the flux required to produce an E. M. F. 120 volts with 1,380 R. P. M. and 480 conductors?

4. What should be the number of conductors on the surface of the armature of a bipolar dynamo, if the flux is 1,000,000, the R. P. M. 1,200, and the voltage 100?

5. What should be the R. P. M. of a dynamo, if the number of conductors is 3,200, the E. M. F. is 1,000, and the flux is 947,000?

6. What is the E. M. F. of a bipolar dynamo, if the R. P. M. is 1,200, the conductors 500, and the flux 1,000,000?

7. What is the E. M. F. of a bipolar machine, if the R. P. M. is 2,400, the flux 2,000,000, and the conductors 400?

8. What should the number of conductors be on the armature of a bipolar machine, if the E. M. F. is 100, the flux 1,000,000, and the R. P. M. 600?

363. Multipolar dynamos. The E. M. F. of a multipolar dynamo with a series winding is $E = \frac{frequency \times c \times n}{100,000,000}$. The frequency is revolutions per second, the R. P. S., multiplied by $\frac{1}{2}$ the number of poles. The frequency of a 20-pole machine, making 600 R. P. S. is $\frac{1}{2}$ of $20 = 10$, and 10 times $600 = 6,000$. The frequency of a 24-pole machine making 600 R. P. M. is $600 \div 60 = 10$ R. P. S. $24 \div 2 = 12$, and 10 times $12 = 120$ frequency.

The E. M. F. of a multipolar dynamo with multiple winding is $E = \frac{R.\ P.\ S. \times c \times n}{100,000,000}$

PROBLEMS

1. A 10-pole multipolar dynamo has a speed of 10 R. P. S., a flux of 8,200,000, and 580 conductors in series. What is the E. M. F.?

2. An 8-pole multipolar dynamo has a speed of 10 R. P. S., a flux of 600,000 and 450 conductors in series. What is the E. M. F.?

3. An 8-pole multipolar dynamo has a speed of 180 R. P. M., a flux of 7,900,000 lines of force, and 320 con-

ductors. This machine is multiple-wound. What is the E. M. F.?

4. A 6-pole multipolar dynamo with series winding, has 250 conductors, and gives an E. M. F. of 100 volts, at 240 R. P. M. What is the flux?

5. A 4-pole multipolar dynamo with 500 conductors, multiple-wound, is to give 500 volts at 600 R. P. M. What is the flux?

6. A 4-pole multipolar dynamo, multiple-wound, has a flux of 16,000,000 lines and gives 500 volts at 600 R. P. M. What is the number of conductors?

364. To find how much E. M. F. will be required *to drive an electric current through a certain number of arc lights, multiply the voltage required for each lamp by the number of lamps, and add the E. M. F. required to overcome the resistance of the line.*

What is the E. M. F. of a dynamo to light 100 arc lights, if each of the lamps requires 50 volts and 9 amperes, and the resistance of the line is 25 ohms? $100 \times 50 = 5{,}000$, the number of volts required for the 100 lamps. $E = C \times R$; or, $9 \times 25 = 225$, the number of volts for the line. And 5,000 volts + 225 volts = 5,225 volts.

How many watts per lamp are consumed? Watts $= C \times E = 9 \times 50 = 450$ watts consumed by each lamp. The total number of watts for the line and lamps = 5,225 volts × 9 amperes = 47,025 watts.

What is the E. M. F. of a dynamo to light 50 arc lamps, each requiring 110 volts and 4.9 amperes, the resistance of the line being 50 ohms? How many watts are consumed by each lamp?

$110 \times 50 = 5{,}500$, the number of volts required.

$4.9 \times 50 = 245$, the number of volts required for the line. $(E = C \times R.)$

5,500 volts + 245 volts = 5,745 volts required at the terminals of the dynamo, or the E. M. F. of the dynamo.

$110 \times 4.9 = 539$, the number of watts. (Watts $= C \times E$)

365. In incandescent lighting and lamps in parallel, *the current through the dynamo is the sum of the currents through all the lamps, and the E. M. F. through any lamp is less than the E. M. F. through the dynamo by the amount of the drop in potential on the line.*

366. The drop in the line, or the loss of E. M. F., *between any two points on the line in indicated by the $E = C R$; or, is the current in that part of the line multiplied by the resistance of the line between the points.*

Each of a group of 12 lamps 100 feet from a dynamo and a group of 10 lamps 200 feet beyond, takes one half ampere of current at 120 volts and the resistance of the line is .0004 ohm per foot. What is the loss of E. M. F. at the lamps? What is the voltage of each group? How many watts are consumed by each group?

The current at the lamps of the first is 12 lamps $\times \frac{1}{2}$ ampere = 6 amperes. The resistance to the first group is $100 \times .0004$ ohm = .04 ohm.

The drop in potential, or the loss in voltage, at the first group is $(6 \times .04)$ .24 volt. The voltage at the first group is 120 volts − .24 volt = 119.76 volts.

The current at the second group is $(10 \times \frac{1}{2})$ 5 amperes. The resistance from the first group to the second group is $200 \times .0004$ ohm = .08 ohm.

The loss in voltage between the first and second groups is 5 (amperes) $\times$ .08 (ohm) = 4 volt.

The voltage at the second group is 120 volts − (.24 volt + .4 volt) = 119.36 volts. The total drop from dynamo to second group is .64 volt.

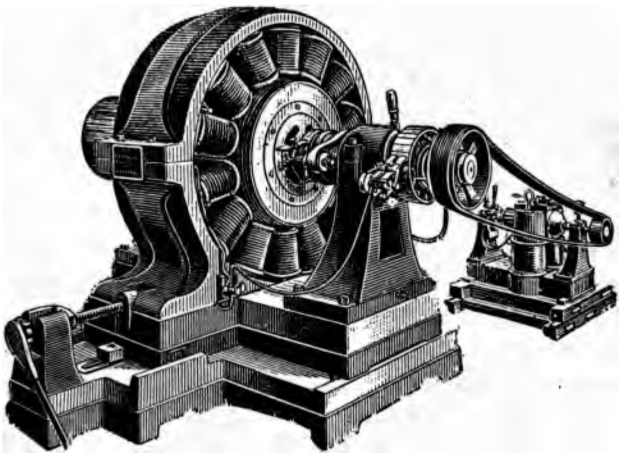

FIGURE 70—ALTERNATING CURRENT DYNAMO AND EXCITUR

367. The watts consumed by each group of lamps *are equal to the volts at the group terminals, multiplied by the current passing through the group.*

$W = C \times E$, or the watts consumed by a lamp equal the volts at the lamp terminals multiplied by the current passing through the lamp.

The current passing through the first group = 6 amperes. The volts at the first group terminals = 119.76 volts.

$6 \times 119.76 = 718.56$, the number of watts consumed by first group.

The current passing through the second group = 5 amperes. The volts at the second group terminals = 119.36 volts.

$5 \times 119.36 = 596.80$, the number of watts consumed by the second group. The power absorbed = watts divided by 746. $H.P. = \frac{W}{746}$

PROBLEMS

1. There are on a circuit forty 16-candle power incandescent lamps, each taking .5 ampere at 110 volts; and 4 arc lights, each taking 6.8 amperes at 50 volts. How many watts and how many horsepower are required to operate these lights?

2. In an electric light circuit there are 40 arc lights, each taking 50 volts, and 10 miles of wire having a resistance of 2 ohms per mile. If the current is 9.6 amperes, how many watts are required to run the lights?

3. An arc light has a circuit of 80 lamps, each requiring 9.6 amperes and 50 volts at the lamp terminals, the resistance of the line being 25 ohms. What is the E. M. F. required for all the lamps? What is the E. M. F. for the line? What is the E. M. F. required at the machine? If 10% of the energy should be lost in the line, what energy should be supplied by the dynamo?

4. An arc light circuit has 100 lamps, requires 4.9 amperes and 110 volts at the lamp terminals, and there is a loss of 147 watts by choke coil. How many volts should be supplied by the dynamo?

5. Dynamo to run 75 (16-*c. p.*) incadescent lamps, each lamp taking 110 volts, will give how many watts effective output? If the maximum voltage is 125 volts, and losses by resistances and by heat are 10%, what will be the efficiency of the machine? What will be the maximum power of the machine in watts? How many amperes will the machine produce? If the engine to drive the dynamo is 80% efficiency, what is its horsepower?

6. Two hundred incandescent lamps are connected in parallel to a dynamo circuit. The resistance of the line is .8 of an ohm, and the resistance of the lamps is 220 ohms each. The P. D. at the dynamo terminals is 112 volts. What current flows through the circuit?

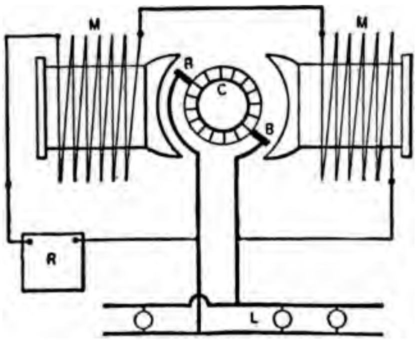

FIGURE 71 DYNAMO AND ELECTRIC LIGHT DIAGRAM

7. Fifty lamps, each requiring $\frac{1}{2}$ ampere and 100 volts, are connected in series. The P.D. at the dyname terminals is 500 volts, Figure 71. How much current flows from the power station?

8. In an electric light station the voltameter shows 115 volts, the ammeter shows 330 amperes. This is a two-wire direct current system and uses 55-watt, 110-volt incandescent lamps. How many lamps are burning?

ELECTRIC RAILWAY

368. A tractive force of 25 lbs. per ton is usually taken for a car on a level road, running at 10 miles per hour. An eleven-ton car will require 11 × 25 lbs. = 275 lbs. tractive force on a level.

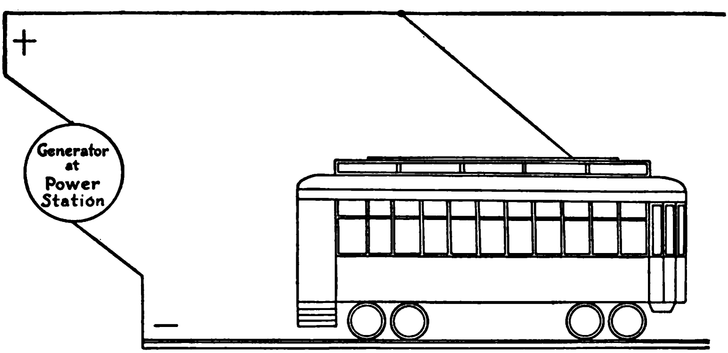

FIGURE 72.—DIAGRAM OF ELECTRIC RAILWAY SYSTEM

369. To find the H. P. required *to propel a car at a certain speed on a level road, multiply the tractive force by the speed in feet per minute and divide by 33,000. To find the watts required, or the watts output, by the motor to propel the car on a level road, multiply the H. P. by 746; and to get the watts required by the motor divide the watts output of the motor by the efficiency of the motor.*

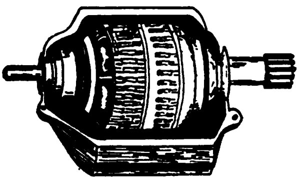
FIGURE 73—ARMATURE OF MOTOR

How many watts are supplied to the motors of a street car weighing 8 tons, tractive force per ton 25 lbs., running 10 miles per hour on a level road? Efficiency is 75%.

Tractive force is $8 \times 25 = 200$.

Feet per minute is $\frac{10 \times 5{,}280}{60} = 880$.

Foot-pounds is $880 \times 200 = 176{,}000$.

Horsepower $= \frac{176{,}000}{33{,}000} = 5.3+$.

Watts $= 5.3 \times 746 = 3{,}953.8$ watts.

Watts supplied to motors $= 3{,}953.8 \div .75 = 5{,}271.73$ watts.

370. To find the energy of a car *to enable it to overcome a grade, find the number of feet it is raised vertically per minute by multiplying the speed in feet per minute by the percentage of the grade. Then multiply by the weight of the car in pounds and divide by 33,000 for the horsepower.*

How many watts must be supplied to the motors of the car in the last example to enable it to climb a 10% grade at 10 miles per hour?

The vertical rise in feet per minute $= 880 \times .10 = 88$.

Weight of car in pounds $= 16{,}000$ lbs.

Foot-pounds $= 16{,}000 \times 88 = 1{,}408{,}000$.

Horsepower $= \frac{1{,}408{,}000}{33{,}000} = 42.66$.

Watts = $42.66 \times 746 = 31{,}824.36$.

F = tractive force = 25 lbs. per ton.

Eff = efficiency of motor in per cent.

Wt = weight of car and passengers in tons.

S = Speed of car in feet per minute.

W = watts required by motor on level road.

$$W = \frac{Wt. \times F \times S \times 746}{33{,}000} \div Eff.$$

PROBLEMS

1. A street car weighs 8 tons and the passengers weigh 2 tons; the tractive force is 25 lbs. per ton; efficiency of motor is 75%. How many watts are supplied to the motor when the car is running at the rate of 10 miles per hour?

2. A street car weighs 12 tons and the passengers weigh 3 tons; the tractive force is 25 lbs. per ton; the efficiency of the motor is 75%. How many watts are supplied to the motor when the car is running at the rate of 10 miles per hour? How many watts must be supplied to the motor to enable the car to climb a grade of 10% at 10 miles per hour?

3. How many horsepower must be supplied to a street car weighing 10 tons, tractive force 25 lbs. per ton, running on a level road at 10 miles per hour?

4. How many watts are required to propel a street car weighing 14 tons, tractive force 30 lbs. per ton, running at 20 miles per hour on a level road, if the efficiency is 70%?

5. How many watts are required to propel a street car weighing 15 tons, tractive force 25 lbs. per ton, up a 10% grade at 15 miles per hour?

6. How many horsepower are necessary to run a 15-ton street car at 20 miles per hour, level road, and 25-lb, tractive force?

TRANSFORMERS

371. A **transformer** is composed of a primary coil, a secondary coil, and a magnetic core.

The induced electromotive force set up in the secondary coil is to that in the primary coil, nearly, as the number of coils in the secondary is to the number of coils in the primary. If the primary has 200 turns of wire and the secondary has 2,000 turns, then the induced E. M. F. in the secondary will be about 10 times as great as that in the primary.

If the secondary contains 100 turns of wire and the primary 200 turns, then the secondary will have $\frac{1}{2}$ the E. M. F. of the primary.

The E. M. F. in either coil of a transformer equals 4.44 times the number of turns in the coil, times the maximum flux, times the frequency divided by 100,000,000. (10^8.)

Let T = the number of primary turns; t = the number of secondary turns; E = the primary E. M. F.; e = the secondary E. M. F.; n = the frequency or the number of cycles; Fm = the maximum flux.

$$\text{Then } E.\ M.\ F. = \frac{4.44 \times T \times Fm \times n}{100{,}000{,}000}.$$

A transformer has 400 turns in the primary coil and 50 turns in the secondary coil, a frequency of 80, and a maximum flux of 30,000 lines. What is the E. M. F. in each coil?

$$E = \frac{4.44 \times 400 \times 80 \times 30{,}000}{100{,}000{,}000} = 42.6, \text{ the number of volts.}$$

$$e = \frac{4.44 \times 50 \times 80 \times 30{,}000}{100{,}000{,}000} = 5.3, \text{ the number of volts.}$$

The ratio of the number of volts in the primary to the number of volts in the secondary is as 42.6 is to 5.3. Or,

the ratio (*Ra.*) of the transformer is, $Ra = \frac{E}{e}$. Also,

$$Ra = \frac{C''}{C'} . \quad \begin{array}{l} C'' = \text{secondary current.} \\ C' = \text{primary current.} \end{array}$$

The maximum flux through the core of a transformer is

$$Fm = \frac{E \times 100{,}000{,}000}{4.44 \times n \times T} .$$

PROBLEMS

1. A transformer has 720 turns in the primary coil and 36 turns in the secondary coil. The frequency is 136 and the maximum flux is 160,000 lines. What is the E. M. F. in each coil?

2. A transformer has 600 turns in the primary coil and 60 turns in the secondary coil. The frequency is 90 and the maximum flux is 60,000 lines. What is the E. M. F.?

3. The E. M. F. of a primary coil is 100 and the ratio of transformation is 40. What is the E. M. F. of the secondary coil?

4. The ratio of transformation is 20, the secondary E. M. F. is 200. What is the E. M. F. of the primary?

5. What is the secondary E. M. F. if the ratio of transformation is 20 and the primary E. M. F is 800?

6. If the ratio of transformation is 20, the secondary E. M. F. is 400, and the secondary turns 40, what is the primary E. M. F.?

7. If a secondary E. M. F. is 60, the primary turns 600, the frequency 60, and the ratio of transformation 30, what is the maximum flux?

8. The ratio of transformation is 30, the secondary E. M. F. is 300, and the secondary turns 60. What is the primary E. M. F.?

side at right angles to the axle of the wheel to overcome the resistance of all the wheels.

A 4-6-0 type of locomotive has 78-in. wheels and 26-in. stroke, and is required to exert a pull of 30,000 lbs. What force is required at each crank pin on one side when the pin on the other side is at the top of the stroke, or at right angles to the pin on this one side?

A pull of 30,000 ÷ 6 (the number of drivers) = 5,000 lbs., the sliding force at each wheel. As the wheel diameter is 78 in. and the piston stroke is 26 in., the diameter of the wheel is 3 times the length of the piston stroke. 3 × 5,000 lbs. = 15,000 lbs. As the crank pins on the opposite sides are at right angles to each other, it will require 2 × 15,000 lbs. = 30,000 lbs. to apply to each of the crank pins to keep the wheels on both sides moving.

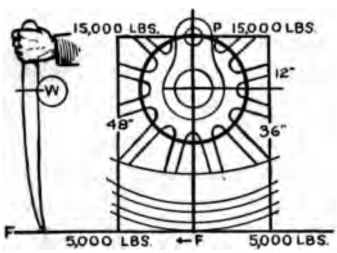

FIGURE 76

In Figure 76 is shown a circle marking the path of the crank pin (to which the power moving the wheel is applied) around the central point of the wheel.

The diameter of this circle is the "stroke of the engine." The diameter of the circle in Figure 76 is 24 in. Therefore, the length of the stroke of the engine is 24 in. The diameter of the driving wheel is 72 in. Or, 2 × 36″ = 72 in. In this case, also, the diameter of the wheel is 3 times the piston stroke.

If a power of 15,000 lbs. is applied at the crank pin, with what force will the axle push forward, if the length of the piston stroke is 24 in. and the diameter of the wheel is 72 in.? The wheel is like a lever, having its fulcrum at *F*, its power at *P*, and its weight at *W*. With levers, the

power multiplied by its distance from the fulcrum equals the weight multiplied by its distance from the fulcrum. The power, 15,000 lbs. $\times$ its distance, 48″ = the weight $W \times$ its distance from the fulcrum, 36 in. $48 \times 15{,}000$ lbs. = 720,000 lbs. And 720,000 lbs. $\div$ 36 = 20,000 lbs. = W. Therefore, when 15,000 lbs. is applied at the crank pin, the axle will push forward with a force of 20,000 lbs., if the diameter of the wheel is 3 times the length of the piston stroke.

One revolution of the driving wheels is one "step of the locomotive" (the iron horse). The distance which a locomotive moves forward at each revolution of the drive wheels equals the circumference of the drive wheels. The circumference equals the diameter multiplied by 3.1416.

PROBLEMS

1. How many feet will a locomotive travel in one revolution of its driving wheels, if the wheels are 78 in. in diameter? 78″ = $6\frac{1}{2}$ft. $3.1416 \times 6\frac{1}{2}$ ft. = 20.42 ft.

2. How many feet will a locomotive travel in one revolution of its driving wheels, if they are 72 in. in diameter?

3. How many feet will a locomotive travel in one revolution of its driving wheels, if they are 66 in. in diameter?

4. How many revolutions will the driving wheels of a locomotive make in going 204.204 ft., if they are 78 in. in diameter?

5. How many feet will a locomotive travel in 20 revolutions of its driving wheels, if they are 72 in. in diameter?

6. How many revolutions will be made per mile by wheels 78 in. in diameter?

7. How many revolutions will be made per mile by wheels 60 in. in diameter?

8. The driving wheels of a locomotive are 78 in. in diameter and make 324 revolutions per minute. How many miles will the locomotive travel per minute?

9. The driving wheels of a locomotive are 60 in. in diameter and make 420 revolutions per minute. How many miles will the locomotive travel per minute?

10. At what speed in miles per hour will a locomotive run, whose driving wheels are 70 in. in diameter, and make 300 revolutions per minute?

11. The drive wheels of a 6-drive wheel locomotive are fast and must slide, and there is a pull of 60,000 lbs. on the drawbar. What is the sliding force on the rail where each wheel rests?

12. The drive wheel is 72 in. in diameter, and the piston stroke is 24 in. How many pounds force must there be at each crank pin, when the crank pins are at right angles to the axles of the wheel, to resist 20,000 lbs. where each wheel rests on the rail?

13. A 4-6-0 type engine has 60-in. drive wheels and 20-in. stroke, and exerts a pull of 60,000 lbs. What force is required at each crank pin on one side, when the pin on the other side is at the top of the stroke?

373. The Pulling Power of Engines. The relation of the position of crank pin, axle and point of contact at the rail is always changing as the wheel revolves. The crank-pin may be at the top or the side, or at the bottom of the stroke. When figuring the pull of an engine the average power of the wheel is considered for the entire revolution. This is about .637 of the maximum tractive force per side. There are several resistances offered to moving trains, as, friction, the resistance offered to being moved, etc.

A horizontal force of about $5\frac{1}{2}$ lbs. is needed to keep one ton (2,000 lbs.) moving at 10 miles per hour, overcoming these resistances. A loaded train of 2,000 tons would require a power of $(2{,}000 \times 5\frac{1}{2})$ 11,000 lbs. to keep it moving.

374. Grade Resistance. A 1% grade is one that has a rise equal to 1% of its length. Or, a rise of 52.8 ft. in a length of 5,280 ft., or one mile.

If a car is raised 1% of one mile (5,280 ft.) it will take 1% of the weight of the car to pull it up the incline. To pull one ton up a grade of $1\frac{1}{2}$% would take a power of $1\frac{1}{2}$% of 2,000 lbs. = 30 lbs. To pull one ton up a grade of $2\frac{1}{2}$% would take $2\frac{1}{2}$% of 2,000 lbs. = 50 lbs.

What force will be needed to haul 1,600 tons up a grade of 1%?

To haul 1 ton up a grade of 1% requires 20 lbs. and to haul 1,600 tons up this grade of 1% will take $1{,}600 \times 20$ lbs. = 32,000 lbs. To these figures must be added $5\frac{1}{2}$ lbs. for friction, per ton, if the train is moved 10 miles per hour up the incline. 20 lbs. per ton + $5\frac{1}{2}$ lbs. per ton = $25\frac{1}{2}$ lbs. per ton. $1{,}600 \times 25\frac{1}{2}$ lbs. per ton = 40,800 lbs. There must be provided sufficient tractive force to overcome this resistance.

375. Tractive Force and Adhesive Weight. Friction is necessary to prevent slipping, and weight is necessary to produce friction. The necessary friction is obtained by giving to the wheels a weight of $4\frac{1}{2}$ times the slipping force. In Figure 76 the pressure at F is $15{,}000 \times 12''$ (the distance from P to the fulcrum F) $\div 36''$ (the distance from W to the fulcrum). Or, the pressure at F is 5,000 lbs. $(15{,}000 \times 12 = 180{,}000$. And $180{,}000 \div 36 = 5{,}000$ lbs.)

This 5,000 lbs. is a backward pressure, or slipping force.

It is, therefore, necessary to provide sufficient friction at F, between the wheel and the rail to withstand this

backward pressure of 5,000 lbs. This is $4\frac{1}{2} \times 5{,}000$ lbs. = 22,500 lbs. This is called "adhesion" and the $4\frac{1}{2}$ is the factor of adhesion.

If the power at the crank pin of a 2-6-0-type engine is 10,000 lbs., the drive wheels are 70 in. in diameter, and the stroke is 28 in., what is the tractive force, and what weight of engine would be required to produce a factor of adhesion of $4\frac{1}{2}$ lbs.? $(10{,}000 \times 14) \div 35 = 4{,}000$. By section 373, we use .637 as a multiplier. $.637 \times 4{,}000$ lbs. = 2,548 lbs. per wheel. $6 \times 2{,}548$ lbs. = 15,288 lbs. tractive force. $4\frac{1}{2} \times 15{,}288$ lbs. = 68,796 lbs. adhesive weight.

Tractive force, when diameter of cylinder and boiler pressure are given.

$$\text{Tractive Force } (T.\ F.) = \frac{.85 \times P \times A \times S \times .637 \times 2}{D}.$$

P = boiler pressure; A = area of cylinder; S = length of stroke; .637 = factor from section 373; D = diameter of drive wheel; .85 is factor to give more correct result, or average pressure.

A locomotive type 2-6-0 has a 20-in. cylinder, a 28-in. stroke, 50-in. diameter driving wheels, and a boiler pressure of 180 lbs. What is the tractive force?

$.85\ (P) \times 180 = 153$ lbs. average pressure. $20'' \times 20'' \times .7854 = 314.16$ sq. in. (A). $314.16 \times 153 = 48{,}066.48$, the average number of pounds of piston pressure.

$28 \times 48{,}066.48$ lbs. = 1,345,861.44 lbs. average piston stroke pound pressure.

1,345,861.44 lbs. $\div$ 50 = 26,917.23 lbs. tractive force per each side drive wheels.

$.637 \times 26{,}917.23$ lbs. = 17,140.2755 lbs. average tractive force per side.

$2 \times 17{,}140.2755$ lbs. = 34,280.55 lbs. total tractive force for both sides.

$$T.\ F. = \frac{.85 \times 20^2 \times .7854 \times 28 \times 180 \times .637 \times 2}{50} = ?$$

A locomotive engine has a cylinder of 19-in. diameter, a 26-in. stroke, a driving wheel 63-in. in diameter, and steam pressure of 190 lbs. per square inch at beginning of the stroke. What is the tractive force of the engine at 10 miles per hour? As in the above problem, let A = the piston area in square inches; P = the steam pressure in pounds per square inches; S = the length of stroke in inches; D = the diameter of the driving wheel in inches.

The formula for tractive force is:

$$T.F. = \frac{1.08 \times P \times A \times S}{D}.$$

$(.85 \times .687 \times 2 = 1.08.)$

$$T.\ F. = \frac{1.08 \times 190 \times 283.53 \times 26}{63} = 24{,}010.94.$$

PROBLEMS

1. With 30,000 lbs. power at the crank pin, the diameter of the wheel 72 in. and length of stroke 24 in., what is the slipping force, and what weight must be given to the wheels to counteract this?

2. With 15,000 lbs. at the crank pin, the diameter of the wheel being 72 in., and the length of stroke being 24 in., what is the slipping force and what weight must be given the wheels to counteract this?

3. With 10,000 lbs. power at the crank pin, the diameter of the wheel 60 in., and the length of stroke 24 in., what is the slipping force and what weight must be given to the wheel to counteract this?

4. At 40 miles per hour, how many revolutions will a drive wheel 60 in. in diameter make per minute?

5. An engine is running at 40 miles per hour. The driving axle is $8\frac{1}{2}$ in. in diameter and the driver is 60 in. in diameter. What velocity will a point on the journal of the axle make in feet per minute?

$\frac{8\frac{1}{2} \times 3.1416}{12} = 2.23$ feet = the circumference = one revolution of the wheel or axle. 60 in. = 5 ft. 3.1416×5 ft. = 15.71 ft. = circumference of the driver.

$5{,}280 \div 15.71 = 336.09$ = times the circumference of the driver is contained in one mile. $\frac{336.09 \times 40}{60} = 224.06$ revolutions per minute of driver. And 2.23 ft. per revolution times the number of revolutions per minute equals $2.23 \times 224.06 = 499.65$, the number of feet per minute.

6. An engine is running 60 miles per hour. How many revolutions will a truck wheel, which is 33 in. in diameter make per minute?

7. A train is running at 44 ft. per second. How many miles will it travel in one hour?

8. A train is traveling at the rate of 3.87 miles in 6 minutes. How many miles per hour is it traveling?

9. A train takes 120 seconds to go one mile. What is its speed in miles per hour?

10. At the rate of 80 seconds per mile, how fast is a train moving in miles per hour?

11. At the rate of 55 miles an hour, how many seconds will there be between mileposts?

12. A watch shows 55 seconds between mileposts. What is the speed in miles per hour?

13. How many feet will a locomotive travel in one revolution of the driving wheel which is 78 in. in diameter?

14. How many revolutions will a driving wheel make which is 60 in. in diameter in going 10 miles?

15. A locomotive engine, type 2-6-0, has a 20-in. cylinder, a 28-in. stroke, a driving wheel 50 in diameter and a boiler pressure of 180 lbs. What is the tractive force?

$$T.\ F.\ \text{(Tractive Force)} = \frac{.85 \times P \times A \times S \times .637 \times 2}{D}$$

376. Curve resistance. The sharpest curve has a radius of 573 ft, measured on the middle of the track between the rails. A railroad curve having a radius of 5,730 ft. is called a one degree curve without regard to length.

The degree of curvature is 5,730 ÷ the radius of the curvature in feet. 5,730 ÷ 573 = 10. A curve having a radius of 2,865 ft., or 5,730 ÷ 2865, is a 2-degree curve.

The train resistance on curves is about .8 lb. per ton of train for each degree of curvature. With a two-degree curve we must provide the additional tractive force to overcome 5½ lbs. per ton of train resistance. This is 2 × .8 lb. = 1.6 lbs. per ton.

What force will be needed to haul a train of 1,600 tons up a grade of 1% and on a curve of 10%?

1,600 × 20 lbs. = 32,000 lbs. If the train moves at 10 miles per hour, we add 5½ lbs. per ton for friction. 1,600 × 5½ lb. = 8,800 lbs. A 10% curve requires 10 × .8 lb. = 8 lbs. per ton. 8 × 1,600 lbs. = 12,800 lbs.

32,000 lbs. + 8,800 lbs. + 12,800 lbs. = 53,600 lbs. Or, adding 20 lbs., 5½ lbs., and 8 lbs., we have 33½ lbs. And 1,600 × 33½ lbs. = 53,600 lbs.

Train resistance for a train moving at 10 miles per hour on a level is about 5½ lbs. per ton.

Train resistance for a train moving up a grade is 20 lbs. per ton for each 1 % of grade.

Train resistance for a train moving on a curve is .8 lb. per ton for each degree of curvature.

377. The number of driving wheels. Wheel loads on rails are based on the weight of the rail per yard of length. The safe load per wheel is 250 times the weight of the rail per yard. From section 375, a wheel must have a weight of 4½ times the slipping force to withstand the necessary backward pressure.

TRANSFORMERS

371. A **transformer** is composed of a primary coil, a secondary coil, and a magnetic core.

The induced electromotive force set up in the secondary coil is to that in the primary coil, nearly, as the number of coils in the secondary is to the number of coils in the primary. If the primary has 200 turns of wire and the secondary has 2,000 turns, then the induced E. M. F. in the secondary will be about 10 times as great as that in the primary.

If the secondary contains 100 turns of wire and the primary 200 turns, then the secondary will have $\frac{1}{2}$ the E. M. F. of the primary.

The E. M. F. in either coil of a transformer equals 4.44 times the number of turns in the coil, times the maximum flux, times the frequency divided by 100,000,000. (10^8.)

Let T = the number of primary turns; t = the number of secondary turns; E = the primary E. M. F.; e = the secondary E. M. F.; n = the frequency or the number of cycles; Fm = the maximum flux.

$$\text{Then } E.\ M.\ F. = \frac{4.44 \times T \times Fm \times n}{100{,}000{,}000}.$$

A transformer has 400 turns in the primary coil and 50 turns in the secondary coil, a frequency of 80, and a maximum flux of 30,000 lines. What is the E. M. F. in each coil?

$$E = \frac{4.44 \times 400 \times 80 \times 30{,}000}{100{,}000{,}000} = 42.6, \text{ the number of volts.}$$

$$e = \frac{4.44 \times 50 \times 80 \times 30{,}000}{100{,}000{,}000} = 5.3, \text{ the number of volts.}$$

The ratio of the number of volts in the primary to the number of volts in the secondary is as 42.6 is to 5.3. Or,

the ratio (*Ra.*) of the transformer is, $Ra = \frac{E}{e}$. Also,

$$Ra = \frac{C''}{C'}. \quad \begin{array}{l} C'' = \text{secondary current.} \\ C' = \text{primary current.} \end{array}$$

The maximum flux through the core of a transformer is

$$Fm = \frac{E \times 100{,}000{,}000}{4.44 \times n \times T}.$$

PROBLEMS

1. A transformer has 720 turns in the primary coil and 36 turns in the secondary coil. The frequency is 136 and the maximum flux is 160,000 lines. What is the E. M. F. in each coil?

2. A transformer has 600 turns in the primary coil and 60 turns in the secondary coil. The frequency is 90 and the maximum flux is 60,000 lines. What is the E. M. F.?

3. The E. M. F. of a primary coil is 100 and the ratio of transformation is 40. What is the E. M. F. of the secondary coil?

4. The ratio of transformation is 20, the secondary E. M. F. is 200. What is the E. M. F. of the primary?

5. What is the secondary E. M. F. if the ratio of transformation is 20 and the primary E. M. F is 800?

6. If the ratio of transformation is 20, the secondary E. M. F. is 400, and the secondary turns 40, what is the primary E. M. F.?

7. If a secondary E. M. F. is 60, the primary turns 600, the frequency 60, and the ratio of transformation 30, what is the maximum flux?

8. The ratio of transformation is 30, the secondary E. M. F. is 300, and the secondary turns 60. What is the primary E. M. F.?

9. A transformer has 840 turns in the primary and 48 in the secondary. The frequency is 140 and the maximum flux is 180,000 lines. What is the E. M. F. in each coil? What is the ratio of transformation?

10. A transformer has 400 turns in the primary coil, a maximum flux of 30,000 lines and a voltage of 42. What is the frequency?

Transformers are used generally to change from very high and dangerous voltage and low current to lower and more safe voltage and larger current.

LOCOMOTIVES

372. Locomotives are classified by the number of wheels from front to back, as the front truck wheels, the drivers,

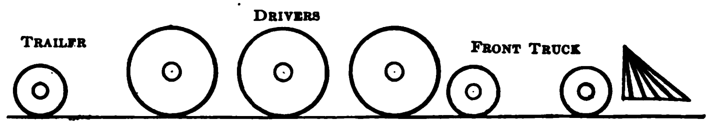

FIGURE 74.

the trailers. In Figure 74, 2 front truck wheels, 3 drivers, and 1 trailer are seen on one side. So there are 4 front truck wheels, 6 drivers, and 2 trailers on this locomotive. The type number is 4-6-2. This type is read "four-six-two type." Read from front.

A locomotive with 2 front trucks, 6 drivers, and no trailers, is a 2-6-0 type, or a "two-six-0" type. The cypher shows no trailers. The weight is given in the nearest even thousandths of pounds. If this locomotive weighs 16,000 lbs., it is given, "2-6-0-16." If it weighs 204,572 lbs., it is given "2-6-0-204."

If the tender is on the engine, T is placed where the dash is, as "2-6-0T204."

If there is a compound cylinder, then C is in place of T, and if a compound cylinder and tank are on the engine. then use both letters, as "2-6-0TC204."

If a 4-drive wheel locomotive has each drive wheel fast so it cannot turn, but must slide, and there is a pull of 40,000 pounds on the drawbar, pulling the locomotive ahead, what is the sliding force on the rail where each wheel rests? 40,000 ÷ 4 = 10,000 lbs. where each wheel rests on the rail.

If the drive wheel is 60 in. in diameter, and the piston stroke is 20 in., how many pounds force would be needed at each crank pin when the crank pins are at right angles to the axle of the wheel, to resist the 10,000 lbs. where each wheel rests on the rail? The diameter of the wheel is three times the length of the piston stroke, or the ratio of the diameter of the wheel to the piston stroke is 3 to 1. Therefore, it would require (3 × 10,000) 30,000 lbs. at each crank pin to counteract the 10,000 lbs. resistance where each wheel rests on the rail. If the right and left crank-pins are placed at right angles to each other, those on the other side of the locomotive would be on the same horizontal line as the axle and would not help to overcome the resistance on the rail. Therefore, it would require 2 × 30,000 lbs., or 60,000 lbs., at each crank pin which is on the

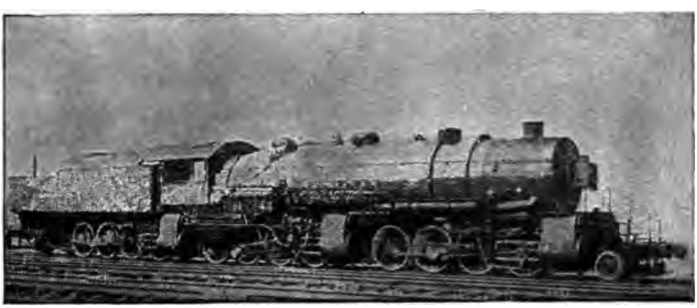

FIGURE 75—"MATT H. SHAY," POWERFUL LOCOMOTIVE CAPABLE OF HAULING 640 FREIGHT CARS IN A TRAIN 4¾ MILES LONG. LENGTH 105 FEET; WEIGHT, 853,050 POUNDS.
Courtesy Erie Railroad.

TRANSFORMERS

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The induced electromotive force set up in the secondary coil is to that in the primary coil, nearly, as the number of coils in the secondary is to the number of coils in the primary. If the primary has 200 turns of wire and the secondary has 2,000 turns, then the induced E. M. F. in the secondary will be about 10 times as great as that in the primary.

If the secondary contains 100 turns of wire and the primary 200 turns, then the secondary will have $\frac{1}{2}$ the E. M. F. of the primary.

The E. M. F. in either coil of a transformer equals 4.44 times the number of turns in the coil, times the maximum flux, times the frequency divided by 100,000,000. (10^8.)

Let T = the number of primary turns; t = the number of secondary turns; E = the primary E. M. F.; e = the secondary E. M. F.; n = the frequency or the number of cycles; Fm = the maximum flux.

$$\text{Then } E.\ M.\ F. = \frac{4.44 \times T \times Fm \times n}{100{,}000{,}000}.$$

A transformer has 400 turns in the primary coil and 50 turns in the secondary coil, a frequency of 80, and a maximum flux of 30,000 lines. What is the E. M. F. in each coil?

$$E = \frac{4.44 \times 400 \times 80 \times 30{,}000}{100{,}000{,}000} = 42.6, \text{ the number of volts.}$$

$$e = \frac{4.44 \times 50 \times 80 \times 30{,}000}{100{,}000{,}000} = 5.3, \text{ the number of volts.}$$

The ratio of the number of volts in the primary to the number of volts in the secondary is as 42.6 is to 5.3. Or,

the ratio (*Ra.*) of the transformer is, $Ra = \frac{E}{e}$. Also,

$$Ra = \frac{C''}{C'}. \quad \begin{array}{l} C'' = \text{secondary current.} \\ C' = \text{primary current.} \end{array}$$

The maximum flux through the core of a transformer is

$$Fm = \frac{E \times 100{,}000{,}000}{4.44 \times n \times T}.$$

PROBLEMS

1. A transformer has 720 turns in the primary coil and 36 turns in the secondary coil. The frequency is 136 and the maximum flux is 160,000 lines. What is the E. M. F. in each coil?

2. A transformer has 600 turns in the primary coil and 60 turns in the secondary coil. The frequency is 90 and the maximum flux is 60,000 lines. What is the E. M. F.?

3. The E. M. F. of a primary coil is 100 and the ratio of transformation is 40. What is the E. M. F. of the secondary coil?

4. The ratio of transformation is 20, the secondary E. M. F. is 200. What is the E. M. F. of the primary?

5. What is the secondary E. M. F. if the ratio of transformation is 20 and the primary E. M. F is 800?

6. If the ratio of transformation is 20, the secondary E. M. F. is 400, and the secondary turns 40, what is the primary E. M. F.?

7. If a secondary E. M. F. is 60, the primary turns 600, the frequency 60, and the ratio of transformation 30, what is the maximum flux?

8. The ratio of transformation is 30, the secondary E. M. F. is 300, and the secondary turns 60. What is the primary E. M. F.?

383. The resistance of the air is proportional to the number of square feet of the aeroplane surface, and to the square of the speed of the plane. Therefore, if the surface of the plane is large enough, and the speed is fast enough, the plane and its load are supported.

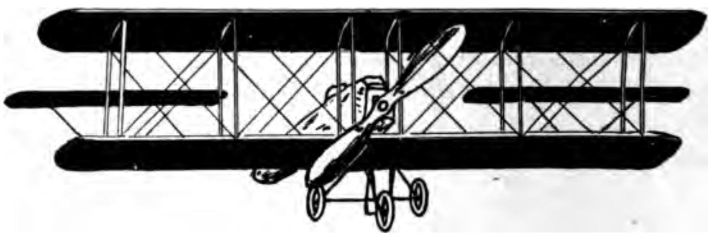

FIGURE 80—A BIPLANE.

An aeroplane should be able to make twice the velocity of any wind that it may encounter.

384. A monoplane is a high speed machine, and must be kept at a higher velocity than the biplane. Its average sustaining weight is about 4 lbs. per square foot. The average sustaining weight of the biplane is about 2 lbs. per square foot. With high speed the sustaining surface may be smaller.

The weight sustained by a plane varies as the area of the plane, the density of the air, and the velocity of the plane in respect to the velocity of the air. The relation between weight and sustaining area is $W = .0012\,A\,V^2$ or,

$$A = \frac{W}{.0012\,V^2}.$$

A = area in square feet, V = velocity in feet per second, W = weight in pounds. .0012 is the density of air.

For an aeroplane weighing 500 lbs. to make 30 miles per hour, or 44 ft. per second, it should have an area of

$$A = \frac{500}{.0012 \times 44^2} = 215 = \text{square feet of surface.}$$

385. Rule: *In aeroplanes the power in foot-pounds per second of the engine is equal to the thrust multiplied by the velocity of the plane.*

Let ft = foot-pounds of power per second and T = the propeller thrust, in pounds.

A propeller thrust of 10 lbs. per horsepower, is $10 = \frac{W}{5} = 50 = W$, weight of machine. That is, one thrust horsepower is required for each 50 lbs. of weight. This depends somewhat upon the type of the machine. Generally the thrust power is taken as $T = \frac{W}{6}$. So that a machine weighing 2,400 lbs. would require $T = \frac{W}{6} = T = \frac{2{,}400}{6} = 400$, the number of pounds of thrust.

386. To find the pitch of a propeller of true-screw form, *divide the speed of the aeroplane in feet per minute by the number of revolutions per minute of the motor.*

If an aeroplane has a speed of 60 miles per hour and the motor is running at 1,500 R. P. M., the pitch of the propeller in feet is $\frac{60 \times 5280}{60 \times 1{,}500} = 3.52$. $60 \times 5{,}280$ = the speed of the plane in feet per minute. As in a screw, the pitch is the distance between the threads of the screw, or the distance the screw advances by making one revolution. This plane advances 3.52 ft. by one revolution of the propeller. The speed, 60 miles per hour, is reduced to feet per minute by multiplying 60, the number of miles, by 5,280 the number of feet in one mile and dividing by 60, the number of minutes in an hour. This result is the number of feet the plane advances in one minute, which divided by 1,500 R. P. M. gives the distance in feet that the plane advances by one revolution of the propeller, or the pitch of the propeller.

387. To find the speed of the tips of the propeller in miles per minute, *find the circumference of the circle in feet described by the tips, multiply this by the number of revolutions per minute of the propeller and divide by the number of feet in a mile.*

If a propeller is 6 feet in diameter and revolves 1,500 times per minute, we have $\dfrac{6 \times \pi \times 1{,}500}{5{,}280} = 5.35$ miles per minute. The diameter, 6 ft., multiplied by π gives the circumference of the circle which is traveled by the blade tips 1,500 times per minute. This result divided by the number of feet in a mile gives the number of miles per minute covered. To find the horsepower of the motor. Fasten one end of a spring balance to the front end of the skid and the other end of the balance to a stake driven in the ground a few feet back. When the motor is started the spring balance will show the pull or thrust. Press a revolution counter into the hole in the hub of the propeller and with a watch count the revolutions per minute. If the thrust is 250 pounds by the balance and the R. P. M. by the revolution counter is 1,500 R. P. M. and the pitch is 3.52 feet.

$$H.\ P. = \frac{T \times P \times R.\ P.\ M.}{33{,}000} = \frac{250 \times 3.52 \times 1{,}500}{33{,}000} = 40.$$

388. The thrust horsepower of a machine *is the thrust in pounds multiplied by the velocity of the machine in feet per minute and divided by 33,000. Thus, if the velocity of the machine is* 30 *miles per hour, and the required thrust is* 400 *lbs., the horsepower is thrust* $H.\ P. = \dfrac{400 \times 30 \times 5{,}280}{60 \times 33{,}000} = 32.$

389. The work of the propeller is to move a mass of air of known weight at a certain rate per minute.

The power of the propeller depends upon the area, the speed of rotation, the pitch, and the resistance.

Let Ft = foot-pounds of work done per second, W = weight of air moved, V = velocity of air in feet per second. (1 cu. ft. of air weighs .073 lb.) $Ft = \frac{W V^2}{2g}$. Let Rs = the number of revolutions per second; A = the area through which the propulsive force is exerted; P = the pitch of the propeller in feet. Then, $Ft = \frac{P A Rs V^2 .073}{2g}$

Or, as $V = P \times Rs.$, $Ft = \frac{P^3 A R^3s .073}{2g}$. $(g = 32 \text{ ft.})$

A 40 H. P. engine drives a propeller which sweeps an area 60 sq. ft. at 20 R. P. S. What is the required pitch of the propeller?

40 H. P. = 40 × 550 = 22,200 foot-pounds. 22,200 foot-pounds $= \frac{P^3 \times 60 \times 8{,}000 \times .073}{2 \times 32}$. $P^3 = \frac{177.1}{4.38}$. $P = 3.4$ ft.

PROBLEMS

1. An aeroplane requires a thrust of 400 lbs. when at a velocity of 35 miles per hour. What is the horsepower required?

2. A machine requires a thrust of 250 lbs. and has a flying speed of 50 miles per hour. What is the horsepower?

3. What weight machine can a 12 horsepower engine drive at 35 miles per hour, if the thrust horsepower is 6.25 lbs.?

4. A 12-horsepower engine is driving an aeroplane at 25 miles per hour. What is the thrust horsepower?

5. A 24-horsepower engine has a thrust of 300 lbs. What velocity can be given the machine?

6. An aeroplane is flying 60 miles per hour and the propeller is making 20 R. P. S. What is the pitch of the propeller?

7. If the pitch is 2.9 ft., the thrust is 232 lbs., and the R. P. S. is 20, what is the H. P.?

8. What is the thrust of a 4-ft. propeller which can make 33 miles per hour?

9. What horsepower is required in horizontal flight to produce a thrust of 160 lbs. at a speed of 40 miles a hour?

10. If a 30-horsepower engine produce 160 pounds thrust at 40 miles an hour, what is the efficiency of the engine?

11. If a biplane is to have a speed of 40 miles per hour in still air with the motor running 1,800 R. P. M. with no slip, or perfect efficiency, what should be the pitch of the propeller?

$\frac{40 \times 5{,}280}{60 \times 1{,}800} = 1.73$, the pitch or number of feet plane advances for each revolution of propeller.

40 (the speed in miles per hour) times 5,280 (feet per mile), divided by 60 (minutes per hour) gives the speed of the plane in feet per minute. Dividing this by 1,800 (revolutions per minute) gives the number of feet the plane will advance for each revolution of the propeller. This is 1.73 ft., or, this is the pitch of the propeller.

12. If a biplane is to have a speed of 60 miles per hour in still air with the motor running 1,200 R. P. M. with no slip, or at an efficiency of 100%, what should be the pitch of the propeller?

13. If a biplane is to have a speed of 60 miles per hour in still air with the motor running 1,800 R. P. M., slip 15%, or an efficiency of 85%, what should be the pitch of the propeller blades?

$$\frac{60 \times 5{,}280 \times 100}{60 \times 1{,}800 \times 85} = 3.45, \text{ the pitch in feet.}$$

14. If an aeroplane is to have a speed of 50 miles per hour in still air with a motor R. P. M. 1,800, and an efficiency of 90%, what should be the pitch of the propeller?

15. If the propeller of an aeroplane is 6 feet in diameter, and makes 1,800 R. P. M., with what velocity will the tips of the propeller blades be traveling through the air?

$$\frac{6 \times \pi \times 1{,}800}{5{,}280} = 6.426, \text{ the number of miles per minute.}$$

16. If the propeller of a plane is 8 feet in diameter, and makes 2,000 R. P. M., with what velocity will the tips of the propeller blades be traveling through the air?

17. If the diameter of a propeller is 12 feet and the number of revolutions per minute is 1,200, what will be the speed of the tips of the propeller in miles per minute?

To determine the horsepower of an aeroplane motor, a speed counter was pressed against the motor hub when in motion and with stop watch in hand 1,200 revolutions were counted in one minute. The thrust of the plane was 270 pounds, the pitch of the propeller 3.5 feet, and the efficiency 90%. What was the horsepower of the motor? (Thrust is pounds pull given the plane by motor.)

$$\frac{1{,}200 \times 270 \times 3.5 \times 100}{33{,}000 \times 90} = 38.18.$$

18. What is the horsepower of an aeroplane motor whose thrust is 300 pounds, pitch 3.5 ft., R. P. M. 1,500, and efficiency 85%?

19. What is the horsepower of biplane motor, if the revolutions per minute are 1,800, the thrust 250 pounds, the slip 15%, and the pitch 3.5 feet?

(A 15% slip gives an efficiency of 85%.)

20. An aeroplane motor has a thrust of 400 pounds, an efficiency of 90%, makes 2,000 R. P. M., and the propeller has a pitch of 3.5 feet. What is the horsepower of the machine?

21. A spring balance had its one end attached to the rear end of an aeroplane and the other end to a near-by tree. The balance indicated a thrust of 250 pounds. A speed counter showed 1,200 R. P. M. If the propeller pitch is 3.8 ft., and the efficiency 85%, what is the horsepower of the motor?

MOVING PICTURE PROJECTORS

390. In moving picture projectors the size of the film pictures is $\frac{3}{4}$ inch high by 1 inch wide, and the aperature plate is $\frac{1}{16}$ inch smaller, making the size of the mat opening, or the actual opening, $\frac{11}{16} \times \frac{15}{16}$ inch.

The throw is the distance from the lens to the screen.

Focal length: Throw : : Film picture: Screen image.

Picture height = height of aperature ($\frac{11}{16}$ inch) $\times$ throw $\div$ focal length of lens.

Focal length of lens = film aperature width ($\frac{15}{16}$ inch) $\times$ throw $\div$ required picture width.

Picture width = aperature width ($\frac{15}{16}$ inch) $\times$ throw $\div$ focal length of lens. Throw = width of desired screen picture $\times$ focal length of lens $\div$ film aperature width ($\frac{15}{16}$). (Dimensions to be in inches.)

To obtain a picture about 9×12 feet with a lens having a focal length of 4 inches. What is the throw? Throw =

$$\frac{12 \times 12 \times 4 \times 16}{15} = 51.2, \text{ the number of feet.}$$

Reduce 12 feet to inches, multiply by 4 inches and divide by $\frac{15}{16}$ inches.

PROBLEMS

1. A certain theatre requires a throw of 70 feet. The picture is to be about 9 feet high by 12 feet wide. What is the focal length of the lens?

(Focal length = width of film aperature ($\frac{15}{16}$ in.) × throw ÷ picture width.) With a lens of 4-inch focal length and a throw of 50 feet, what will be the size of the picture? (Find height and then find width of picture.)

To obtain a 10 × 12 foot picture with a 4-inch focal length lens, what must be the throw?

2. What focal length lens must be used to produce an 8 × 11 foot picture, at a 60-foot throw?

3. With a 6-inch focal length lens what size of picture can be obtained at an 80-foot throw?

4. A moving picture projector has a $4\frac{1}{4}$-inch lens. What size of picture will be thrown on a screen 70 feet away?

5. With a 7-inch focal length lens, what size of picture will be produced at 50 feet?

POULTRY

391. Poultry. There are many classes, or breeds, of poultry. Some are raised largely for eggs, others for meat, and others also for feathers.

1. A flock of 30 hens are fed daily 2 lbs. of wheat worth $1 per bushel. If the hens average 140 eggs per year each, when eggs are worth 20c. per dozen. what is the profit on the flock?

2. A hen averages 140 eggs a year. Allowing 1,000 eggs for hatching, what is the income from 100 hens when eggs are worth 24c. per dozen, and 1,500 lbs. of young chickens, live weight, is worth 18c. per pound?

3. A man bought 50 fowls at 70c. each. During the year he paid $80 for feed. He sold during the year 5,000 eggs at 24c. per dozen, 90 chickens at 50c. each. At the end of the year he had on hand 75 fowls valued at 90c. each. What was his gain for the year?

4. A man had at the beginning of the year 40 hens worth 80c. each, and at the end of the year 60 hens worth 85c. each. He sold during the year $120 worth of eggs, and $45 worth of chickens. What were his net profits for the year?

5. A dozen Plymouth Rock eggs weigh 22 oz. and a dozen Leghorn eggs weigh 16 oz. How many Plymouth Rock eggs will it take to equal 24 dozen Leghorn eggs?

6. What is the difference in the value of two hens, one laying 160 eggs and the other laying 120 eggs per year, when the average price of eggs per dozen is 20c.?

7. A flock of 50 hens averages 95 eggs per year each. If it costs $20 per year to feed this flock, and the average price of eggs is 16c. per dozen, what is the profit on this flock?

8. Hens on a farm average 80 eggs per year each. If by selecting the best layers an average of 130 eggs a year each could be obtained, what would this difference amount to with a flock of 100 hens, when eggs are worth 18c. per dozen?

9. One out of every five eggs bought during the summer is bad, and a price is made per dozen to allow for this waste. If a man pays 14c. a dozen for these eggs, what is the actual value of good eggs per dozen?

10. Which is the cheapest food for producing fat and heat: eggs which are 10% fat, at 24c. per dozen (9 eggs equals 1 lb.), or steak, which is 13% fat and cost 15c. per pound?

DAIRYING

392. Dairying is a branch of farming which deals with the production of milk, butter and cheese.

One gallon of milk weighs about 8.5 lbs. The *amount* of butter-fat in milk varies from about 2% to 6%.

One pound of butter-fat will make about $1\frac{1}{6}$ lbs. of butter. Butter-fat is the richest part of the milk from which butter is made. A 4% milk contains 4 lbs. of butter-fat to every 100 lbs. of milk.

PROBLEMS

1. A certain cow gave 200 lbs. of 4.2% milk in one week in the month of August. How many pounds of butter did the cow produce that week?

4.2% milk is 4.2 lbs. of butter-fat to every 100 lbs. of milk. $4.2 \times 2 = 8.4$ lbs. of butter-fat. 1 lb. of butter-fat will make about $1\frac{1}{6}$ lbs. of butter. $8.4 \times 1\frac{1}{6} = 9.8$ lbs. of butter.

2. How many pounds of butter will 200 lbs. of butter-fat make?

3. How many pounds of butter-fat in 250 lbs. of butter?

4. If 4c. per pound will cover the expense of making butter, what is the profit per pound in selling butter for 40c. per pound, when you can sell the butter-fat to the creamery for 20c. per pound?

5. If a cow averages 4 gal. of milk daily, which is 4% butter-fat, how much butter does she produce per year?

6. A cow gives daily 4 gal. of milk testing 4% butter-fat, and costs \$1.50 per month for pasture. What is gained over the cost of pasture when butter is 45c. per pound and skim milk is 10c. per gallon?

7 If it costs \$40 per year to keep a cow that gives in the same time 1,100 gal. of milk of 4% butter-fat, what is the average cost per pound of the butter?

8. A cow whose milk contains 4.3% butter-fat must give how many gallons of milk to yield 90 lbs. of butter?

9. A \$30 cow gives 12 lbs. of 2% milk per day, and a \$70 cow gives 25 lbs. of $4\frac{1}{2}$% milk per day, when butter is

worth 25c. per pound. Which cow gives the greater profit, and how much?

10. A cow when fed 1 bu. of corn and 1 shock of fodder in 5 days gives 6 gal. of milk in that time. This is 4% milk. How much butter is produced?

11. A dairyman has 5% milk and $3\frac{1}{2}$% milk which he wishes to mix to make 200 gal. of 4% milk. How many gallons of each should he take?

Let x = the number of gallons of 5% milk, and $200 - x$ = the number of gallons of $3\frac{1}{2}$% milk. Then

$$\frac{5x}{100} + \frac{3\frac{1}{2}}{100}(200 - x) = \frac{4}{100}200.$$

x, the number of gallons of 5% milk, $= 66\frac{2}{3}$ gal. $200 - x = 200 - 66\frac{2}{3}$ gal. $= 133\frac{1}{3}$ gal. of $3\frac{1}{2}$% milk.

12. A dairyman has 6% milk and skim milk and wishes to make 150 lbs. of 3.5% milk. How much skim milk and how much 6% milk must be used? (Skim milk is 0% milk.) (0 times the second term in the equation gives what for the second term?)

13. A dairyman has an order for 200 gallons of 4% milk. He has skim milk and 5% milk. How many gallons of each should he use?

14. A dairyman wishes to make 150 lbs. of cream testing 18%. How many pounds of 5% milk and of 25% cream must he use?

15. A cow gave in one year 14,562 lbs. of milk. Of this quantity 910 lbs. was butter-fat. What was the per cent of butter-fat?

16. One cow gives 4 gal. of 3.2% milk per day, and another gives 3 gal. of 4.6% milk per day. How much more butter-fat does one produce than the other per day?

17. A cow produced in one year 17,280 lbs. of milk containing 1,050 lbs. of butter-fat. If the butter is $1\frac{1}{8}$ times

the weight of the butter-fat, how many pounds of butter did the cow produce in the year?

18. If a quart of milk weighs 2.15 lbs., how many quarts per day will 17,280 lbs. of milk for a year of 365 days average?

SILOS

393. Silos are buildings used for preserving in their green state cornstalks, sugar cane, alfalfa, etc.

They are built air-tight to prevent fermentation of the ensilage.

In small silos, one cubic foot of corn ensilage weighs about 30 lbs. In silos 25 to 38 ft. high, one cubic foot of ensilage weighs about 40 lbs.

One cubic foot, or about 40 lbs., of ensilage is the usual daily ration for a cow. Ensilage, or silage, is finely cut vegetable material, as cornstalks, alfalfa, grass, etc.

PROBLEMS

1. If a cow is fed 40 lbs. of corn ensilage per day for 180 days, what should be the capacity in cubic feet of a round silo for 20 cows?

2. If one acre will produce 10 tons of corn fodder, how many acres will be required to produce corn silage to fill the silo in problem 1?

3. If this silo in problem 1 is 25 ft. high, what is its diameter?

4. How many tons of silage in a silo 10 ft. in diameter and 30 ft. high?

5. How many acres of corn ensilage, 10 tons to the acre, will a silo 10 ft. in diameter and 30 ft. high hold?

6. How many acres of good corn land are required to fill a silo 12 ft. in diameter and 36 ft. high? (10 tons to the acre.)

7. A round silo is 20 ft. in diameter and contains 180 tons of ensilage. If this averages 30 lbs. per cubic foot, what is the depth of the silo?

8. If a silo is 16 ft. in diameter and 40 ft. high, how many tons of corn ensilage will it hold? How many cows can be supplied from it for 180 days?

9. A silo is 15 ft. square, inside measurement, and 36 ft. high. At 40 lbs. to the cubic foot, how many tons of ensilage will it hold?

10. It is desired to build a silo 40 ft. high. What should be its diameter to contain the corn ensilage from 20 acres of good corn land?

11. What is the ratio of the nutrients in 1 bu. (32 lbs.) of oats to those in 1 bu. (56 lbs.) of corn?

12. What is the nutritive ratio of 10 lbs. of clover hay to 20 lbs. of corn ensilage?

13. What is the nutritive ratio of 10 lbs. of corn fodder to 10 lbs. of corn ensilage?

14. A work horse should have for every 100 lbs. of weight 1 lb. of grain and 1.5 lbs. of hay per day. How many ears of corn (100 ears make 1 bu. of 56 lbs.) and how many pounds of hay should be fed daily to a horse weighing 1,200 lbs.?

RATIO OF NUTRIENTS IN FEEDS

394. Ratio of Nutrients in Feeds. It is important to know the ratio of digestible flesh forming nutrients to the best forming substances in all kinds of feed for animals.

What is the ratio of the nutrition in 10 lbs. of corn to that in 5 lbs. of cottonseed meal? Corn contains 7.9 lbs. of protein in 100 lbs. of corn, and 66.7 lbs. of carbohydrates.

In 10 lbs. of corn there would be .10 of 7.9 lbs. of protein or .79 lb. of protein. In 10 lbs. of corn there would be .10 of 66.7 lbs. of carbohydrates, or 6.67 lbs. of carbohydrates.

Cottonseed meal contains 37.6 lbs. of protein in 100 lbs. and 21.4 lbs. of carbohydrates.

In 5 lbs. of cottonseed meal there would be .05 of 37.6 lbs. of protein, or 1.88 lbs. of protein. In 5 lbs. of cottonseed meal there would be .05 of 21.4 lbs. of carbohydrates, or 1.07 lbs. of carbohydrates.

.79 lb. of protein in the corn + 1.07 lbs. of protein in the cottonseed meal = 1.85 lbs. of protein.

6.67 lbs. of carbohydrates in the corn + 1.07 lbs of carbohydrates in the cottonseed meal = 7.74 lbs. of carbohydrates.

The amount of protein in both the corn and the cottonseed is to the amount of carbohydrates in both the corn and the cottonseed as 1.85 : 7.74 = 4.1+. The ratio is 1 to 4.1+.

VALUE OF FEEDS

395. The value of any feed lies in the amount of food elements, as the amount of fats, carbohydrates, and protein.

Amount of Food Elements in Grain, Fodder and Hay

Foods	Different Food Elements per Cwt.		
	Proteins	Carbohydrates	Fats
Corn fodder	3.70 lbs.	41.4 lbs.	1.4 lbs.
Corn	7.90 lbs.	66.7 lbs.	4.3 lbs.
Corn ensilage	1.25 lbs.	14.2 lbs.	.7 lb.
Oats	11.07 lbs.	50.3 lbs.	3.8 lbs.
Clover	7.10 lbs.	37.8 lbs.	1.8 lbs.
Timothy	2.80 lbs.	43.4 lbs.	1.4 lbs.

A 1,200-lb. horse at moderate work requires daily about 2 lbs. of protein and 12 lbs. of carbohydrates. A 1,000-lb.

cow not giving milk requires daily about .7 lb. of protein and 7 lbs. of carbohydrates. To produce 1 lb. of 4½% milk requires .049 lb. of protein, .25 lb. carbohydrates and .020 lb. of fat.

How much corn ensilage must be fed a cow that she may get 1.8 lbs. of protein?

We find what part or per cent the element required is of the amount contained in 100 lbs. and take this part or per cent of 100 lbs.

PROBLEMS

1. Timothy and clover hay are selling at the same price per 100 lbs. Which is the cheaper feed for horses?

2. How much cottonseed meal must a cow eat that she may get 2 lbs. of protein?

3. How much clover hay must a cow be fed that she may get 2 lbs. of protein?

4. How many pounds of oats must a horse be fed that he may get 2 lbs. of protein?

5. What is the cost per pound of protein in clover hay at $15 per ton?

6. How many pounds of corn must be fed a horse that he may get 2 lbs. of protein?

7. How many pounds of cottonseed meal must be fed a cow that she may get 2 lbs. of carbohydrates? (Cottonseed meal contains 37.6% protein and 21.4% carbohydrates.)

8. How many pounds of protein and carbohydrates would a 1,000-lb. cow require daily to support herself and give 10 lbs. of 4½% milk?

WEIGHTS OF ANIMALS

296. Calves at birth weigh from 40 to 120 lbs. About 1½ gal. of milk make 1 lb. of flesh gain of live weight.

Hogs and poultry lose about 20% of live weight when dressed. Calves lose about 40% of live weight when dressed, and cattle lose, on an average, about 47%. Fat beef cattle lose the least. A sucking calf should gain about 2 lbs. of weight a day.

PROBLEMS

1. A calf weighing 50 lbs. at birth should weigh how much at the end of 4 months?

2. How many gallons of milk will it take to make a calf weighing 60 lbs. at birth weigh 150 lbs.?

3. A calf at birth weighs 80 lbs. and at the end of 90 days weighs 280 lbs. What is the per cent of gain for the time?

4. If a calf dressed weighs 60% of its live weight, what was the live weight of a calf which when dressed weighed 180 lbs.?

5. If a beef dressed is 55% of its live weight, what was the live weight of a steer whose dressed weight was 800 lbs.?

6. If 1 bu. of corn produces 11 lbs. of flesh, when corn is 40c. per bushel, what must be the price of fat hogs to realize 40c. per bushel for the corn?

7. What is the dressed weight of a crate of chickens weighing 400 lbs.?

8. A 400-lb. live hog is purchased at $8.00 a hundred and the meat sold at 16c. a pound. What is the profit?

9. If 50 lbs. of corn increases the weight of hogs 10 lbs., and dressed pork is worth $12 per hundred, how much does a farmer gain or lose by feeding 350 bu. of corn worth 60c. per bushel to his hogs? (1 bu. weighs 56 lbs.)

10. Is it better to sell 4 hams of 40 lbs. each at 12c. per pound.; 4 shoulders of 30 lbs. each at 12c. per pound; 4 sides of 30 lbs. each at 12c. per pound; or to country cure and sell the hams and shoulders at 18c. per pound and the

sides at 12c. per pound? (Country-cured meat shrinks $\frac{1}{3}$ of its weight.)

AMOUNT OF WATER TO MATURE A CROP

397. It requires about 300 lbs. of water to mature 1 lb. of corn.

It requires about 500 lbs. of water to mature 1 lb. of oats.

It requires about 400 lbs. to mature 1 lb. of clover, barley or wheat.

In a field of corn producing 50 bu. to the acre how much water would be required to mature the crop? 50 bu. of corn at 70 lbs. to the bushel makes 3,600 lbs. The stalks, etc., amount to 3,600 lbs. Total = 7,200 lbs. The amount of water required for each pound of corn is about 300 lbs.

7,200 lbs. × 300 lbs. = 2,160,000 lbs. of water used by 50 bu. of corn or one acre of corn.

To find the amount of rainfall used by 50 bu., or on one acre of corn. A cubic foot of water weighs 62.5 lbs. A rainfall of 1 in. would be $\frac{1}{12}$ of 62.5 lbs. = 5.208 lbs. per square foot of surface. For 1 acre, or 43,560 sq. ft., of rainfall 1 in. deep there would be 43,560 × 5.208 = 226,860.48 lbs. of water. Dividing 2,160,000 by 226,860.48 = 9.52, the number of inches deep that would be required for 50 bu. of corn per acre.

PROBLEMS

1. How much water is required to mature 40 acres of corn averaging 50 bu. of corn per acre?

2. How much water is required to mature 10 acres of oats averaging 40 bu. per acre? (1 bu. of oats weigh 32 lbs.)

3. What depth of rainfall would be required to mature 10 acres of oats averaging 40 bu. per acre?

4. A 10-acre corn field averaging 80 bu. per acre receives 6 in. of rainfall. How many more cubic feet of water are required to mature the crop? (70 lbs. = 1 bu. of corn on cob.)

5. Find the difference between the amount of water to mature a 10-acre field of corn averaging 80 bu. per acre and a field of wheat of the same size producing 40 bu. per acre?

PLANT FOOD REQUIRED

398. The amount of plant food required for different crops. The most important plant foods in the soil are nitrogen, potash and phosphoric acid. Clover and a few other crops draw their nitrogen from the air.

Amount of Plant Food Required for Different Crops.

	To produce straw	To produce bu. of grain	Extracts from the Soil		
			Nitrogen	Phosphoric Acid	Potash
Corn	3,000 lbs.	50 bu.	74 lbs.	26 lbs.	42 lbs.
Wheat	2,000 lbs.	20 bu.	38 lbs.	15 lbs.	23 lbs.
Oats	2,500 lbs.	50 bu.	48 lbs.	18 lbs.	40 lbs.
Clover hay	2,000 lbs.	....	*	11 lbs.	36 lbs.
Timothy hay	2,000 lbs.	....	24 lbs.	10 lbs.	17 lbs.

Corn to produce 3,000 lbs. of straw and 50 bu. of grain, extracts from the soil 74 lbs. of nitrogen, 26 lbs. of phosphoric acid, and 42 lbs. of potash.

Wheat, to produce 2,000 lbs. of straw and 20 bu. of grain, extracts from the soil 38 lbs. of nitrogen, 15 lbs. of phosphoric acid, and 23 lbs. of potash.

Oats, to produce 2,500 lbs. of straw and 50 bu. of grain, extracts from the soil 48 lbs. of nitrogen, 18 lbs. of phosphoric acid, and 40 lbs. of potash.

Clover, to produce 2,000 lbs. of straw, extracts from the soil 11 lbs. of phosphoric acid and 36 lbs. of potash.

*Clover extracts its nitrogen from the air.

PROBLEMS

1. How many pounds of plant food are required to mature 20 acres of corn, averaging 50 bu. to the acre? 20 acres will produce 20 × 3,000 straw + 20 × 50 bu. This will give 20 × 74 lbs. nitrogen + 20 × 26 lbs. phosphoric acid + 20 × 42 lbs. of potash = 2,840 lbs. of plant food.

2. How many pounds of plant food are required to grow 10 acres of wheat averaging 20 bu. to the acre?

3. How many pounds of plant food are required to grow 10 acres of oats averaging 50 bu. to the acre?

4. How many pounds of plant food are required to grow 30 acres of clover averaging 2,000 lbs. of straw to the acre?

5. How many pounds of plant food are required to grow 20 acres of timothy hay averaging 2,000 lbs. per acre?

PLANT FOOD RESTORED

399. The amount of plant food restored to the soil by different fertilizers.

Amount of Plant Food Restored to the Soil by Different Fertilizers.

	Nitrogen	Phosphoric Acid	Potash
One ton of manure..........	10 lbs.	5 lbs.	10 lbs.
One ton of commercial fertilizer.....................	33 lbs.	33 lbs.	33 lbs.
One ton of cornstalks.......	20 lbs.	6 lbs.	28 lbs.
One ton of oat straw.........	12 lbs.	4 lbs.	24 lbs.
One ton of wheat straw......	12 lbs.	2 lbs.	10 lbs.
One ton of clover hay........	40 lbs.	7 lbs.	44 lbs.

PROBLEMS

1. How many pounds of plant food will 20 tons of wheat straw furnish to the soil? 1 ton will furnish 12 lbs. nitrogen, 2 lbs. phosphoric acid, and 10 lbs. of potash. *20 tons* × 12 + 20 × 2 + 20 × 10 = 480 lbs. of plant food.

2. How many pounds of plant food will 20 tons of manure furnish to the soil?

3. How many pounds of plant food will 15 tons of cornstalks furnish to the soil?

4. How many pounds of plant food are returned to the soil when 20 acres of clover are plowed under, averaging 1 ton per acre? What is the value of this plant food, if nitrogen is \$2 per ton; phosphoric acid is \$25, and potash \$50 per ton?

5. What is the value of 4,500 lbs. of cornstalks, if the values of the plant foods are as given in problem 4?

FARM PRODUCTS IN BULK

400. One bushel of corn weighs 56 lbs. When in the husk, $3\frac{1}{2}$ cu. ft. of corn is 1 bu. On the cob $2\frac{1}{2}$ cu. ft. of corn is 1 bu. 1 bu. = 2,150.4 cu. in.

1. A corncrib is 12 ft. long, 8 ft. wide, and 8 ft. high. How many bushels of corn in the husk will it hold?

2. How high must a corncrib 14 ft. long, 8 ft. wide be to hold 200 bu. of corn on the cob?

3. How many bushels of corn in husk are there in a round pile 16 ft. across, and tapering to a point on top 8 ft. high?

4. How many bushels of corn on cob will a wagon bed hold, that is 10 ft. long, 3 ft. wide and 2 ft. deep?

5. How many bushels of potatoes can be put in a wagon 10 ft. long, 3 ft. wide, and 18 in. deep? Apples, potatoes, turnips, etc., are measured by the heaped bushel (2,747.8 cu. in., or $1\frac{3}{5}$ cu. ft.).

6. How many bushels of apples can be put in a box 8 ft. long, 4 ft. wide, and 4 ft. deep?

7. How many bushels of potatoes in a round pile, 10 ft. across the bottom, and tapering to a point 5 ft. above the middle?

8. How many tons of timothy hay in a loft 30 ft. long, 20 ft. wide, and 8 ft. deep? (Packed timothy hay contains 515 cu. ft. to the ton. Clover hay contains 450 cu. ft.)

9. How many tons of clover hay in a space 20 ft. long, 18 ft. wide, and 8 ft. high?

10. How many tons of clover hay in a haystack which measures 100 ft. around it at about one half way from ground to top, and is 20 ft. high?

Square $\frac{1}{4}$ the distance around the stack, measured halfway from ground to top of stack, and multiply this by height of the stack. This gives the cubic feet in the stack.

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